TAX-REVENUE VOLATILITY

NATHAN SEEGERT

Abstract. Governments around the world are experiencing budget crises. The severity of these budget crises are magnified by the volatility of their tax revenue which has increased dramatically in the last decade. For example, U.S. state governments have recently experienced a 500 percent increase in volatility of their tax revenue, relative to previous decades. The inability of state governments to smooth volatile revenue streams causes them to make large cuts in expenditures. The theoretical model demonstrates that tax-revenue volatility is a function of three sets of factors; business cycle factors, tax policy factors, and tax base factors such as the level of taxable consumption. I adapt empirical decomposition methods by Oaxaca (1979), Blinder (1979), and Dinardo, Fortin, and Lemieux (1996) to quantify the contribution of these three groups. The key insight is by adapting these methods I am able to quantify tax base changes, which I do not observe, in a similar way as they quantify discrimination in pay or the effect of unions, which are also unobserved. Despite amplified business cycles in the 2000s and important changes affecting tax bases such as e-commerce, I find that changes in tax policy explains 70 percent of the increase in tax-revenue volatility. Motivated by this result I create a normative model of taxation to determine the optimal tax policy when volatility of tax revenue is considered. The theoretical model produces a volatility adjusted Ramsey rule that can be estimated to determine whether a state’s tax portfolio is balanced or overweight the income or sales tax. I find that 12 states overweight the sales tax and 16 overweight the income tax and an imbalanced tax portfolio is positively correlated with increased volatility in the tax revenues in the 2000s. Therefore, I find strong evidence the increase in tax-revenue volatility state government’s recently experienced is due to changes in state tax portfolios that have caused them to expose their revenues to unnecessary levels of risk.

JEL Numbers: H21, H7, H68, R51

I would like to thank seminar participants at the Michigan Public Finance Seminar for providing valuable feedback, and in particular Jim Hines, David Albouy, Steve Salant, Scott Page, Joel Slemrod, and Dan Silverman for their help and advice. Any mistakes are my own. Please contact the author at seegert@umich.edu.
Governments around the world are experiencing budget crises. The severity of these budget crises are magnified by the volatility of their tax revenue which has increased dramatically. For example, U.S. state governments have recently experienced a 500 percent increase in volatility of their tax revenue, relative to previous decades. The inability of state governments to smooth volatile revenue streams causes them to make large cuts in expenditures.\textsuperscript{1} The nearly $20 billion increase in average yearly shocks to U.S. state tax revenues increase uncertainty in the economy and increase uncertainty of individuals’ tax payments.\textsuperscript{2} This paper analyzes the increase in tax-revenue volatility experienced by U.S. state governments and the policy mechanisms that exist to stabilize tax revenues.

The theoretical model demonstrates tax-revenue volatility is a function of three sets of factors; business cycle factors, tax policy factors, and tax base factors such as the level of taxable consumption. I collect data on tax policy and business cycle factors but am unable to observe tax base factors because they are a complex combination of changes in the economy, such as the increase in e-commerce, and tax law. I adapt empirical decomposition methods by Oaxaca (1979), Blinder (1979), and Dinardo, Fortin, and Lemieux (1996) to quantify the contribution of these three groups. The key insight is these methods, appropriately adapted, allow me to quantify tax base changes, which I do not observe.\textsuperscript{3} Despite amplified business cycles in the 2000s and important changes in the economy such as e-commerce, I find changes in tax policy explain a significant proportion of the increase in tax-revenue volatility.

The empirical decomposition conducts policy experiments to address the question “What would tax-revenue volatility in the 2000s have been under the tax policies of prior decades?” Intuitively, the empirical decomposition uses these policy experiments to quantify the contribution of each of the three groups of factors. Theoretically, they motivate the question, “What should tax policy be when the welfare costs of volatility in tax revenue is considered?”

\textsuperscript{1}State governments’ inability to smooth volatile revenue is a consequence of self-imposed balanced budget rules which all states impose, though with varying strictness.

