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ABSTRACT

Benefit taxation is a system in which individuals are taxed according to the benefits they receive from public expenditures. This paper describes an alternative to the standard Lindahl method of determining the distribution of individual benefits from government-provided public goods, and uses this alternative to calculate benefit taxes. This new method of benefit taxation avoids some of the paradoxical features of Lindahl pricing, and generates outcomes in which all consumers benefit (or at least none are made worse off) from reduced costs of providing public goods. An illustrative calculation shows that benefit taxes defined in this way may be quite regressive.

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1. Introduction.

Governments distribute tax burdens and select levels of public goods to provide. The benefit principle says that taxes should be levied on individuals according to the benefits they receive from government expenditures. In order to apply the benefit principle it is necessary to establish the distribution of benefits from public goods. The traditional approach to benefit determination relies on Lindahl prices. Taxes based on Lindahl prices can, however, have perverse features: high-income individuals may be required to pay so much more in taxes than are low-income individuals (with identical preferences) that after-tax incomes become inversely related to pre-tax incomes. While such a welfare ordering reversal arises only in special cases, it illustrates the more general problem that the welfare properties of Lindahl prices differ fundamentally from the welfare properties of private good prices.

This paper describes an alternative to Lindahl pricing of public goods, one that, if used to distribute tax burdens, guarantees that the welfares of individuals with identical preferences increase with their pre-tax incomes. This method of assigning tax burdens also ensures that consumers prefer identical levels of public good provision, and that they are made uniformly better off by reduced costs of providing public goods. In order to determine the distribution of benefit taxes under this alternative, it is first necessary to establish the money equivalent of benefits that individuals receive from government provision of public goods. This benefit calculation is designed so that taxes set equal to benefits would cover the cost of public good expenditures. Since in practice tax systems seldom, if ever, assign taxes according to benefits so defined, deviations from benefit taxes can then be interpreted as net redistributions due to the fiscal system inclusive of both taxes and expenditures.

Section 2 of this paper describes two methods of valuing public good benefits for purposes of benefit taxation: Lindahl pricing and the egalitarian-equivalent solution proposed by Moulin. Section 3 presents an alternative benefit determination scheme. Section 4 applies the alternative method to fiscal data for the United States, and compares its redistributive implications to those of Lindahl pricing.

2. Lindahl Pricing and the Egalitarian-Equivalent Solution.

The Lindahl theory of public expenditure and taxation is the most well-known method of taxing according to benefits received. The idea is that any public good provision level G can be supported by individual-specific prices, as long as those prices equal marginal values in money terms.[1] Aaron and McGuire (1970) apply the Lindahl framework to attribute the benefits of public goods among individuals according to their marginal valuations. This method is used extensively in the applied literature on government redistributions.[2] One odd aspect of benefit taxes implied by this method is that individuals with high (pre-tax) incomes may be made worse off than low-income individuals with the same preferences.

This feature of Lindahl pricing can be understood by representing consumer i's utility function in semi-indirect form $\Psi^i(\mathbf{p}, y_i, G^*)$, in which the vector $\mathbf{p} = (p_1, \ldots, p_m)$ represents the market prices of private goods and G^* is the (common) level of public good provision. y_i is consumer i's private after-tax income, equal to $(Y_i - T_i)$, in which Y_i is consumer i's before-tax income and T_i his tax obligation. The Lindahl method, as applied by Aaron and McGuire, proceeds from the observation that this utility level is identical to

[1]See Lindahl (1919), Samuelson (1954), and their interpretation by Musgrave (1959). Foley (1970) first established the general equilibrium properties of Lindahl equilibria; for an analysis of some of the limitations of the Lindahl concept, see Johansen (1963) and Roberts (1974).

[2]Besides Aaron and McGuire, those who apply the Lindahl method to estimate the distribution of benefits from public goods include Maital (1973), Dodge (1975), Reynolds and Smolensky (1977), Gillespie and LaBelle (1978), Martinez-Vazquez (1982), and Lambert and Pfahler (1988).

one in which consumers receive individual-specific lump-sum transfers but must purchase the public goods they consume at individual-specific prices. Properly chosen, these individual-specific transfers and prices satisfy:

$$\Psi^i(\mathbf{p},\ y_i,\ G^*) \quad = \quad V^i(\mathbf{p},\ \rho_i,\ y_i + M_i) \qquad \forall \quad i \tag{1}$$

