Flight to Quality, Flight to Liquidity, and the Pricing of Risk

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VERY PRELIMINARY PRESENTATION NOTES

Abstract

We propose a dynamic equilibrium model of a multi-asset market with exogenous transaction costs. Our model's key feature is that investors' preference for liquid versus illiquid assets changes over time. In particular, when volatility is high, investors are more concerned that they might be forced to liquidate their portfolios after a bad draw of poor performance, and this makes them less willing to hold illiquid assets. We show that when volatility increases: (i) the discount of illiquid assets relative to comparable liquid ones increases, (ii) the market betas of illiquid assets increase, and (iii) the market becomes more risk averse. Moroever, a CAPM analysis that does not condition on the volatility factor would understate the risk inherent in illiquid assets.

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INTRODUCTION

- Liquidity premia seem to vary substantially over time.
- Clean experiments:
  - T-notes vs. T-bills. (Amihud and Mendelson (1991), Kamara (1994), Strebulaev (2002))
  - Off- vs. on-the run bonds. (Krishnamurthy (2002))
  - Refcorp vs. government bonds. (Longstaff (2002))
  - Municipal vs. government bonds. (Chalmers, Kadlec, and Vayanos (2003))
- Effects can be quite strong.
  - Variation in spread equals 4-6 times average spread.
  - Variation in relative prices can be up to 15%.
  - Flight to liquidity.
- Effects could be stronger for other asset classes.
  - Corporate bonds.
  - Emerging market bonds.
  - Stocks.

Why Do Liquidity Premia Vary?

- Liquidity premia seem to be high when
  - Interest-rate volatility is high. (Kamara (1994))
  - Consumer confidence is low. (Longstaff (2002))
  - Money flows away from equity funds, into money market funds. (Longstaff (2002))
  - Stock market goes down. (Chalmers, Kadlec, and Vayanos (2003))
- Variation in liquidity premia seems to be correlated across markets.
  - On-the-run premium is correlated with commercial-paper spread. (Krishnamurthy (2002))

Questions

- Questions:
  - What is economic mechanism driving variation in liquidity premia?
  - How is risk associated to changes in liquidity premia priced?
- Answers could be important for understanding
  - An important component of asset price volatility.
  - An important component of pricing of asset risk. (Especially for illiquid assets.)

This Paper

- Proposes a theory of time-varying liquidity premia.
- Explores its asset-pricing implications.
- Dynamic multi-asset equilibrium model.

Main Assumptions

Transaction costs.

- Exogenous, constant over time, different across assets.
- Illiquid assets $\equiv$ High TC.
- Time-variation in liquidity premia will not caused by TC.
  - Seems realistic: Variation in bid-ask spreads is often small relative to that in liquidity premia.
    - Off-the-run bonds.
    - Refcorp bonds. (Longstaff (2002))

Main Assumptions (cont'd)

Stochastic volatility.

- Asset payoffs have stochastic volatility.
- Volatility will drive liquidity premia.
- Seems realistic.
  - Empirical evidence.
  - Anecdotal evidence: Traders value liquidity more at times of "uncertainty."

Delegated money management.

- Portfolio decisions are made by fund managers.
- When return falls below a threshold, fund is liquidated.
- High volatility $\Rightarrow$ High liquidation probability $\Rightarrow$ Short investment horizons $\Rightarrow$ High liquidity premia.

Results / Empirical Implications

- Liquidity premia ($\equiv$ price differentials between assets which are identical except for TC) increase with volatility.
  - Flight to liquidity.
- Market's effective risk aversion ($\equiv$ expected return per unit of variance) increases with volatility.
  - Flight to quality.
  - Risk aversion varies not because of stochastic utility functions, but because of concern of liquidation.
- Betas depend on volatility.
  - Betas of illiquid assets increase with volatility.
- Correlations depend on volatility.
  - Correlations between similar assets increase with volatility.
- Unconditional two-factor CAPM (factors = market, volatility) does not price assets correctly.
  - Understates risk of illiquid assets.
- Implications for price of volatility factor.

