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ABSTRACT

The optimal progressivity of the tax system depends inversely on the compensated elasticity of taxable income with respect to the net-of-tax rate. Recent empirical evidence suggests that this elasticity is subject to manipulation by government policy with respect to tax administration and enforcement, as well as the choice of tax policy. This paper formalizes this notion, first in a general model and then in a particular example in which the elasticity of taxable income is determined by how broad the tax base is. In the context of the example, we show that a larger tax base implies a higher optimal degree of progressivity, and vice versa. Moreover, more egalitarian societies will have lower taxable income elasticities. This notion can help explain the pattern of income tax changes and empirical results of the past decade in the United States.

The Optimal Elasticity of Taxable Income

1. Introduction

Behavioral elasticities are key to understanding the efficiency cost of non-lump-sum taxes. Standard treatments of optimal income taxation take as given behavioral elasticities derived from immutable preferences, and characterize the tradeoff between redistribution and the deadweight loss of progressive taxes. Loosely speaking, larger elasticities imply that less progressive tax systems are optimal. Most of these models have focused on the elasticity of labor supply, in the belief that the relative price of leisure and goods is the most important margin affected by taxes. Empirical evidence suggests that, at least for hours worked, the aggregate (compensated) labor supply elasticity is quite small. However, increasing leisure is by no means the only possible margin of response to marginal tax rates. When personal tax rates on ordinary income rise, evasion may increase, businesses may shift to corporate form, there may be a rise in the consumption of deductible activities such as charitable giving, and individuals may rearrange their portfolios and compensation packages to receive more income as tax-preferred capital gains. All of these responses to higher taxes will show up in declines in taxable income, and there is a growing body of evidence that, at least for high-income individuals, the elasticity of taxable income is substantial.

Under certain assumptions, it is the compensated elasticity of taxable income, regardless of whether its origin is responsiveness of labor supply or some other behavior, that summarizes the efficiency cost of taxation and therefore is the crucial parameter in models of optimal progressivity. The idea is that all tax-induced behavior entails costs that will be incurred until, at the margin, the private cost equals the tax saving. Certain qualifications to this assertion are discussed in Slemrod (1998), including the importance of accounting for shifts across tax bases (individual to corporate, for example) and across time (e.g., a shift to deferred compensation or greater use of retirement accounts). One critical qualification is that the elasticity of taxable income is not only a function of preferences, and is therefore not immutable. With regard to labor supply, it is not too egregious to assume its elasticity is a primitive value based on preferences[1], but this assumption is very dubious when considering the other margins of response to taxation. On the contrary, it depends on a host of policy decisions about the tax base and enforcement. The responsiveness of evasion depends on the enforcement regime, the ability to shift the form of business organization depends on the roles governing the choice, the capital gains realizaton elasticity depends on how carryover at death is treated, and the size of the tax shelter industry created by high tax rates depends on the passive loss limitation provisions.

Recognizing that the elasticity of taxable income is subject to policy control has intriguing policy implications. For example, Slemrod (1994) demonstrated that the optimal degree of progressivity given a suboptimal setting of tax enforcement can be substantially below the globally optimal progressivity. The paper adopts the metaphor of Okun (1975), in which redistribution is characterized as transferring water from the rich to the poor in a leaky bucket, the degree of leakage representing the efficiency cost of progressive taxes. Using Okun's language, Slemrod (1994) raises the possibility that the leak can be fixed, albeit at some cost. Put less metaphorically, there are multiple instruments to an optimal tax system problem. However, Slemrod (1994) does not explicitly characterize the global optimum when the enforcement intensity can be chosen jointly with the degree of progressivity.

[1] Although it is more heroic to assume, as is standard, that the elasticity is constant across individuals and time. In practice, the labor supply elasticity may depend on labor market institutions such as work-week regulations.

This paper addresses that problem. First, we characterize the optimal linear income tax solution when the government has an instrument which affects the elasticity of taxable income. Next, we investigate at length a special case in which the policy instrument which determines the taxable income elasticity is the broadness of the tax base. We demonstrate several propositions about the optimal tax system and the optimal elasticity of taxable income.

2. The Basic Idea

2.1 A Simple Model of Optimal Leakage

The idea of the degree of leakage as a policy instrument can be illustrated simply in a model of two persons with unequal endowments, in which government wishes to maximize a concave social welfare function. Its only instrument is a tax on the higher-endowed, or "rich," person, the proceeds of which is transferred to the lower-endowed, or "poor," person. There is leakage in the tax-transfer procedure, however, so that the transfer of amount G equals (1 - δ) times the tax, T, where δ ($0<\delta<1$) is the rate of leakage; for now, we do not inquire as to the nature of the leakage, but formally model it below. The key element of the problem is that δ is subject to control by the government. Reducing δ is costly, so that $\delta = \delta(C)$, where C is the resource cost of administering and enforcing the tax system.[2] Assume, as is natural, that $\partial\delta / \partial C < 0$, and $\partial^2\delta/\partial C^2 > 0$.