in which ρ_i is individual i's price of public good consumption and M_i the lump-sum transfer he receives from the government. The indirect utility function $V^i(\cdot)$ represents consumer i's utility under the assumption that he must pay for the public good at price ρ_i; the consumer then has a demand curve for G that is a function of income $(y_i + M_i)$ and prices. There are two necessary conditions for these prices and income transfers to support a desired public good consumption level of G* by individual i: $\rho_i = (\partial\Psi^i/\partial G)/(\partial\Psi^i/\partial y_i)$ (price equals marginal willingness to pay) and $M_i = \rho_i G^*$ (budget balance).[3] Assuming these conditions to hold, then every consumer i, if given an income supplement of M_i and required to purchase government services at price ρ_i, would choose quantity G*. M_i represents consumer i's public good benefit in the sense that M_i is the income supplement necessary to support this choice. The Lindahl equilibrium is one in which taxes equal benefits, so it represents the fixed point at which M_i equals T_i for every consumer.[4] In practice taxes are not usually set that way, so Aaron and McGuire measure consumer i's net fiscal redistribution as $(M_i - T_i)$.

The problem with this interpretation of M_i as consumer i's income-equivalent of benefits from provision of G is the endogeneity of the reference

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[3]These conditions also imply that $\partial V^i(\cdot)/\partial(y_i + B_i) = \partial\Psi^i(\cdot)/\partial y_i$. In addition, it is necessary for the preference set to be convex and preferences to exhibit continuity and monotonicity, as Neary and Roberts (1980) note in a related context.

[4]The Lindahl equilibrium is one in which benefit taxes finance the cost of public good provision. Kaneko (1977) and Mas-Colell and Silvestre (1989) analyze a simple modification (proportional scaling of tax burdens) for cases in which public goods are produced with nonconstant returns to scale.

price vector ($\mathbf{p}$, ρ_i) to preferences and incomes. Consumers generally face different ρ_is, since their tastes and y_is differ. As a result, their benefit-inclusive incomes ($y_i + M_i$) are not generally comparable because their real incomes depend on a price level that varies from consumer to consumer.[5] As an illustration, consider two individuals, j and k, with *identical* preferences but different after-tax incomes ($y_j > y_k$). If the public good is a Giffen good, then ($y_k + M_k$) exceeds ($y_j + M_j$).[6] The Aaron and McGuire procedure would attribute greater total income to k than to j, even though they have identical tastes, consume the same quantity of the public good, and k has lower income than j. This rank reversal is possible because prices are endogenous to preferences and incomes, and therefore income has different meaning to different consumers.

Moulin (1987) proposes the egalitarian-equivalent solution as an alternative method of allocating tax burdens (and thereby implicitly allocating public good benefits) in a way that avoids the problem of endogenous prices. Suppose that the government provides G*; in the egalitarian-equivalent scheme every consumer is treated as if he received an endowment of g* (< G*) public goods. The money equivalent of the utility difference between consuming G* and consuming g* represents the consumer's benefits; hence benefits equal Q_i defined so that

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[5]Slivinski (1983) characterizes the extremely limited set of circumstances in which it is meaningful to compare the incomes of consumers facing different price vectors.

[6]To prove this property, totally differentiate the consumer's demand function $G(\rho_i, y_i + M_i)$ with respect to y_i; at a fixed G* it must be the case that $(\partial G/\partial y_i)[1 + dM_i/dy_i] + (\partial G/\partial \rho_i)(d\rho_i/dy_i) = 0$. Since $M_i = G^*\rho_i$, then $d\rho_i/dy_i = (dM_i/dy_i)/G^*$; making this substitution yields $dM_i/dy_i = -1 + (\partial G/\partial \rho_i)/[G^*(\partial G/\partial y_i) + \partial G/\partial \rho_i]$. The anomaly described in the text is the case in which $d(y_i + M_i)/dy_i < 0$; this arises, therefore, if $(\partial G/\partial \rho_i)/[G^*(\partial G/\partial y_i) + \partial G/\partial \rho_i] < 0$. By the Slutsky decomposition the term in brackets is negative, so if $(\partial G/\partial \rho_i) > 0$ then $d(y_i + M_i)/dy_i < 0$. It is noteworthy that this phenomenon arises only when the public good is a Giffen good. Kovenock and Sadka (1981) and Snow and Warren (1983) analyze the conditions under which Lindahl pricing generates progressive tax schedules, but do not consider the possibility that tax burdens may increase faster than one-for-one with income.

$$\Psi^i(\mathbf{p},\ y_i,\ G^*) \quad = \quad \Psi^i(\mathbf{p},\ y_i + Q_i,\ g^*) \quad \forall \quad i \tag{2}$$

and g^* is chosen so that the sum of Q_is just equals the cost of producing G^*. Moulin shows that an allocation in which $T_i = Q_i\ \forall\ i$ has the property that every consumer would benefit from reduced costs of producing public goods, as long as every consumer has a positive marginal valuation of the public good. Moulin also shows that the resulting allocation is in the core, again under the requirement that marginal valuations of the public good are nonnegative.