MODEL

- Infinite horizon, discrete time. Time between periods is h.

Assets.

- Riskless asset, return rh. r is exogenous.
- N risky assets.
- Volatility v_t evolves according to

$$v_{t+h} = v_t + \gamma(\overline{v} - v_t)h + \sigma\sqrt{v_t h}\eta_{t+h}.$$

- Asset n pays dividend $\delta_{n,t}h$. $\delta_{n,t}$ evolves according to

$$\delta_{n,t+h} = \delta_{n,t} + \kappa(\overline{\delta} - \delta_{n,t})h + \sqrt{v_t h}\left(\phi_n\zeta_{t+h} + \psi_n\eta_{t+h} + \xi_{n,t+h}\right).$$

  - ζ_{t+h}: systematic shock, independent of η_{t+h}.
  - $\xi_{n,t+h}$: idiosyncratic shock.

- Supply S_n.
- Transaction costs per share ϵ_n.
- Price (average of bid and ask) $p_{n,t}$.

Fund Management and Liquidation

- Each investor "manages" a fund of size W_t.
- Performance-based liquidation:
  - In each period, a fund can be "monitored" by its owners with probability μh.
  - Fund is liquidated if $W_{t+h} - W_t < -L\sqrt{h}$, for $L > 0$.
- Random liquidation:
  - In each period, a fund can be liquidated with probability λh, regardless of performance.
- At liquidation:
  - Investment in each asset is sold in the market.
  - Manager can find employment in a new fund, whose size is equal to old fund's liquidation value.

Fund Managers

- Infinitely lived, continuum, mass one.
- Manager decides how much to allocate in each asset.
- Exogenous management fee $(aW_t + b)h$.
- Exogenous withdrawal by fund's owners $(\hat{a}W_t + \hat{b})h$.
- Assumptions: $a > 0$, $a + \hat{a} = r/(1 + rh)$.
- Manager maximizes

$$-E \sum_{k=0}^{\infty} \exp\left(-\alpha c_{t+kh} - \beta kh\right).$$

- Consumption is derived from fee.
- Manager can save/borrow in riskless asset.

EQUILIBRIUM

Basic Properties

- Managers buy and hold the market portfolio.
- Price of asset n is

$$p_{n,t} = q_n(v_t) + \frac{1-\kappa h}{r+\kappa}(\delta_{n,t} - \overline{\delta}).$$

- Value function of a manager is

$$-\exp\left[-A[W_t + zw_t + Z(v_t)]\right],$$

  where $A \equiv \alpha a$, $z \equiv r/[(1+rh)a]$, and w_t are manager's private savings.
- When h goes to zero, functions $\{q_n(v)\}_{n=1,..,N}$ and $Z(v)$ can be characterized by a system of $N+1$ ODEs.
  - ODEs are derived from Bellman equation.
  - They are second order.

Preliminaries

- Notation:
  - $\phi_M \equiv \sum_{n=1}^{N} S_n \phi_n$,
  - $\psi_M \equiv \sum_{n=1}^{N} S_n \psi_n$,
  - $q_M(v) \equiv \sum_{n=1}^{N} S_n q_n(v)$,
  - $\epsilon_M \equiv \sum_{n=1}^{N} S_n \epsilon_n$.
- Number of assets N becomes large, while $\sum_{n=1}^{N} S_n$ stays fixed.
  - S_n goes to zero.
  - Idiosyncratic shocks do not matter.

NO PERFORMANCE-BASED LIQUIDATION

- ODEs for $\{q_n(v)\}_{n=1,..,N}$ and $Z(v)$ have linear solution:
  - $q_n(v) = q_{n0} - q_{n1}v$.
  - $q_M(v) = q_{M0} - q_{M1}v$.
  - $Z(v) = Z_0 + Z_1 v$.
- Properties:
  - $q_{M1} > 0$: Market goes down when volatility increases.
  - $\partial q_{n0}/\partial \epsilon_n < 0$: Liquidity premia are positive.
  - $\partial q_{n1}/\partial \epsilon_n = 0$: Liquidity premia are independent of volatility.