\textsuperscript{2}The second of four maxims described by Adam Smith with regards to taxation is certainty. He claims, “The certainty of what each individual out to pay is, in taxation, a matter of so great importance, that a very considerable degree of inequality, it appears, I believe, from the experience of all nations, is not near so great an evil as a very small degree of uncertainty.” The Wealth of Nations p. 778.

\textsuperscript{3}In other contexts, the change I quantify as tax base changes is the structural change or “treatment effect” in the empirical decomposition.
To determine the optimal tax policy when volatility of tax revenue is considered I develop a normative model of taxation. Standard optimal taxation models constrain a welfare-maximizing government to collect an exogenous level of expected revenue, irrespective of the variance.\textsuperscript{4} In my model the variance of tax revenue enters the government’s objective function because the representative individual has utility over a public good financed by tax revenues.\textsuperscript{5} The normative model produces conditions for the optimal mixture of tax rates across the income, sales, and corporate tax bases.

The government distributes the production risk in the economy between public and private consumption by setting different tax rates on different tax bases. By changing the tax bases the government relies on it is able to absorb some of the production risk at the cost of increased volatility in the public good but to the benefit of lower volatility in private consumption. Lump sum taxes which bring in certain levels of tax revenue are not optimal because they cause the production risk to be concentrated in the private sector. The government by taxing income and consumption is able to absorb some of the production risk but with the additional cost of deadweight loss. Therefore, the efficient tax rates the government sets must trade off the benefits and costs of volatility with the additional cost of deadweight loss. In this tradeoff the government makes, volatility is of first-order importance while deadweight loss is of second order.\textsuperscript{6} If a government is able to decrease the costs of volatility and deadweight loss without affecting the expected level of tax revenue the tax portfolio held by the government is imbalanced.

Empirically, state government tax portfolios have changed over the last 60 years, with an increased reliance on personal income taxes. For example, in 1951 total sales taxes accounted for 59 percent and personal income accounted for 9 percent of total tax revenue aggregate across all states. In 2010 total sales taxes accounted for 49 percent of total taxes and personal income for 34 percent.\textsuperscript{7}

\textsuperscript{4}Constraining the welfare-maximizing government to collect an exogenous level of expected revenue abstracts from costs from higher moments including variance. The focus of this paper is the variance but the analysis is done for higher moments in the appendix.

\textsuperscript{5}The model allows governments to smooth some of the volatility of tax revenue. This model nests the standard optimal taxation model because in the limit when the government is able to perfectly smooth tax revenue this model reduces to a standard optimal taxation model.

\textsuperscript{6}Seegert (2011) demonstrates volatility is of first-order importance by taking a Taylor series expansion of the expected utility function of the representative individual’s utility (the government’s objective function).

\textsuperscript{7}General sales taxes, a subset of total sales taxes, changed from 22 percent in 1951 to 32 percent in 2010. Total sales tax includes taxes such as alcohol and cigarette taxes.
In aggregate these changes appear to have resulted in more balanced state tax portfolios. However, at the state level my research indicates some states still rely too heavily on sales taxes and some have over-corrected and now rely too heavily on income taxes.

To determine if state tax portfolios are balanced, or overweight one of the tax bases, I estimate this sufficient condition derived from the volatility adjusted Ramsey rule. I find 12 states overweight the sales tax, 16 overweight the income tax, and 20 have balanced portfolios.\footnote{Alaska and New Hampshire are not included because they do not have an income or sales tax.} I also find an imbalanced portfolio is positively correlated with increased volatility in the tax revenues in the 2000s, while having a balanced portfolio is negatively correlated. This research finds strong evidence the 500 percent increase in tax-revenue volatility state government’s recently experienced is due to changes in state tax portfolios that have caused them to expose their revenues to unnecessary levels of risk.