The egalitarian-equivalent method of assigning public good benefits does not share the curious feature of Lindahl pricing that taxation according to benefits may reverse the welfare ordering of two individuals with identical preferences. But the requirement that marginal valuations of the public good be positive in order for all consumers to benefit (or at least not be harmed) by reduced costs of producing the public good is, in practice, quite restrictive. In the absence of reliable information on consumer preferences it is difficult to make conclusive statements,[7] but it appears that certain individuals assign negative valuations - quite apart from the associated tax burdens - to marginal government expenditures on national defense, development of nuclear power, support for modern art, various law enforcement activities, and other categories of spending. Consequently, it is useful to consider methods of assigning benefit taxes for which consumers benefit from reduced costs of government provision of public goods even if marginal valuations are not uniformly positive.

[7]The analysis in this paper presumes that consumer preferences for public goods are known with certainty. In practice, the information revelation problem (surveyed by Laffont (1987)) implies that it may be impossible for the government to elicit from consumers sufficient information to impose benefit taxes precisely according to preferences for public goods. Instead, the government may have to estimate consumer preferences in some manner. While it would be useful to incorporate information problems directly in the analysis of benefit taxation, it is important first to characterize benefit taxation in the presence of full information.

3. Benefit Taxation: An Alternative.

This section describes an alternative to Lindahl pricing that incorporates the egalitarian-equivalent solution as a special case. This alternative has three important properties. The first property is that, for a fixed allocation of real resources, individual utility increases in the sum of private after-tax income and public good benefits. This property implies that two consumers with identical preferences prefer the consumption bundle of whomever has greater benefit-inclusive income.

The second property is that the sum of measured public good benefits equals the cost of providing public goods. This is a nontrivial normalization of aggregate public good benefits. The idea is that taxes assigned equal to benefits could be used to pay for the public good. An alternative would be to set the total value of public goods equal to the surplus consumers receive relative to zero government provision. Consider, however, the case of food: consumers would die without food, yet the income necessary to feed them is not infinity, but the income required to buy the food. Property two fixes the income equivalent of total consumer benefits at the cost of providing the public good. Although this property is not intrinsic to the Lindahl scheme it is commonly assumed to hold.[8]

The third property is that all consumers benefit from - or, more specifically, that none are harmed by - lower costs of providing public goods.

Together, these properties imply that benefits equal B_i, when:

$$\Psi^i(\mathbf{p},\ y_i,\ G^*) \;=\; V^i(\mathbf{p},\ \rho^*,\ y_i + B_i) \qquad \forall \quad i \tag{3}$$

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[8]This property implicitly assumes that every additional unit of public good creates an obligation that must be met with benefit taxes, hence measured benefits must rise dollar for dollar with expenditures on public goods. Clearly such a method is inappropriate for cost/benefit calculations that seek to measure the benefits of additional spending. However, for the purpose of distributing a fixed amount of benefits between different consumers, this property is required for budget balance.

in which ρ^* is common for all consumers, and chosen so that the sum of the B_is equals the cost of providing G*. In equation (3), ρ^* is the (possibly nonlinear) common price schedule for public goods; as will be explored shortly, however, consumers do not actually receive the levels of G that they would demand at the common price ρ^*.

The scheme of ρ^* and B_is described by (3) satisfies the second property by construction and must satisfy the first since indirect utility functions are increasing in income. But more than one scheme satisfies (3).

With linear pricing benefits are measured as if each consumer could purchase as much of the public good as he wants at (linear) price ρ^*. Since he cannot do so - instead, the government provides G* for free (once taxes are paid) - he is compensated for the difference between the utility he would have obtained in a market for G at price ρ^* and the utility produced by government provision of G* for free.

To calculate this benefit, note that the consumer's utility can (from (1)) be represented by $V^i(\mathbf{p}, \rho_i, y_i + M_i)$, in which ρ_i is his personalized price; this utility level also equals $V^i(\mathbf{p}, \rho^*, y_i + B_i)$. Denote the consumer's utility level U. Since, from the expenditure function identity, $E^i(\mathbf{p}, \rho_i, U) = y_i + M_i$ and $E^i(\mathbf{p}, \rho^*, U) = y_i + B_i$, in which $E^i(\cdot)$ is consumer i's expenditure function, then:

$$B_i = M_i + E^i(\mathbf{p}, \rho^*, U) - E^i(\mathbf{p}, \rho_i, U) \tag{4}$$

thereby illustrating the difference between benefits as calculated by the linear pricing method (B_i) and those calculated by the Lindahl method (M_i). Applying the fundamental theorem of the calculus to the right side of (4),

$$B_i = M_i - \int_{\rho^*}^{\rho_i} \frac{\partial E^i(\cdot)}{\partial \rho} dz = M_i - \int_{\rho^*}^{\rho_i} G^c(\mathbf{p}, z, U)\, dz \tag{5}$$

in which G^c denotes compensated demand and the last is derived from the identity that $\partial E^i(\cdot)/\partial \rho = G^c(\cdot)$.