CAPM

- ODE for $q_n(v)$ can be written as

$$E_t(R_{n,t+h}) = A\mathsf{Cov}_t(R_{n,t+h}, R_{M,t+h}) + AZ'(v_t)\mathsf{Cov}_t(R_{n,t+h}, R_{v,t+h}) + \left[r + 2\lambda \exp(2A\epsilon_M)\right]\epsilon_n h,$$

where

 - $R_{n,t+h}$: excess return on asset n.
 - $R_{M,t+h}$: excess return on market portfolio.
 - $R_{v,t+h}$: excess return on volatility portfolio.
 - Excess returns are per share, and between t and $t+h$.

- Conditional two-factor CAPM, adjusted for transaction costs.
- Linear solution $\Rightarrow Z'(v_t) = Z_1$.
- Taking expectations, we find

$$E(R_{n,t+h}) = A\mathsf{Cov}(R_{n,t+h}, R_{M,t+h}) + AZ_1\mathsf{Cov}(R_{n,t+h}, R_{v,t+h}) + \left[r + 2\lambda \exp(2A\epsilon_M)\right]\epsilon_n h,$$

i.e., unconditional two-factor CAPM, adjusted for transaction costs.

Volatility Factor

- Recent research has considered a volatility/liquidity factor.
- Pastor and Stambaugh (2001)
  - Liquidity factor $\equiv$ Price reversals.
- Acharya and Pedersen (2002)
  - Liquidity factor $\equiv$ Aggregate price impact.
- Liquidity factor:
  - Negative risk premium.
  - Significant explanatory power.
  - In our model, aggregate price impact is proportional to volatility $\Rightarrow$ Volatility factor corresponds to PS-AP liquidity factor.
- Ang and Hodrick (2002)
  - Volatility factor.

Volatility Risk Premium

- $Z_1 > 0 \Rightarrow$ Volatility factor carries positive risk premium.
  - Holding market beta constant, investors prefer assets that pay off when volatility is low.
  - Opposite to empirical findings.
- Intuition:
  - Holding wealth constant, investors are better off at times of high volatility.

PERFORMANCE-BASED LIQUIDATION

- Liquidation condition is $W_{t+h} - W_t < -L\sqrt{h}$.
- Probability of this event depends on tails of ζ_{t+h} and η_{t+h}.
- Normal distribution is not tractable.
- Power laws are, however, very tractable.

Power Laws

- Assume that lower tail of ζ_{t+h} and upper tail of η_{t+h} follow power laws with exponent b, i.e.,

$$\text{Prob}(\zeta_{t+h} < -y) \sim \frac{c_\zeta}{y^b},$$

$$\text{Prob}(\eta_{t+h} > y) \sim \frac{c_\eta}{y^b}.$$

- $b = 2$: Liquidation probability is

$$\pi(v_t) = \frac{v_t}{L^2}\left[c_\zeta \frac{\phi_M^2}{(r+\kappa)^2} + c_\eta \left[\frac{\psi_M}{r+\kappa} + q_M'(v_t)\sigma\right]^2\right].$$

  Linear in v_t.

- $b = 4$: Liquidation probability is

$$\pi(v_t) = \frac{v_t^2}{L^4}\left[c_\zeta \frac{\phi_M^4}{(r+\kappa)^4} + c_\eta \left[\frac{\psi_M}{r+\kappa} + q_M'(v_t)\sigma\right]^4\right].$$

  Quadratic in v_t.

- Equations are valid for L large relative to v_t.