Formally, the government's problem can be written as

$$\max_{T,C} \quad w(v_P,v_R) - \lambda[G - (1 - \delta(C))T + C] , \qquad (1)$$

where w is a social welfare function and v_i refers to the utility of the ith person, rich (R) or poor (P).

The first-order conditions for G and T reduce to

[2] We assume that the net transfer by the rich person received is T(1 - δ) - C.

$$\frac{w_R(y_R - T)\frac{\partial v_R}{\partial E}}{w_P(y_P + G)\frac{\partial v_P}{\partial E}} = (1-\delta), \tag{2}$$

where y_R and y_P are the exogenous incomes of the rich and poor, respectively, and E is exogenous income. The first-order condition for C is simply

$$T\frac{\partial \delta}{\partial C} - 1 = 0, \text{ or} \tag{3}$$

$$1\Big/\frac{\partial \delta}{\partial C} = T. \tag{4}$$

If, because either the social welfare function gets more concave or the distribution of income becomes more unequal, the optimal value of T increases, so will the optimal expenditure on reducing δ via expenditure on C. The payoff to a reduction in δ is proportional to T, and is therefore higher the more redistributional is the chosen policy.

In this simple example δT represents the standard measure of excess burden, although the total resource cost of the tax system is in fact δ(C)T + C. In richer models, the source of the standard excess burden is explicit, arising because taxpayers substitute away from taxed activities to untaxed activities. In a model of fixed producer prices in which the consumer chooses only leisure and a single composite consumption good, the ratio of the standard excess burden to revenue raised, δ in the simple example above, is approximately $\frac{1}{2}\varepsilon t$, where t is the rate of income tax and ε is the compensated elasticity of demand (or supply, in the case of labor). Thus, the standard excess burden is proportional to the compensated elasticity of labor supply; in addition, the marginal excess burden is increasing with t. The simple example presented here has neither of these features, nor does it illustrate the idea of an endogenous, policy-driven, elasticity of taxable income.

Next we analyze a model in which the standard excess burden does arise from consumer substitution, and where the degree of substitution can, to some extent and at some cost, be controlled by the government.

2.2 The Optimal Linear Income Tax With A Policy Instrument That Affects the Elasticity of Response

We consider the problem of choosing the optimal linear income tax in a multi-person economy. In this problem the government must raise R in revenue.[3] It can choose two parameters of a linear income tax, the marginal tax rate (t) and a uniform lump-sum tax (T). Raising all of the required revenue using the lump-sum tax would minimize excess burden, but the population is comprised of people with a distribution of abilities to earn income, and the government objective function is such that it would be willing to accept some inefficiency in exchange for less inequality in the distribution of welfare. We assume that the society consists of a large number of individuals indexed by the parameter y -- a measure of ability equal to the wage rate, with the distribution function F(y) normalized so that $\int F(y)dy = 1$.

We modify the standard problem by assuming that the government controls an instrument z, which reduces the elasticity of taxable income with respect to the tax rate.[4] It has a cost A(z), $\partial A/\partial z > 0$, with the property that it is prohibitively costly to completely eliminate leakage from taxable income. Given the parameters of the tax system, each consumer chooses consumption and leisure in such a way that gives rise to the indirect utility function v(t,T,z) and taxable income I(t,T,z). The tax paid by the consumer is equal to tI + T.

The government's problem can be expressed as

[3] We do not inquire, nor model, how the government disposes of R.

[4] The idea of expanding the optimal tax problem to include tax instruments other than tax rates is nicely illustrated in Mayshar (1991).