Figure one illustrates the application of (5) in a market with two consumers. D_1 in Figure 1 represents consumer one's Marshallian demand for G, while D_1^c is his compensated demand curve at the utility he would have obtained if able to consume Q_1 units of G at price ρ^*. Analogous schedules are presented for consumer two. In the linear pricing interpretation of public good benefits, as described by (5), consumer one's benefit equals the area of the rectangle JKEB minus the area of the trapezoid underneath the compensated demand curve as the price falls from ρ_1 to ρ^*; the difference is the area of the rectangle AFEB minus the area of the shaded triangle KDF. Consumer two's benefit is the area of LMEB plus the area under the compensated trapezoid AHML; hence the total is the area of AFEB minus the area of the shaded triangle HFM.

How is ρ^* determined? The second requirement is that benefits sum to the cost of providing the public good; it is met by adjusting ρ^* upward to raise benefits or downward to lower them. This property follows from (3): fix the utility level $\Psi^i(\cdot)$ on the left side, and since $\partial V^i(\cdot)/\partial \rho^* < 0$ for every i by virtue of the fact that private individuals are consumers and not suppliers of public goods, and $\partial V^i(\cdot)/\partial B_i > 0$, it is clear that $dB_i/d\rho^* > 0$ for every consumer and hence for the sum of consumers as well. Since benefits are monotonically increasing in ρ^*, their sum reaches its maximum at $\rho^* = \infty$ and its minimum at $\rho^* = 0$. At $\rho^* = 0$ every $B_i \leq 0$, since consumers always prefer to choose as much as they want at zero price rather than consume G^* units of the public good. At $\rho^* = \infty$ consumers demand zero public goods; hence B_i equals the amount consumer i is willing to pay in order to have the government provide G^* rather than zero units of G. Continuity of demand then implies that linear pricing can support any provision level at which costs do not exceed the sum of consumer benefits.

Different levels of G imply different ρ^*s, of course. It is straightforward to show that $d\rho^*/dG < 0$ when the public good is "underprovided" in the sense that the sum of consumers' marginal consumption benefits exceeds the marginal cost of the public good - and that $d\rho^*/dG > 0$ when it is overprovided. Consider a small variation in G, holding ρ^* constant. From (3), $[\partial V^i(\cdot)/\partial(y_i + B_i)]\ dB_i/dG = \partial\Psi^i(\cdot)/\partial G$. Since $\partial V^i(\cdot)/\partial(y_i + B_i) = \partial\Psi^i(\cdot)/\partial y_i$, $dB_i/dG = (\partial\Psi^i(\cdot)/\partial G)/(\partial\Psi^i(\cdot)/\partial y_i)$, or $dB_i/dG = \rho_i$. If the sum of ρ_is, the marginal valuations of the public good, exceeds its marginal cost, then by holding ρ^* constant an increase in G raises the sum of B_is by more than it raises the total cost of providing the public good. Since benefits must sum to costs, and $dB_i/d\rho^* > 0$, then higher G must therefore be accompanied by a reduced ρ^* when pubic goods are underprovided. If instead the marginal cost of providing the public good exceeds the sum of the ρ_is, so the public good is overprovided, then by analogous reasoning $d\rho^*/dG > 0$.

The solid line in figure 2 illustrates a typical ρ^* - G locus. At every level of G < G' the public good is underprovided, while for G > G' the public good is overprovided.

Benefit taxation describes an outcome in which financing of the public good implies that $T_i = B_i$ for each consumer. From (3), consumer i's utility under benefit taxation equals $V^i(\mathbf{p}, \rho^*, Y_i)$, recalling that Y_i is his before-tax income. Since $\partial V^i(\cdot)/\partial\rho^*$ is always nonpositive, <u>every</u> consumer prefers the government to adopt policies that minimize ρ^*; any other choice is Pareto-inferior. In figure 2, G' is the optimal provision level. This method of benefit taxation would, therefore, resolve conflicts over expenditure policy, since it is in every individual's interest that the government chooses an efficient expenditure level.

Linear pricing is not the only application of (3). Consider, for example, the egalitarian-equivalent scheme: it can be interpreted as a nonlinear pricing method in which ρ^* is zero for the first g^* units of the public good and infinite thereafter. Assuming that, if faced with such a price schedule, every consumer would demand g* units of public goods, then, from (3), B_i must satisfy $\Psi^i(\mathbf{p}, y_i, G^*) = \Psi^i(\mathbf{p}, y_i + B_i, g^*)$. As long as consumers have positive marginal valuations of the public good, and g* is chosen so that the B_is sum to the cost of provision, then the egalitarian-equivalent mechanism satisfies both criteria and represents one form of (3).