\section{1. Literature Review}

This paper is related to literatures on optimal taxation, taxation with uncertainty, and optimal tax portfolios. How to finance public goods is a classic problem in public finance. The literature has progressed extending the basic model that minimizes aggregate deadweight loss for a given level of tax revenue to account for distributional considerations \cite{Mirrlees1971}, externalities and complementarities \cite{CorlettHague1953,DiamondMirrlees1971,GreenSheshinksi1974}, administrative costs and tax avoidance \cite{AllinghamSandmo1971,Yitzhaki1974,AndreoniErardFeinstein1998}, and dynamic considerations \cite{Chamley1986,Judd1985,Summers1981}. My paper extends the basic model by considering costs from volatility. In this environment lump sum taxes do not produce the Pareto optimum and distortionary taxes can be more efficient.

My paper considers the cost of volatile tax revenue when uncertainty exists which is related to work on optimal taxation with uncertainty. Stigltiz \cite{Stiglitz1982} demonstrates using random tax rates can decreases excess burden, if the excess burden for an individual as a function of revenue raised is concave. Barro \cite{Barro1979} making the assumption excess burden is convex in the amount of revenue raised demonstrates the expected value of a tax rate tomorrow should be equal to the current tax rate. Skinner \cite{Skinner1988} demonstrates the welfare gain from removing all uncertainty about future tax
policy is 0.4 percent of national income.\textsuperscript{9} This literature focuses on the accumulated deadweight loss occurring with uncertainty. My paper’s focuses on the costs of volatility in public and private consumption as a result of tax policy and trading these costs off with the costs from deadweight loss.

My paper is also related to the optimal tax portfolio literature started by Groves and Kahn (1952) which has focused on U.S. state governments. This literature considers the choice of tax rates as an optimal portfolio problem for the government. Early work focused on the short-run elasticity of different tax revenue streams with respect to personal income (Wilford, 1965; Legler and Shapiro, 1968; Mikesell, 1977) which is a measure of variability. Later work considered growth in revenues or the long-run elasticity of different tax revenue streams with respect to personal income (Williams et al., 1973; White, 1983; Fox and Campbell, 1984). Recent work has focused on improving these estimates (Dye and McGuire, 1991; Bruce, Fox, and Tuttle, 2006). My paper extends Groves and Kahn’s theoretical model to produce a volatility-adjusted Ramsey rule.

2. DESCRIPTIVE STATISTICS

This section describes the data and demonstrates state tax revenue became more volatile in the 2000s compared to earlier decades. Figures 1 and 2 demonstrate the increase in volatility in the 2000s by graphing tax revenue aggregated across states and its deviations from a time trend.\textsuperscript{10} For the rest of the paper, I define volatility as the squared deviations from trend which is a short-run measure of variability.\textsuperscript{11} This measure produces a data point for each state-year observation. The absolute value of deviations from trend increased by $19.1 billion in the 2000s and volatility increased by $712 billion.

\textsuperscript{9}Kaplow 1986b and Weiss 1976 are other important papers in the literature of random taxation.
\textsuperscript{10}The time trend is estimated using a cubic time trend.
\textsuperscript{11}Volatility in variable $x$ is defined as $\tilde{x} = (x_t - \bar{x}_{\text{time trend}})^2$. This measures the short-run variability which is the focus of the paper. The variance of tax revenue, $\sigma_R = (R_t - \bar{R})^2$ conflates short-run variability and differences due to a time trend. For example, making a state’s time trend steeper would increase the variance but would not change the short-run variability. The time trend, estimated for each state separately, in the baseline case is a cubic time trend. The results are robust to different time trends including a Hodrick-Prescott filter with a bandwidth of 6.25, as recommended by Ravn and Uhlig (2002) for yearly data, which is shown in table 3 and time trends with autoregressive processes, semi-parametric power series estimators, and moving averages.
Tax-revenue volatility increased per person in the 2000s for all fifty states, in levels. As a percent of tax revenue, volatility increased in the 2000s for forty five states mapped in figure 4. Finally, if state tax revenues became more correlated in the 2000s this could explain the increase in volatility. However, as figure 3 demonstrates tax revenues became less correlated in the 2000s. Figure 3 graphs the moving average of the coefficient of variation across states for each year between 1951 and 2010, demonstrating tax revenues began converging in the 1960s but in the late 1990s began diverging.\footnote{The moving average uses a seven year window on either side and includes the specific year.}