$$\underset{T,t,z}{\text{Max}} \int w(v(t,T,z)dF(y), \qquad (5)$$

where w is the social welfare function, subject to the budget constraint

$$\int [T + tI(t,T,z)]dF(y) = R + A(z). \qquad (6)$$

The Lagrangian for this problem is

$$L = \int w(v(t,T,z)) + \lambda[T + tI(t,T,z) - R - A(z)]dF(y). \qquad (7)$$

The three first-order conditions, rearranged using the standard results of consumer optimization, and the fact that $\partial I/\partial T = -1 - (1-t)\,\partial I/\partial E,$[5] are as follows

$$T: \frac{\partial L}{\partial T} = \int w' v_T + \lambda(1 + t\frac{\partial I}{\partial T})dF(y) = 0 \qquad (8)$$

or $$\int - w'\mu + \lambda(1 - t\frac{\partial I}{\partial E})dF(y) = 0 \qquad (9)$$

$$t: \frac{\partial L}{\partial t} = \int w' v_t + \lambda(I + t\frac{\partial I}{\partial t})dF(y) = 0, \qquad (10)$$

or $$\int - Iw'\mu + \lambda I(1 - t\frac{\partial I}{\partial E} - \frac{t}{1-t}\varepsilon)dF(y) = 0 \qquad (11)$$

$$z: \frac{\partial L}{\partial z} = \int w' v_z + \lambda\left(t\frac{\partial I}{\partial z} - A'(z)\right)dF(y) = 0, \qquad (12)$$

where μ denotes the marginal utility of exogenous, non-taxable income (the Lagrange multiplier from the consumer maximization problem), $\varepsilon = \left(\frac{\partial I^c}{\partial(1-t)}\right)\left(\frac{(1-t)}{I}\right)$ is the

[5] We assume that the lump-sum tax is deductible from taxable income.

compensated elasticity of taxable income with respect to the net-of-tax rate, and E denotes exogenous income (so that the Slutsky equation is $\frac{\partial I}{\partial t} = -\frac{I}{1-t}\varepsilon - \frac{\partial I}{\partial E}I$).

For given z, conditions (8), (10) and the budget constraint determine the optimal tax system. The standard (e.g., Atkinson and Stiglitz, 1980) approach is to combine them to form

$$\frac{t}{1-t} = -\frac{\operatorname{cov}(I, g)}{\int I\varepsilon dF(y)}, \tag{13}$$

where I is labor income, and $g = \frac{w'\mu}{\lambda} + t\frac{\partial I}{\partial E}$ is the marginal social valuation of a (non-taxable) lump-sum transfer to an individual. This expression implicitly determines the optimal marginal tax rate as inversely related to the compensated elasticity of labor supply which, in this model, is equivalent to the compensated elasticity of taxable income.

In the complete model ε depends on the choice of z, so that the optimal progressivity of the tax system cannot be determined independently of z which, in turn, affects the compensated elasticity of taxable income. To see it more clearly notice that we can derive the Slutsky-type equation for the effect of z to be

$$\frac{\partial I}{\partial z} = \frac{\partial I^c}{\partial z} + \frac{\partial I}{\partial E}\left(\frac{v_z}{\mu}\right), \tag{14}$$

where I^c refers to the compensated taxable income. Now write the first-order condition for z, expression (12), as

$$\int\left[\frac{v_z}{\mu}\left(\frac{w'\mu}{\lambda} + t\frac{\partial I}{\partial E}\right) + t\frac{\partial I^c}{\partial z}\right]dF(y) = \int\left[\frac{v_z}{\mu}g + t\frac{\partial I^c}{\partial z}\right]dF(y) = A'(z). \tag{15}$$

It is insightful to further decompose the compensated impact of z on taxable income. To do that, notice that, for any t=t*,

$t^* I^c = \int_0^{t^*} \left(t \frac{\partial I^c}{\partial t} + I^c \right) dt = \int_0^{t^*} I^c \left(1 - \frac{t\varepsilon}{1-t} \right) dt$. Increasing the tax rate by dt increases

revenue by $I^c dt$, but it also stimulates a substitution response that reduces taxable income by $\frac{dt}{1-t}\varepsilon$ percent for each percent increase in the tax rate. Now we can write

$$t^* \frac{\partial I^c}{\partial z} = \frac{\partial}{\partial z} \left(\int_0^{t^*} I^c \left(1 - \frac{t\varepsilon}{1-t} \right) dt \right) \tag{16}$$

$$= \int_0^{t^*} \frac{\partial I^c}{\partial z} \left(1 - \frac{t\varepsilon}{1-t} \right) dt - \int_0^{t^*} \frac{tI^c}{1-t} \frac{\partial \varepsilon}{\partial z} dt \ .$$

The first term of expression (16) represents the direct effect on tax revenue of changing z. The second term represents the effect on revenue of changing the compensated elasticity of taxable income. The impact of this change is positively related to the tax rate.