This interpretation of the egalitarian-equivalent mechanism illustrates the role played by the assumption that consumers have positive marginal valuations of the public good. A consumer with a negative marginal valuation of the public good at provision level g* would not choose to purchase g* units if faced with a price schedule of $\rho^* = 0$ for all units up to g* and $\rho^* = \infty$ thereafter. Instead, such a consumer would either purchase zero units of the public good, or else purchase an amount at which his marginal valuation were zero. In either case, government provision of g* units implies that such consumers, who are unable to dispose of the public good, are made worse off by higher provision levels and better off by lower provision levels. In these circumstances, consumers facing tax burdens determined by the egalitarian-equivalent mechanism will not unanimously prefer a single public good provision level.

Moulin demonstrates that the egalitarian-equivalent mechanism (2), if used to assign taxes equal to benefits, has the feature that a reduced cost of providing the public good, holding the provision level fixed, results in a Pareto improvement. The reason is that reduced costs of public good provision are associated with higher levels of g*. Consumers with negative marginal valuations of the public good are, however, thereby made worse off, so cost reductions do not generate Pareto improvements under all preference structures.

Linear pricing of public goods implies that cost reductions generate Pareto improvements under any specification of consumer preferences. With linear benefit taxation every consumer's utility equals $V^i(\mathbf{p}, \rho^*, Y_i)$; hence if ρ^* increases with a reduction in costs all consumers are worse off and if ρ^* decreases then all consumers are better off. But a cost reduction implies a total tax reduction, so at least one consumer is better off, since taxes must be reduced on *someone* while provision of public goods stays constant. Hence ρ^* must decline with a cost reduction, so a reduced cost of providing the public good shifts the ρ^* - G locus in figure 2 from the solid line to the dotted line, improving the welfare of every consumer at every level of public good provision. More generally, any linear or nonlinear pricing variant of (3) in which cost reductions are associated with reduced ρ^* at all public good provision levels imply that reduced costs generate Pareto improvements.

Moulin demonstrates that the mechanism (2) is a core selection: if the government chooses an efficient quantity of public goods along with benefit taxes then no subset of consumers can form a superior coalition. This result requires that marginal valuations of the public good always be nonnegative, since if $B_i < 0$ then consumer i receives a subsidy, and a coalition of all other consumers can do better by excluding him. The nonlinear pricing interpretation of the egalitarian-equivalent mechanism is somewhat more general, since it permits negative marginal valuations - and in some cases this removes the core property of the outcome. The linear pricing allocation also is not always in the core, since (as illustrated by the example in section 4) benefit taxes on some consumers may be negative, so other consumers would do better by excluding those they subsidize. Given the very real possibility of negative marginal valuations of public goods, there is a fundamental inconsistency between the core property and the requirement that consumers not be harmed by reduced costs of public good provision.

Which of these methods is the most compelling way to measure public good benefits and to assign taxes corresponding to benefits? The goal of benefit taxation is to allocate taxes in a manner akin to market prices for public services; since no markets exist,[9] it is necessary to mimic a market's properties. The law of one price is a conspicuous feature of markets, but the difficulty with its application to public goods is that at identical prices consumers generally select different quantities of public goods. The linear pricing method of benefit taxation ensures that every consumer obtains the utility he would have achieved if public goods had, in fact, been available at linear prices. Alternatives such as Lindahl pricing or the egalitarian-equivalent mechanism do not appear to distribute tax burdens in ways that share such similarities to market processes.[10]

4. Benefit Taxes and Redistribution in Practice.

It is instructive to apply the linear pricing method of benefit taxation to models and data that previous authors analyze using the Lindahl method. Consider a model with a single private good with price normalized to unity and consumers with identical utility functions of the form: $U^i = y_i^{\alpha} G^{1-\alpha}$. Suppose that the government provides Γ units of the public good at unit cost. The Lindahl price for consumer i, ρ_i, is $[\partial U^i/\partial G]/[\partial U^i/\partial y_i] = (1-\alpha)y_i/\alpha\ \Gamma$. Benefits as measured by the Lindahl method, M_i, equal $\rho_i\Gamma$, or $y_i(1-\alpha)/\alpha$. Note that these benefits are unaffected by the public good provision level.

To apply the linear pricing method it is necessary to choose a ρ^* that generates a sum of B_is equal to Γ. The constant shares property of Cobb-Douglas demand functions yields the indirect utility function:

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[9]Private markets can provide public goods in some special circumstances, of course. See, for example, Demsetz (1970), Oakland (1974), Brito and Oakland (1980), Brennan and Walsh (1981), Burns and Walsh (1981), Bergstrom, Blume and Varian (1986), Andreoni (1988), Varian (1994), Buchholz and Konrad (1995), Bilodeau and Slivinski (1996) and Fraser (1996).