The increase in tax-revenue volatility is especially important for state governments because of their self-imposed balanced budget rules. The rules differ in strictness and in some cases restrict the use a rainy day funds to smooth volatile revenue streams. The inability of state governments to smooth volatile tax revenues is demonstrated in figure 5 which plots the deviations from trend of aggregate state expenditures and tax revenues. Tax-revenue volatility leads expenditure volatility, which is confirmed by a Granger causality test.\footnote{While tax-revenue volatility Granger causes expenditure volatility the reverse is not true.}
state level because due to prior commitments expenditure volatility is concentrated in a few items such as education, the timing causes state expenditures to be pro-cyclical which is costly to the extent state expenditures should be counter-cyclical, and swings in state government expenditures adds salient uncertainty to the economy.
To empirically decompose this increase in tax-revenue volatility data on tax policies and economic conditions are collected. Income, corporate, and sales tax rates for all states between the years 1950-2010 are collected from the Book of States, the World Tax Database, the Advisory Commission on Intergovernmental Relations biannual report “Significant Features in Fiscal Federalism,” and the Tax Foundation. Data on tax revenues for all states and years 1950 through 2010 are collected from the Book of States and the U.S. Census of Governments.\textsuperscript{16} Data on state level economic conditions

\textsuperscript{16}Approximately a dozen inconsistencies between the Book of States and the U.S. Census of Governments were found. When inconsistencies were found the data from the Book of States is used, though the analysis is robust to using the other sources.
such as state level GDP and personal income are collected from the Bureau of Economic Analysis and exist for all states in all years between 1963 and 2010.

Table 1 and figure ?? demonstrates the frequency and balance of the 1108 tax rate changes across 3000 state-year observations in the sample. These changes are roughly evenly divided between the tax bases; the sales tax rate changes the fewest times (252 times) and the top income tax rate changes the most (326 times). Of these changes, 603 are tax rate increases and 505 are tax rate decreases. The tax rate changes are spread across the years in the sample such that there is a tax rate change by at least one state for each tax base in over ninety-percent of the years observed. Furthermore, in about half of the years observed, at least one state increases a given tax rate and another state decreases the same tax rate.

Despite the political climate figure ?? provides little evidence state governments changed tax rates fewer times in the 2000s relative to other decades. The number increase and decreases in tax rates are misleading because tax rate increases tend to be larger than tax rate decreases. For example, the bottom income tax rate is increased 130 times and decreased 128 times but the average tax rate increased from 3.15 in 1950-2000 to 4.81 in 2000-2010. Similarly the top income tax rate increased from 4.86 to 5.34, the sales tax from 3.15 to 4.81, and the corporate rate from 5.18 to
Table 1. State Tax Rate Changes

<table>
<thead>
<tr>
<th>Tax Rate</th>
<th>Observations</th>
<th>Years with</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Changes</td>
<td>Increases</td>
</tr>
<tr>
<td>Sales Tax</td>
<td>252</td>
<td>214</td>
</tr>
<tr>
<td>Corporate Tax</td>
<td>272</td>
<td>94</td>
</tr>
<tr>
<td>Top Income Tax</td>
<td>326</td>
<td>165</td>
</tr>
<tr>
<td>Bottom Income Tax</td>
<td>358</td>
<td>130</td>
</tr>
</tbody>
</table>

Data 1950-2010 tax rates by state collected by author from Book of States, the World Tax Database, the Advisory Commission on Intergovernmental Relations biannual report and the Tax Foundation. The 1108 tax rate changes across 3000 state-year observations demonstrate the variation used in the empirical analysis.