First-order condition (15) can now be rewritten as

$$\int \left\{ \frac{v_z}{\mu} g + \int_0^{t^*} \frac{\partial I^c}{\partial z} \left(1 - \frac{t\varepsilon}{1-t} \right) dt - \int_0^{t^*} \frac{tI^c}{1-t} \frac{\partial \varepsilon}{\partial z} dt \right\} dF(y) = A'(z) \tag{17}$$

Expression (17) says that, at the optimum, the benefit of increasing z should equal the marginal administrative cost. Its benefit is the sum of its direct effect on social welfare (denominated in dollars), the revenue effect holding the elasticity constant, and the benefit of changing the elasticity. This last term, $-\int_0^{t^*} \frac{tI^c}{1-t} \frac{\partial \varepsilon}{\partial z} dt$, represents the fact that the more an instrument decreases the taxable income elasticity, the more it is valuable to the social planner (holding other effects constant). Also, the benefit of a lower elasticity increases with the tax rate and taxable income. Thus, the more egalitarian is the social objective function (which manifests itself in higher progressivity), the greater is the benefit from using this instrument, because the reduction of the compensated taxable income elasticity is more valuable.

An interesting special case occurs when the elasticity does not depend on the marginal tax rate. We can then derive $\int_0^{t^*} \frac{tI^c}{1-t} dt = \varepsilon^{-1}\left(\int_0^{t^*} I^c dt - t^* I^c \right)$. Interpreting I^c as the demand for taxable commodities and t as their price, the term in brackets can be thought of as the standard (i.e., not including A(z)) excess burden from taxation: it is (minus) the difference between the amount collected in distortionary taxes and the amount that a "discriminating" (i.e., being able to impose a person-specific, arbitrary non-linear marginal tax schedule[6]) tax authority could collect. Then the benefit of reducing the taxable income elasticity is

$$-\int_0^{t^*} \frac{tI^C}{1-t} \frac{\partial \varepsilon}{\partial z} dt = \varepsilon_z \left(\int_0^{t^*} I^C dt - t^* I^C \right), \tag{18}$$

where $\varepsilon_z \equiv \dfrac{-\frac{\partial \varepsilon}{\partial z}}{\varepsilon}$ is the proportionate change in elasticity resulting from an increase in z, defined to be a positive number. Equation (18) says that the value to the social planner of reducing the compensated taxable income elasticity is proportional to the excess burden.

This model isolates the role of "leakage control" in the optimal setting of a tax system instrument. However, the generality of the instrument z makes it difficult to establish insightful propositions about the expanded optimal progressivity problem. For this reason, we next study a concrete example of this issue.

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[6]With a non-linear tax system this term would still be non-zero, because the social planner has to impose the same tax system on all individuals.

3. A Model of Optimal Progressivity and the Optimal Tax Base

In this section we investigate an example in which the social planner chooses the taxable income elasticity by means of selecting the size of the income tax base. Under certain assumptions about the utility function, the broader is the tax base, the lower is the compensated elasticity of taxable income with respect to the net-of-tax rate.

The structure of the problem is similar to that described in Section 2.2. The government must raise revenue of R and can choose three instruments: a lump-sum tax T, a marginal tax rate t, and the size of the income tax base, denoted by m. The choice of the tax base determines the administrative cost A(m). Following Wilson (1989, p. 1199) we assume that, for given m, those commodities which minimize A are included in the tax base, so that A(m) is a convex function of m, (A'(m)>0).[7]

3.1 The Consumer's Problem

Assume that there is a large number of commodities, so that we can use a continuous approximation of the tax base. Commodities are indexed by a number $i \in [0,1]$, and they are denoted by C_i. The bundle of commodities is written as C ($C : [0,1] \rightarrow R_+$). The size of the tax base is denoted by m, meaning that commodity i is taxed as long as $i < m$. To facilitate the analysis, we use the convention that the lump-sum tax is

[7] The assumption of $A'(m) > 0$ was introduced by Yitzhaki (1979), who provided a simple justification for this shape. We make this assumption for convenience, but it is not required for our results. In particular we know that, as long as a global optimum exists, the administrative costs must be increasing in its neighborhood – otherwise one could extend the tax base and decrease both distortions and administrative costs. Consequently, without this assumption, our propositions should be read as describing the situation in the neighborhood of the optimum only.

We recognize, but do not address, the possibility that, depending how the system is administered, A(1) could be less than A(m*), m*<1. In other words, a completely broad base may be less costly to administer than one with exclusions.

deductible from taxable income (i.e., the higher is the lump-sum tax, the lower the taxable income), so that taxable income is[8] $I = y - T - \int_m^1 C_i di$.