[10]Other methods of distributing the cost of public good provision include those analyzed by Moulin (1990), Otsuki (1992), Moulin and Shenker (1992), and Moulin (1994), none of which embody the features of the allocation characterized by (3).

$$V^i(1,\ \rho^*,\ y_i + B_i) \quad = \quad (1-\alpha)^{1-\alpha}\ \alpha^{\alpha}\ (y_i + B_i)\ \rho^{*\alpha-1} \tag{6}$$

for any ρ^*. Consumer i's expenditure function is:

$$E^i(1,\ \rho_i,\ U) \quad = \quad U\ (1-\alpha)^{\alpha-1}\ \alpha^{-\alpha}\ \rho_i^{1-\alpha} \tag{7}$$

for consumers facing personalized prices ρ_i. Substituting (6) into (7):

$$E^i[1,\ \rho_i,\ V^i(1,\ \rho^*,\ y_i + B_i)] \quad = \quad (y_i + B_i)\ (\rho_i/\rho^*)^{1-\alpha} \tag{8}$$

Then combining (8), (4), and the definitions of ρ_i and M_i yields:

$$y_i + B_i \quad = \quad [\alpha^{-\alpha}(1-\alpha)^{\alpha-1}\ \Gamma^{1-\alpha}\ \rho^{*1-\alpha}]\ y_i^{\alpha} \tag{9}$$

Solving for ρ^* by summing (9) over i and imposing that benefits sum to the cost of the public good, Γ, yields:

$$B_i \quad = \quad \{[\Gamma + \sum_{i=1}^{n} y_i]/[\sum_{i=1}^{n} y_i^{\alpha}]\}\ y_i^{\alpha} \quad - \quad y_i \tag{10}$$

It is straightforward to use measured benefits from (10) to calculate individual tax obligations under benefit taxation. Since B_i in (10) represents consumer i's public good benefits, and in a system of benefit taxes the government sets $T_i = B_i$, then with benefits measured by linear prices (and recalling that $Y_i - T_i = y_i$), one obtains:

$$T_i \quad = \quad Y_i - Y_i^{1/\alpha}\ \mu \tag{11}$$

in which μ is constant across consumers, chosen to balance the budget with taxes set according to (11). Since the average tax rate (T_i/Y_i) from (11) is a decreasing function of Y_i, benefit taxes are regressive. Furthermore, tax burdens rise with income only up to the point at which $Y_i = [\alpha/\mu]^{\alpha/(1-\alpha)}$, after which taxes actually decline with income and become <u>negative</u> at incomes exceeding $\mu^{\alpha/(\alpha-1)}$.

This dramatic regressivity of benefit taxes is the outcome of the government's commitment to provide each consumer the utility he would obtain if able to purchase public goods at a linear price of ρ^*. With Cobb-Douglas preferences G is a normal good, so the wealthiest consumers, if offered unrestricted quantities at price ρ^*, would demand much more of it than the government is likely to provide. Under benefit taxation the tax system performs two roles: it raises revenue sufficient to pay for the public good, and it redistributes income to compensate consumers for differences between actual and desired levels of public good provision. The wealthiest consumers will receive the greatest compensation if this difference is, for them, sufficiently large.

In order to illustrate the novel concept of benefits captured in the measure given by (10), it is instructive to compare it to Aaron and McGuire's estimates of net fiscal incidence for the United States in 1961. Aaron and McGuire use Lindahl valuations of public goods to distribute their benefits among consumers in 1961; they assume that consumers have Cobb-Douglas preferences of the form just analyzed, that public goods are efficiently provided, and that they are produced at constant cost. Their data from the Tax Foundation (1967) allocates tax obligations and benefits from "specific" government expenditures to nine different income groups in the United States in 1961.[11]

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[11]Since in practice the government devotes only a fraction of its budget to public goods provision, it is important to identify the total quantity of public goods provided and to assign other expenditures to the consumers who benefit from them. Aaron and McGuire add imputed externalities from some government expenditures to numbers calculated by the Tax Foundation to arrive at a figure for "high total quantity of public goods"; by this calculation,

(continued...)

Table 1 presents two measures of the distribution of public good benefits in 1961: Aaron and McGuire's calculation, and benefits calculated as in (10) from the linear pricing model. These benefits are calculated for representative indiviuals in each of nine income categories, and can be compared to the actual tax burdens (net of specific benefits).

As calculated, the numbers reveal the tax code to be sharply progressive. Particularly striking are the heavy taxes paid by the wealthiest taxpayers: in return for -$852 in public good benefits, they pay on net $15,363 per family. Benefits rise and then decline with income, reflecting the low money values poor consumers attach to public goods and the losses felt by rich consumers who value public goods highly but whose government does not provide them. Aaron and McGuire, using the Lindahl method, conclude that benefits from public goods rise with income. Nonetheless they find the 1961 fiscal system to be progressive, but much less strongly so than the benefit calculation from (10) indicates.