6.64.17 These tax rate changes changed the relative importance of the income and sales tax in total tax revenues. For example, in 1951 total sales taxes accounted for 59 percent and personal income accounted for 9 percent of total tax revenue aggregate across all states while in 2010 total sales taxes accounted for 49 percent of total taxes and personal income for 34 percent.18

The empirical design in this paper groups observations into years before the increase in volatility and those after. The groups are defined by a structural break found using Quandt likelihood ratio (QLR) test. Formally, the QLR, or sup-Wald, test statistic identifies structural breaks without presupposing in which year they occurred by performing repeated Chow tests, typically on all dates in the inner seventy-percent.1920 The maximum QLR occurs in 2002 for the sales and corporate tax revenues and in 2000 for the income tax revenue. For all three tax revenues, the maximum QLR value (12.26 corporate, 17.78 sales, and 31.09 income) are larger than the critical value at the one percent level, 3.57. For the following analysis, the before years are defined as 1963-2001 for the sales and corporate tax revenue and 1963-1999 for the income tax revenue.

17 For more details see the online appendix where tax rates through time are graphed and discussed.
18 General sales taxes, a subset of total sales taxes, changed from 22 percent in 1951 to 32 percent in 2010. Total sales tax includes taxes such as alcohol and cigarette taxes.
19 Figure 11 in the appendix plots the QLR for the income, sales, and corporate tax revenues for all years between 1970 and 2003.
20 The inner seventy-percent of years correspond to the years between 1970 and 2003 which is the suggested amount of observations for the QLR test.
3. Model

In this section’s model of government finance the government uses taxes to produce a public good in order to maximize a representative individual’s utility in an economy with uncertainty. A technology shock generates uncertainty in the model and the representative individual has rational expectations over this shock. The extent to which the individual believes the technology shock is permanent determines the amount the individual saves, consistent with the permanent income hypothesis. The correlation between the technology shock and these beliefs determine the correlation between income, savings, and consumption in the model. The fact income and consumption are not perfectly correlated produces an incentive for the government to hedge income and consumption specific risk by taxing both sources. The purpose of this model is twofold; first to derive an equation for the variance of tax revenue which can be used in the empirical decomposition (section 4) and
second to setup a normative model to determine the optimal tax policy in the environment where tax-revenue volatility is costly (section 6).

A. Technology. The single-intermediary good, $X$, in the model is assumed to be produced by a single-input factor, $I$, and costlessly transformed into private and public consumption goods. The efficiency with which a representative firm converts the single-input factor into the intermediary-output differs with the state of nature, $\theta$.\footnote{Writing intermediate production in this way implicitly assumes an increase in the input increases the output by the same percentage in all states of nature.}

$$X(I, \theta) = \theta f(I)$$

The single-input factor is assumed to be paid its marginal product $w(\theta) = \theta f'(I)$ which is state dependent but known to the individual before their input-supply choice is made. The representative individual owns and receives the profits of the representative firm. The following analysis makes the stronger assumption of stochastic constant-returns-to-scale, which implies profits are zero in all states. This stronger assumption is made for expositional reasons to limit attention to wage income and consumption taxation; all of the results are extended to the case where profits are nonzero in the appendix.\footnote{The mean and variance of the wage are characterized by the following functions of the mean and variance of the production efficiency state of nature, $\theta$.}

$$E[w(\theta)] = \bar{w} = \bar{\theta} f'(I) \quad \sigma_w^2 = (f'(I))^2 \sigma_\theta^2$$

B. Individual Behavior. The individual has utility over the supply of the input factor $I$, the public good $G$, and total private consumption $c$, which is split between taxed goods, $\beta c$, and untaxed goods, $(1 - \beta)c$. The individual’s beliefs of whether the production shock is permanent or temporary determines the amount the individual saves. Total private consumption by the individual can be written as the difference between income, which depends on the technology state $\theta$, and savings, which depends on the belief state $\mu$. Thus the individual chooses $c$, $I$, and $\beta$ to maximize utility

$$\max_{c, I, \beta} u = U(c(\theta, \mu), \beta, G) + L(-I) \quad L' > 0 \quad L'' < 0$$

subject to

$$c(\theta, \mu) = (1 - \tau_c \beta)((1 - \tau_w)w(\theta)I - s(\mu))$$
where $\tau_c$ and $\tau_w$ are the tax rates on consumption and wage income respectively. The belief over the permanence of the technology shock determines the correlation between consumption and income, according to figure 8(b), where $y(\theta)$ represents wage income.