The consumer's budget constraint is

$$\int_0^m C_i di + (1-t)\int_m^1 C_i di = (1-t)(y-T) = (1-t)y - (1-t)T \text{ , or} \tag{19}$$

$$\int_0^1 C_i di = y - t(y - \int_m^1 C_i di - T) - T . \tag{20}$$

The consumer maximizes utility u(C) subject to the budget constraint. Let the indirect utility function be

$$v(t,T,m) = \max u(C) + \mu\left((1-t)(y-T) - \int_0^m C_i di - (1-t)\int_1^m C_i di \right) . \tag{21}$$

Using the envelope theorem to find the derivatives of the indirect utility function yields

$$v_T = -(1-t)\mu \tag{22}$$

$$v_t = -\mu(y - T - \int_m^1 C_i di) = -\mu I \tag{23}$$

$$v_m = -\mu(C_m - (1-t)C_m) = -t\mu C_m . \tag{24}$$

One can also show that Slutsky equations for individual commodities can be combined to give the following relation for taxable income

[8]It should be clear that this is equivalent to the standard formation in which the lump-sum tax does not affect taxable income; the treatment used here permits a simpler solution in the CES example below. The budget constraint, as in Wilson (1989), can also be expressed as

$$\int_0^1 q_i C_i di = y - T,$$

where

$$q_i = \begin{cases} \dfrac{1}{1-t}, & i \le m \\ 1 \ , & i > m \end{cases} .$$

$$\frac{\partial I}{\partial t} = -\alpha_{I,(1-t)} - \frac{\partial I}{\partial E} I \ , \tag{25}$$

where E denotes exogenous, non-taxable income, and $\alpha_{I,(1\text{-}t)}$ is the substitution response of taxable income with respect to a change in the net-of-tax rate (1-t); it consists of the substitution responses of all exempted commodities, and can shown to be positive (i.e., the substitution response with respect to the tax rate is negative). For the sake of future reference note that

$$\frac{\partial I}{\partial T} = -1 - (1-t)\frac{\partial I}{\partial E} , \tag{26}$$

so that taxable income depends on T both directly and indirectly. The indirect effect (through the change in demand for exempt commodities) is the same as the effect of a decrease of exogenous income of (1-t)dT; i.e., this is what remains after taxes from the increase in T.

In what follows we will assume that the consumer maximizes the CES utility function

$$u(C) = \left(\frac{\int_0^1 \eta_i C_i^{\frac{\sigma-1}{\sigma}}}{\int_0^1 \eta_i^{\sigma} di} \right)^{\frac{\sigma}{\sigma-1}} , \tag{27}$$

where σ is the elasticity of substitution between any two commodities. This is a useful special case because, with a CES utility function, excess burden is minimized for any m by having a uniform tax on all taxable commodities (Sadka, 1977)[9].

The consumer maximizes expression (27) subject to the budget constraint (20).

3.3 The Government's Problem

The government's problem is to choose t, T, and m to maximize the social welfare function, subject to its budget constraint. To be precise, it is

$$\max_{t,T,m} \int w(v(t,T,m,y))dF(y)$$

subject to

$$\int [T + tI(t,T,m)]dF(y) = T + t\int \left(y - \int_m^1 C_i di - T \right) dF(y), \qquad (28)$$

where T is the revenue collected using lump-sum taxes, and $X \equiv tI = t\int \left(y - \int_m^1 C_i di - T \right) dF(y)$ will be called the revenue from the distortionary taxes.[10]

The standard optimal linear tax problem can be thought of as solving for the optimal t and T, for a given m.[11] In this model, the value of t by itself is not a

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[9] As elaborated on below, it is also useful because its homotheticity implies that there are no distributional implications to whether a given amount of distortionary tax is raised via a broad or narrow base (high m, low t versus low m, high t), and because the compensated elasticity of taxable income is a (simple) function of only m and σ. One cost of assuming homotheticity is that the model cannot address the possibility that base broadening might bring into the tax base commodities consumed relatively more heavily by higher-income individuals.

[10] Given our assumptions of a concave welfare function and CES utility, given γ, an optimum tax system exists. The case in which γ is 0 is not optimal as long as A'(0) is not too high (γ=0 means a uniform lump sum tax). The case in which γ is1 is not optimal as long as the administrative cost of taxing all commodities (A(1)) is sufficiently high. We make these assumptions to avoid a corner solution for γ.

[11] One difference is that this model does not allow labor supply response, i.e., no good (leisure) has a price of y(1-t).

meaningful indicator of progressivity (because progressivity depends on γ, as well), so instead we will solve for the optimal amount of X, which is a meaningful measure of the progressivity of the tax system.