Asset Pricing

- Expected returns are given by

$$
\begin{aligned}
E_t(R_{n,t+h}) \;=\; & A\mathsf{Cov}_t(R_{n,t+h}, R_{M,t+h}) + AZ'(v_t)\mathsf{Cov}_t(R_{n,t+h}, R_{v,t+h}) \\
& +\mu \left.\frac{\partial \pi(v_t, x)}{\partial x_n}\right|_{x=S} \frac{\exp(2A\epsilon_M) - 1}{A} h \\
& + \left[r + 2\left[\lambda + \mu\pi(v_t)\right]\exp(2A\epsilon_M)\right]\epsilon_n h.
\end{aligned}
$$

- Third term is “liquidation” risk premium.

LINEAR LIQUIDATION PROBABILITY

- Solution is still linear.
  - $q_n(v) = q_{n0} - q_{n1}v$
  - $q_M(v) = q_{M0} - q_{M1}v$
  - $Z(v) = Z_0 + Z_1 v$.
- New property:
  - $\partial q_{n1}/\partial \epsilon_n > 0$: Liquidity premia increase with volatility.

CAPM

- Liquidation risk premium depends on covariance between asset n, and market and volatility portfolios.
- $\Rightarrow$ Can incorporate liquidation risk premium into CAPM risk premium.
- $\Rightarrow$ Conditional two-factor CAPM, with modified risk-aversion coefficients:

$$\begin{aligned} E_t(R_{n,t+h}) &= A_M(v_t)\mathsf{Cov}_t(R_{n,t+h}, R_{M,t+h}) \\ &\quad + A_v(v_t)\mathsf{Cov}_t(R_{n,t+h}, R_{v,t+h}) \\ &\quad + \left[r + 2\left[\lambda + \mu\pi(v_t)\right]\exp(2A\epsilon_M)\right]\epsilon_n h. \end{aligned}$$

- In linear case, modified risk-aversion coefficients $A_M(v_t)$ and $A_v(v_t)$ are constants.
- $\Rightarrow$ Taking expectations, we can derive unconditional two-factor CAPM.

Risk-Aversion Coefficients

- For market portfolio:

$$A_M \equiv A + 2\frac{\mu}{L^2} c_\zeta \frac{\exp(2A\epsilon_M) - 1}{A}.$$

  - Greater than without performance-based liquidation.

- For volatility portfolio:

$$A_v \equiv AZ_1 + 2\frac{\mu}{L^2}(c_\zeta - c_\eta)\left[q_{M1} - \frac{\psi_M}{\sigma(r+\kappa)}\right]\frac{\exp(2A\epsilon_M) - 1}{A}.$$

  - Can become negative if $c_\eta > c_\zeta$ (volatility tails fatter than dividend tails).
  - $\Rightarrow$ Volatility factor can carry negative risk premium.
  - Holding market beta constant, investors prefer assets that pay off when volatility is high.
  - Intuition: Holding such assets reduces probability of liquidation.

Betas and Correlations

- Conditional betas are constant (independent of volatility) and equal to unconditional betas.
- Same for correlations.

QUADRATIC LIQUIDATION PROBABILITY

- Solution is nonlinear.
- Highest order new term (for large L) is quadratic in v:
  - $q_n(v) = q_{n0} - q_{n1}v - q_{n2}v^2$
  - $q_M(v) = q_{M0} - q_{M1}v - q_{M2}v^2$
  - $Z(v) = Z_0 + Z_1 v + Z_2 v^2$.
- $\partial q_{n2}/\partial \epsilon_n > 0$: Liquidity premia increase with volatility.

Conditional CAPM

- Conditional two-factor CAPM:

$$\begin{aligned} E_t(R_{n,t+h}) &= A_M(v_t)\mathsf{Cov}_t(R_{n,t+h}, R_{M,t+h}) \\ &\quad + A_v(v_t)\mathsf{Cov}_t(R_{n,t+h}, R_{v,t+h}) \\ &\quad + \left[r + 2\left[\lambda + \mu\pi(v_t)\right]\exp(2A\epsilon_M)\right]\epsilon_n h. \end{aligned}$$

- In quadratic case, modified risk-aversion coefficients $A_M(v_t)$ and $A_v(v_t)$ depend on v_t.