Utility maximization requires: i) the marginal disutility from supplying the input-factor equals the marginal utility of the income it produces and ii) the ratio of marginal utilities from total consumption $c$ and the consumption composition parameter $\beta$ is equal to the consumption tax rate times income net taxes and savings. When the consumption tax rate is zero there is no distortion between consumption goods, and the expected marginal utility with respect to $\beta$ is zero. Composing utility in terms of total consumption $c$ and a composition parameter $\beta$ simplifies the exposition of deadweight loss because $\beta$ encompasses all behavioral responses between goods.\footnote{For more details see the appendix.}

\begin{align}
U_1(c(\theta, \mu), \beta, G)(1 - \tau_c \beta)(1 - \tau_w)w &= L' \\
\frac{U_2}{U_1} &= \tau_c((1 - \tau_w)wI - s)
\end{align}

C. Government The government produces the public good $G$ and finances its production with taxes on consumption and wage income. Two assumptions are made for expository convenience: i) the supply of the public good is set equal to the tax revenue and ii) the utility function is additive
such that $U_{1,2} = 0$.\textsuperscript{24} The expected utility of the individual can be completely characterized by the moments of private and public consumption. The analysis below focuses on the first two moments, which is sufficient if the production and belief shocks have a normal joint distribution or if the utility function is quadratic, but the results are consistent with cases where expected utility is characterized by higher moments.\textsuperscript{25,26} The level of social welfare can be written as

$$ E[u] = \int U(c(\theta), G) f(C, R, \sigma^2_C, \sigma^2_R) \equiv M(C, \sigma^2_C, \beta) + G(R, \sigma^2_R) + L(-I) $$

$$ M_1 \geq 0, G_1 \geq 0, L_1 \geq 0, M_2 \leq 0, G_2 \leq 0, L_{1,1} > 0 $$

where $C$ and $R$ are the mean levels of the private and public consumption, $\sigma^2_C$ and $\sigma^2_R$ are the variances of private and public consumption respectively, and $G$ represents the expected utility from public consumption.\textsuperscript{27} The variance of tax revenue is a function of the tax rates, the tax bases, and the economic conditions.\textsuperscript{28,29}

$$ \sigma^2_R = \tau_w^2 I^2 \sigma_w^2 + \tau_c^2 \beta^2 \sigma_c^2 + 2 \tau_w \tau_c \beta I \sigma_{c,w} $$

The variance of tax revenue given in equation 4 provides a structural equation for the empirical decomposition. First, aggregate tax revenue can be decomposed into its parts; income tax-revenue volatility, sales tax-revenue volatility, and the covariance of income and sales tax revenue. Second, each of these parts can be decomposed into its parts; the tax rate, the tax base, and the economic conditions as demonstrated in equation 5 for the sales tax. In equation 5 the sales tax-revenue volatility, the sales tax rate, and the volatility of consumption are observed but the base $\beta$ is unobserved because it is a complex combination of economic conditions, tax rates, and tax laws. The base is estimated in equation 5.1 as a function of tax variables $\tau$ and business cycle variables.