The government's budget constraint can be written as

$$T + X = A(m) + R. \tag{29}$$

At this point it is useful to note the relationship of this problem to the ones solved by Yitzhaki (1979) and Wilson (1989). In a representative consumer model, they characterized the optimal t and m for a given value of T (Wilson assumes T = 0). Thus, in their problem there is no issue of optimal progressivity, only one of trading off the lower excess burden from a larger tax base against the lower administrative cost of a smaller tax base. Although they characterize their concern as the optimal commodity tax base, it is clear that in a one-period setting in which uniform commodity taxes are always optimal, the optimal commodity tax base problem is equivalent to the optimal income tax base problem.

Yitzhaki assumes that the consumer has a Cobb-Douglas utility function, while Wilson generalizes this to a CES utility function. Their characterization of the optimal base can be stated as follows:

$$\text{(Yitzhaki)} \quad ty + [\log(1-t)](1-t)y = A'(\gamma) \tag{30}$$

(Wilson) $$\frac{t(1-b)y}{1-\gamma}+\frac{(b-\gamma)(1-t)y}{\gamma(1-\gamma)(\sigma-1)}=A'(\gamma)\ , \tag{31}$$

where $A(\gamma)=A(m(\gamma))$, $\gamma=\dfrac{\int_0^m \eta_i^\sigma di}{\int_0^1 \eta_i^\sigma di}$ and $b=\dfrac{\gamma}{\gamma+(1-\gamma)(1-t)^{1-\sigma}}$. Note that γ is a monotone transformation of m, and is equal to m if η_i is a constant for all i. It can be shown that I = b(y – T), so that b is the share of income, after lump-sum taxes, that is spent on taxable commodities. Notably, $\dfrac{\partial b}{\partial\gamma}=\dfrac{b(1-b)}{\gamma(1-\gamma)}>0$ and $\dfrac{\partial b}{\partial t}=\dfrac{b(1-b)(1-\sigma)}{1-t}\lesseqgtr 0$ as $\sigma \gtreqless 1$.

In both cases the left-hand side is the marginal social benefit of expanding the tax base, which is equal to the marginal reduction in the excess burden from raising a given amount of revenue with a larger tax base. The right-hand side is simply the marginal resource cost of expanding the tax base.

In the Yitzhaki and Wilson problems, the amount of revenue that must be raised by distortionary taxes is fixed, so that the only problem is whether to use a narrow-base, high-rate system or a broader-base, lower-rate system. In the problem considered here, the government can also reduce distortionary taxes (and, therefore, redistribution) by making greater use of lump-sum taxes (i.e., reducing the lump-sum grant). Because with a CES utility function the income elasticity of all goods is exactly one, there are no distributional implications to whether a given amount of distortionary tax is raised via a narrow or broader base; thus, for a given R the value of T is a sufficient statistic for evaluating the tax system's progressivity, as is the value of X.

4. Characteristics of the Optimal Tax System

We begin by demonstrating two propositions about how the optimal tax base and optimal progressivity aspects of policy are related. Intuitively, we presume that progressivity and the tax base are determined by two separate agencies, and we investigate how each agency will behave if it believes the policy of the other to be fixed.

PROPOSITION 1: For given R, the less the reliance on lump-sum taxes, the larger will be the optimal value of γ.

We take T as given and characterize the solution to the first-order conditions for t and γ, and the budget constraint. Note that the first-order conditions for γ and t can be combined to yield

$$\left(\frac{t(1-b)}{1-\gamma}+\frac{(b-\gamma)(1-t)}{\gamma(1-\gamma)(\sigma-1)}\right)\int(y-T)dF(y)=A'(\gamma)\,, \tag{32}$$

which is a slightly modified version of expression (31).

This implies that the marginal benefit of an increased γ is positively related to the amount of revenue to be raised by distortionary taxes, and inversely related to the revenue raised by the lump-sum tax. We represent this relationship between the optimal γ and X in Figure 1 as the VT curve.[12] To be precise, it is the set of optimal pairs of γ and X for different values of T, or (γ*(T), X*(T)). The value of T increases as one moves closer to the origin. In the appendix, we prove that this curve must be upward sloping, i.e., higher values of T are associated with lower values of both X and γ.[13] Thus, ceteris paribus, more progressive tax systems will feature higher optimal tax bases.

Next, we show that a higher value of γ makes it optimal to make the tax system more progressive:

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[12]Because of the budget constraint T + X = A(m(γ)) + R, for any R a pair of X and γ uniquely determines t and T. Because $A'(\gamma) > 0$, the iso-T curves are upwardly sloping; higher curves correspond to lower values of T.

[13]The standard model can be interpreted as an extreme case in which the VT curve is vertical.

PROPOSITION 2: For given R, a compensated increase in γ implies that the amount of revenue raised through distortionary taxes increases.