This exercise represents only one interpretation of the pattern of redistribution in the United States in 1961, since it rests on the untested assumption that consumers have Cobb-Douglas utility functions. Still, it illustrates the general point that governments cannot provide every consumer with the quantity of public goods he desires, and that, in compromising, governments impose costs on those consumers whose demands are least well met. This paper offers a new method of measuring benefits that incorporates such costs.

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the U.S. government spent $106.6 billion on public goods out of a total national income of $474.8 billion. The Tax Foundation assigns "specific" benefits, such as transfer payments, to income groups on the basis of group income and characteristics such as average family sizes.

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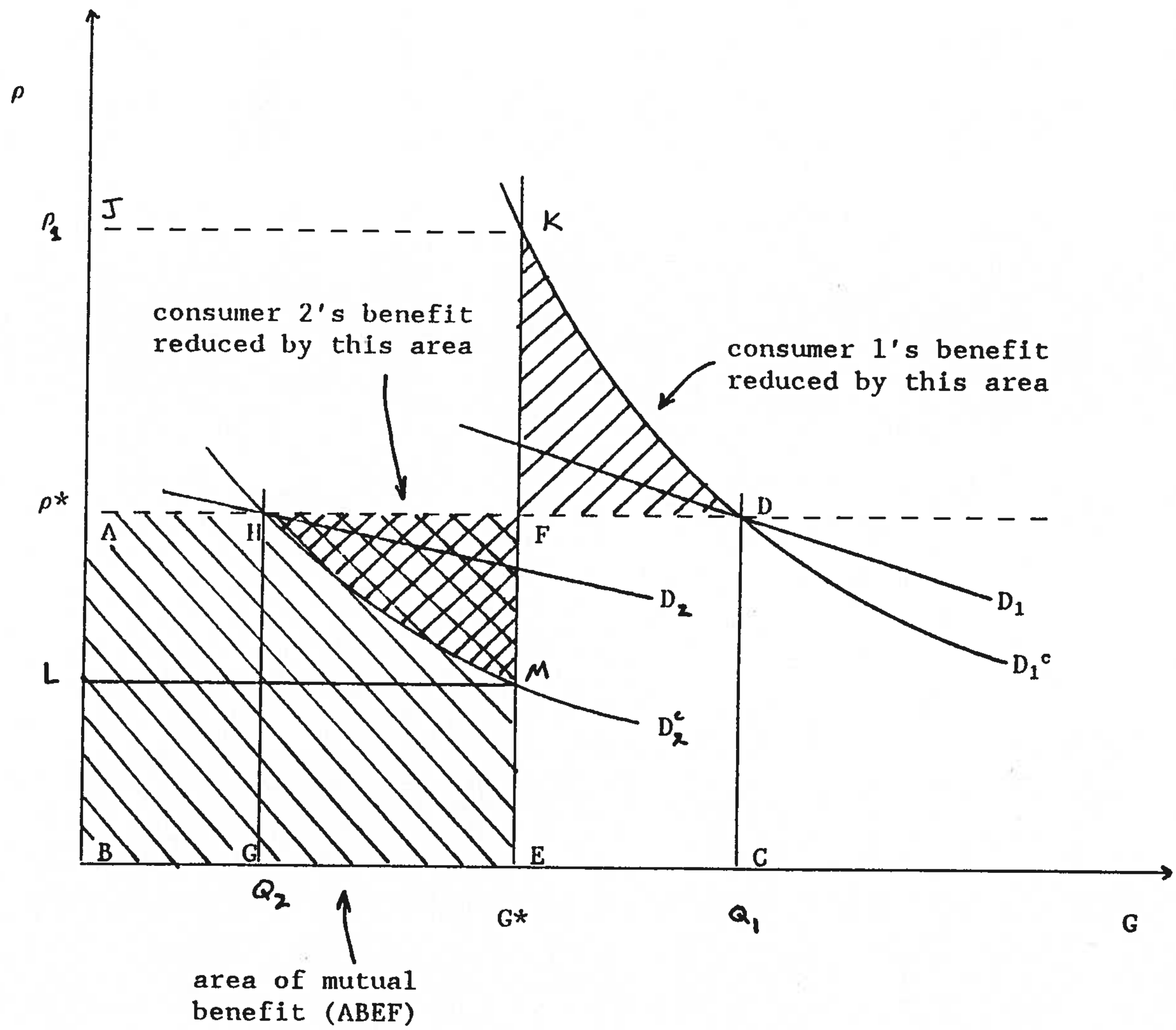
Figure 1