- For market portfolio:

$$A_M(v_t) \equiv A + 4\frac{\mu v_t}{L^4}\frac{c_\zeta \phi_M^2}{(r+\kappa)^2}\frac{\exp(2A\epsilon_M) - 1}{A}.$$

  - Market is more risk averse when volatility is high.
  - Stochastic risk aversion is not because of stochastic utility functions, but because liquidation probability is convex in volatility.

Betas

- Conditional betas are stochastic.
- An asset's market beta increases with volatility if

$$\frac{1}{2}\left[\frac{\sigma q_{n1} - \frac{\psi_n}{r+\kappa}}{\sigma q_{M1} - \frac{\psi_M}{r+\kappa}} + \frac{q_{n2}}{q_{M2}}\right] > \frac{\frac{\phi_M \phi_n}{(r+\kappa)^2} + \left[\frac{\psi_M}{r+\kappa} - \sigma q_{M1}\right]\left[\frac{\psi_n}{r+\kappa} - \sigma q_{n1}\right]}{\frac{\phi_M^2}{(r+\kappa)^2} + \left[\frac{\psi_M}{r+\kappa} - \sigma q_{M1}\right]^2}.$$

- Holds if asset is more price-sensitive to volatility than average.
- Holds if
  - Asset is more payoff-sensitive to volatility than average. (ψ_n small)
  - Asset has higher transaction costs than average. (ϵ_n large)

Correlations

- Conditional correlations are stochastic.
- For two assets n_1 and n_2 such that $\phi_{n_1} = \phi_{n_2}$, $\psi_{n_1} = \psi_{n_2}$, $\epsilon_{n_1} = \epsilon_{n_2}$, correlation increases with volatility.

Unconditional CAPM

- Taking expectations in conditional CAPM, we find

$$\begin{aligned} E(R_{n,t+h}) &= E[A_M(v_t)]\mathsf{Cov}(R_{n,t+h}, R_{M,t+h}) \\ &+E[A_v(v_t)]\mathsf{Cov}(R_{n,t+h}, R_{v,t+h}) \\ &+\mathsf{Cov}\left[A_M(v_t), \mathsf{Cov}_t(R_{n,t+h}, R_{M,t+h})\right] \\ &+\mathsf{Cov}\left[A_M(v_t), \mathsf{Cov}_t(R_{n,t+h}, R_{M,t+h})\right] \\ &+\left[r + 2\left[\lambda + \mu E(\pi(v_t))\right] \exp(2A\epsilon_M)\right] \epsilon_n h. \end{aligned}$$

- New terms: Covariances between stochastic risk-aversion coefficients and conditional covariances.
  - "Risk-aversion/beta" covariances.

CAPM Pricing

- Can two assets n_1 and n_2 have same unconditional covariances, but different sum of risk-aversion/beta covariances?
  - Yes.
  - Sum of risk-aversion/beta covariances is greater for n_1 if $q_{n_1 2} > q_{n_2 2}$.
  - This is equivalent to $\epsilon_{n_1} > \epsilon_{n_2}$.
- Unconditional CAPM understates risk of illiquid assets.
- Intuition:
  - Illiquid assets have high betas at times of high volatility.
  - At these times, market is most risk averse.

Conclusion

This paper:

- Proposes a theory of time-varying liquidity premia.
- Explores its asset-pricing implications.

Basic idea:

- Fund managers face liquidation after poor performance.
- High volatility $\Rightarrow$ High liquidation probability $\Rightarrow$ Short investment horizons $\Rightarrow$ High liquidity premia.

Main contributions:

- Several phenomena associated to crises / volatile times are related.
  - Large liquidity premia. (Flight to liquidity)
  - Increased market risk aversion. (Flight to quality)
  - Increased riskiness of illiquid assets.
  - Increased correlations.
- CAPM understates risk of illiquid assets.
  - These assets become very risky in crises, when market is most risk averse.