The appendix presents a proof of this proposition.[14] To investigate this, we assume the social welfare function is isoelastic,[15] and characterize the first-order conditions for the optimal values of t and T, subject to the budget constraint, for given values of γ, ignoring the direct cost of changing γ. In Figure 1 we denote the relationship between the optimal value of X and γ as $X^*(\gamma)$. Proposition 2 implies that this curve is upward-sloping.[16]

The overall optimum occurs at the intersection of the VT and $X^*(\gamma)$ curves. This determines both the optimal progressivity of the tax system and the optimal size of the tax base. We can make some specific assertions about how the optimum depends on the parameters of the problem. We offer two propositions:

PROPOSITION 3: A more egalitarian society will feature a larger tax base and a more progressive tax system.

PROPOSITION 4: An increase in the marginal cost of administering a broad tax base will decrease progressivity as well as the tax base.

These propositions are proven in the appendix. In terms of Figure 1, both of these propositions depend on the fact that the VT curve is steeper than the $X^*(\gamma)$ curve. An increase in egalitarianism is represented by an upward shift in the $X^*(\gamma)$ c rve, and an

[14]The appendix is available upon request from the authors.
[15]We expect that the proposition holds for any social welfare function with an increasing elasticity of social marginal utility.
[16]Note that demonstrating that X rises with γ does not settle whether t rises with γ. We have not proven that it is impossible that although γ rises, t declines, but not so much that X also declines.

increase in the cost of base broadening by a upward shift of the VT curve in the neighborhood of the optimum.

5. The Optimal Elasticity of Taxable Income

Straightforward calculations, detailed in the appendix, show that in the CES case the compensated elasticity of taxable income with respect to the net-of-tax rate, denoted ε, is equal to

$$\varepsilon = \frac{\sigma(1-\gamma)}{(1-t)^{\sigma-1}\gamma + (1-\gamma)} = \sigma(1-b)\,, \tag{34}$$

where b is the expenditure share of taxable income for taxable commodities; for given σ, the larger the expenditure share on non-taxable commodities, the larger is the elasticity. Although this expression for ε depends on σ, an aspect of preferences assumed to be immutable, it also depends on policy instruments of the government, γ and t, whose optimal setting depends, inter alia, on the egalitarianism of the society and the technology of tax administration. This implies that, across countries or within a country across periods of time, the optimal elasticity of taxable income will vary. For example, we can show that, under certain conditions, the taxable income elasticity will be lower in more egalitarian societies. We summarize this as follows:

PROPOSITION 5: As long as $\sigma<1$, a more egalitarian society will feature a lower compensated elasticity of taxable income.

More egalitarianism increases the optimal value γ, which decreases the elasticity. But it also affects the optimal value of t, which has its own effect on ε. We prove in the appendix that, as long as $\sigma < 1$, the net effect will be to lower the optimal compensated elasticity. But this need not be the case. The case where the VT curve is vertical and $\sigma >$

1 is a counterexample. When egalitarianism increases, optimal γ does not change, so X increases solely via an increase in t. But, when $\sigma > 1$, the increase in t increases the expenditure share of non-taxable commodities, (1 - b), and therefore increases the elasticity of taxable income.

6. Conclusions and Implications

The optimal progressivity of the tax system depends inversely on the compensated elasticity of taxable income with respect to the net-of-tax rate. That elasticity is not entirely an immutable function of consumer preferences and production technologies, but is subject to manipulation by government policy with respect to tax administration and enforcement as well as the choice of tax base.

In this paper we formalize this notion, first in a general model and then in a particular example in which the elasticity of taxable income is determined by the choice of how broad the income tax base is. In the context of the example, we show that a larger tax base implies a higher degree of progressivity, and vice versa. Moreover, more egalitarian societies will have a broader income tax base as well as a more progressive tax system. Under plausible conditions, more egalitarian societies will have lower taxable income elasticities.

This notion can help to explain the pattern of income tax changes and empirical results of the past decade in the United States. The Tax Reform Act of 1986 broadened the tax base and removed some particular loopholes in the law such as tax shelters, both of which arguably reduced the elasticity of taxable income. This is consistent with empirical estimates of taxable income elasticity based on the 1986 (Feldstein, 1995; Moffitt and Wilhelm, 1998; and Auten and Carroll, 1998) versus those based on the 1990 and 1993 tax changes (Carroll, 1998); estimates in the first case are larger. Although differences in methodology[17] and offsetting biases due to unobserved trends may be somewhat responsible for these differences, this paper raises the possibility that the base broadening provisions of TRA86 reduced the true taxable income elasticity.