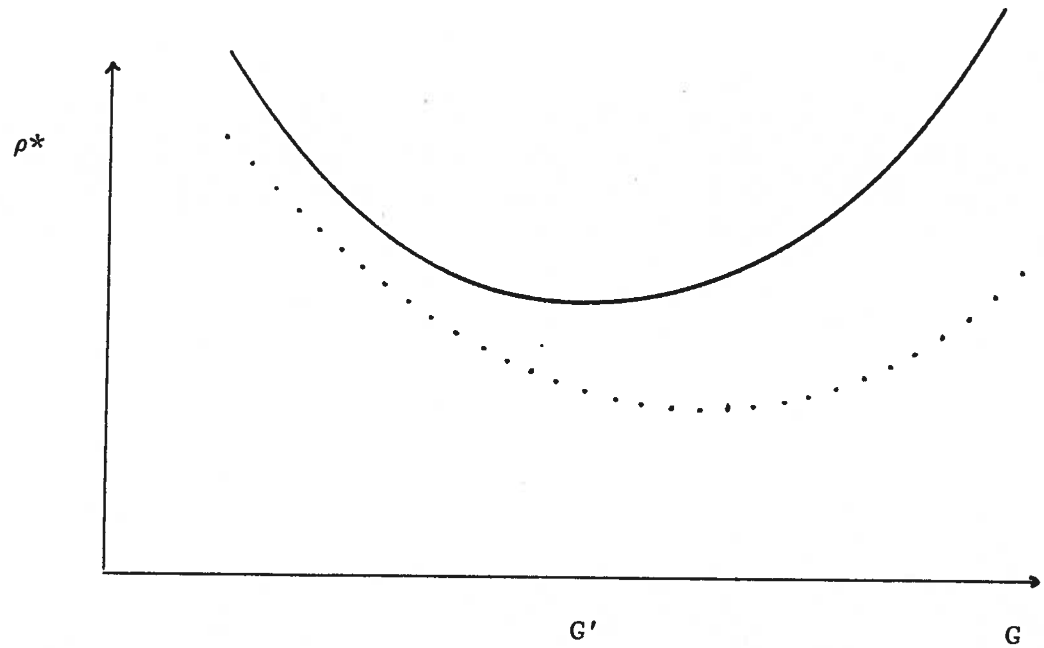
Benefit Calculation with Linear Pricing



Note to Figure 1: The figure depicts benefit calculations using the linear pricing method described in the text. D_1 represents consumer one's Marshallian demand for G, while D_1^c is his compensated demand curve at the utility he would have obtained if able to consume Q_1 units of G at price ρ^*. Consumer one's benefit equals the area of the rectangle JKEB minus the area of the trapezoid underneath the compensated demand curve as the price falls from ρ_1 to ρ^*; the difference is the area of the rectangle ABEF minus the area of the shaded triangle KDF. Consumer two's benefit is the area of LMEB plus the area under the compensated trapezoid AHML, which sums to the area of ABEF minus the area of the shaded triangle HFM.

Figure 2

The Relationship between ρ* and G with Linear Pricing



Note to Figure 2: The Figure illustrates the effect of different levels of G on implied values of ρ* in the linear pricing method described in the text. For the preference and technology combination described by the solid locus in Figure 2, the public good is underprovided (in the sense that the sum of marginal consumer valuations of the public good exceed the marginal cost of providing the public good) at any level below G', and is overprovided at any level above G'. The value of ρ* is minimized at the efficient public good provision level, G'.

The dotted locus in Figure 2 depicts the effect of a technology improvement that reduces the cost of providing any level of the public good. Such a cost reduction lowers the value of ρ* associated with each level of G, and thereby improves the welfare of all consumers.

Table 1

Two Interpretations of Net Fiscal Incidence in the United States, 1961

Mean Pretax Family Income	Public Good Benefits: Lindahl Pricing (Aaron/McGuire, 1970)	Public Good Benefits: Linear Pricing (Equation 10)	Taxes Paid (Net of "Specific" Benefits)
$1,046	$375	$1,243	$ - 265
2,801	798	1,797	12
4,674	1,151	2,069	652
6,561	1,503	2,239	1,309
8,328	1,875	2,341	1,775
10,148	2,321	2,381	2,305
13,482	2,971	2,308	3,101
19,453	4,207	1,908	4,481
44,520	8,344	- 852	15,363

Note: All entries are dollars per year per family. Column one presents mean family income for each of ten deciles in 1961. Column two indicates public good benefits calculated by Aaron and McGuire (1970) based on Lindahl pricing. Column three describes public good benefits based on the linear pricing method presented in the text. Column four presents taxes paid (net of "specific" benefits received from government spending) for each decile in 1961 as calculated by the Tax Foundation (1967).