If one takes seriously the idea that actual tax policy decisions reflect a solution to a maximization problem like that posed here, many intriguing predictions arise. For example, one could argue that, once the Tax Reform Act of 1986 was passed and policy makers become aware that the taxable income elasticity, and therefore the marginal social cost of increasing progressivity had declined, they appropriately increased the progressivity of the tax system via the increased top rates in 1990 and 1993. Across countries, one should observe those countries with more progressive tax systems also having broader tax bases.

The principal message to policy makers is to be wary of making decisions about progressivity on the basis of the existing elasticity of taxable income, which is itself subject to control and may be suboptimal. The global degree of progressivity and all of the other aspects of a tax system must be jointly optimized.

[17] It also calls into question a key assumption of the empirical studies of TRA86: that the taxable income elasticity was the same before and after the tax law change.

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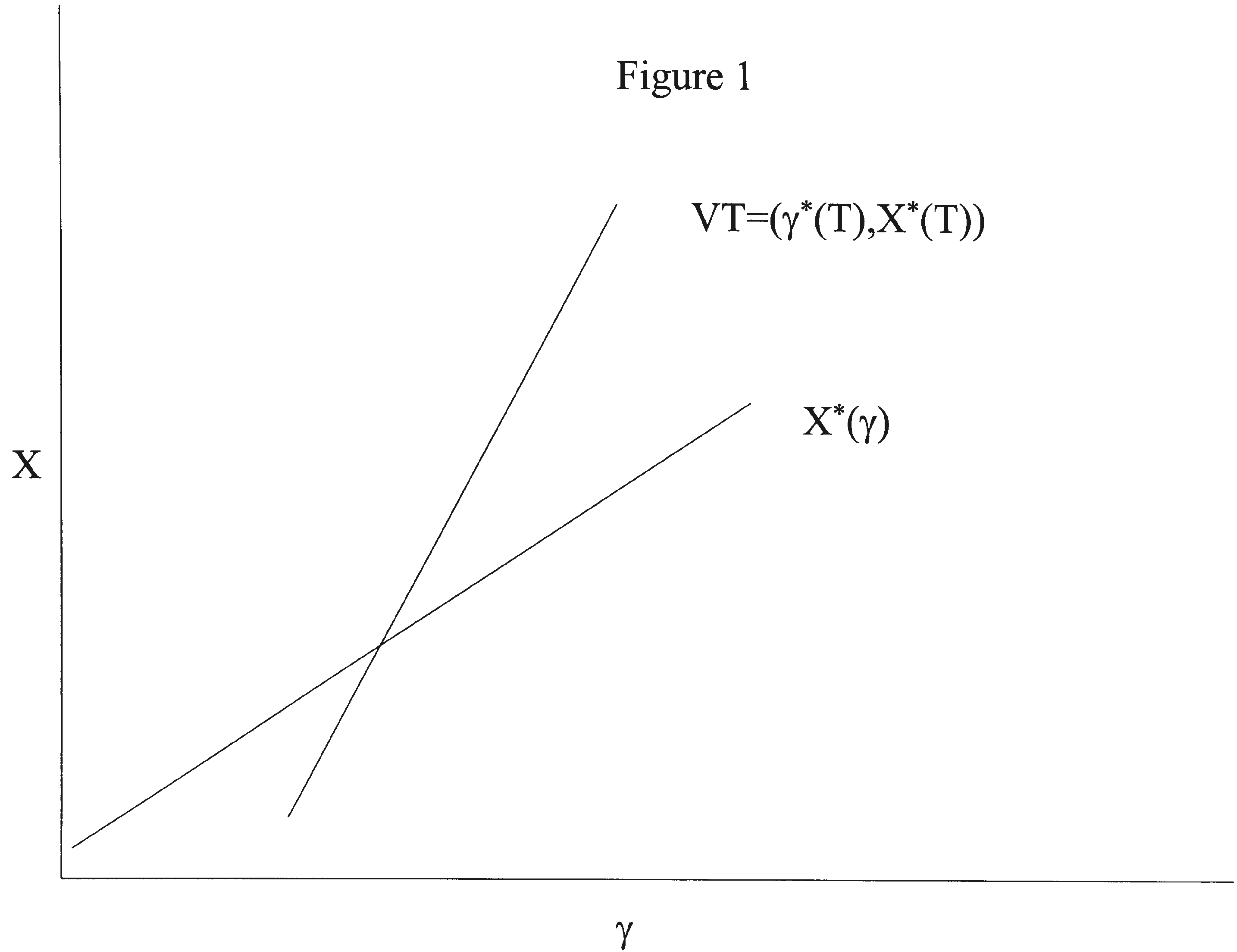
Figure 1



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