\textsuperscript{24}Assuming the government must have a balanced budget abstracts away from debt issues which are not the focus of this paper. This assumption may be less of an abstraction for state governments, forty-nine of which have balanced budget requirements. In practice these balanced budget requirements do not preclude state debt but they do add additional costs. In this model the ability of the government to smooth revenue is modeled in its risk attitude.\textsuperscript{25}In the case where two moments are sufficient, the indifference curves can be shown to be quasi-concave as long as $U'' < 0$.\textsuperscript{26}Analysis in the appendix considers expected utility which is characterized by higher moments.\textsuperscript{27}The shape of $M$ can differ from the shape of $G$, allowing for different attitudes of risk in public and private consumption.\textsuperscript{28}With $I = I - \tau_w W - s$, $\sigma^2_c = \frac{1}{\theta^2} [(1-\tau_w)^2 W^2 \sigma_\theta^2 + \sigma_\theta^2 - \frac{2}{\theta^2} (1-\tau_w) W \sigma_{\theta,s}]$, $\Theta = f(I) \tau_c \beta + f'(I) I (1-\tau_c \beta) \tau_w$\textsuperscript{29}Base factors $I, \beta$ are choice variables allowed to vary with the state of nature. For expository ease they have been treated as constants but their variance can be included.
The tax variables include tax rates from other bases (to account for tax shifting), information on the tax base (such as the number of brackets in the tax schedule), and $\tau_c$. The economic variables include the volatility of state level GDP, personal income, population.

\[
\log(\sigma_{Rc}^2) = 2\log(\tau_c) + 2\log(\beta) + \log(\sigma_c^2)
\]

\[
\log(\beta) = \delta_0 + \log(\tau)\psi_1 + \log(x)\psi_2 + \nu
\]

\[
\log(\sigma_{Rc}^2) = \delta_0 + \log(\tau)\delta_1 + \log(x)\delta_2 + \varepsilon
\]

For the empirical analysis the volatility is measured as the squared deviations from trend to focus on the short-run variability, discussed previously in the descriptive statistics section. Therefore, the volatility of state level GDP included in $x$ in equation 5.2 is given by $\sigma_{gdp,t}^2 = (gdp_t - gdp_{time~trend})^2$.

4. **Empirical Decomposition**

The theoretical model demonstrates the increase in tax-revenue volatility is due to changes in tax rates, amplified business cycles, or tax base changes. Tax rates, business cycle variables, and tax revenues are observable however, tax base changes, such as the increase in e-commerce, are unobservable complicating the empirical decomposition. Adapting empirical decomposition methods pioneered by Oaxaca (1973), Blinder (1973), and DiNardo, Fortin, and Lemieux (1996) allow me to quantify tax base changes in a similar way as they quantify discrimination in pay or the effect of unions, which are also unobserved. The baseline model is estimated using a weighting method similar to DiNardo, Fortin, and Lemieux (1996) which can be thought of as a weighted extension of the decomposition method described by Oaxaca (1973) and Blinder (1973). For this reason I explain the method in terms similar to Oaxaca (1973).\textsuperscript{31}

\textsuperscript{30}The equation for the income tax base assumes the unobservable characteristics $\epsilon$ is additively separable from the observable characteristics. This assumption is loosened in the empirical decomposition by using a weighting method.

\textsuperscript{31}The weighting method, described in the appendix, is chosen as the baseline case because a test of nonlinearity in the Oaxaca (1973) estimate suggests nonlinearities exist. In this case the weighting method is preferred because it controls for nonlinearities and is asymptotically more efficient than matching or regression models, Hirano et al. (2000). In this context controlling for nonlinearities will decrease the upward bias in the structural factor estimates from the Oaxaca (1973) analysis.
Intuitively, the contribution of these three groups of factors are determined by comparing predicted tax-revenue volatility in different counterfactual scenarios. For example, the contribution of tax factors is quantified by the difference between the actual tax-revenue volatility in the 2000s with the predicted tax-revenue volatility in the 2000s if the tax factors in the 2000s were equal to their values in the previous decades.\textsuperscript{32} Similarly the contribution of amplified business cycles can be quantified using the observed difference in economic volatility. Changes in the tax base are captured by changes in the regression coefficients of the tax and business cycle factors, which are not residuals.\textsuperscript{33} The difference between the coefficients estimated in the before and after periods estimate the change in the relationship between tax-revenue volatility and the explanatory tax and business cycle factors, which is the difference in the tax base.

Equation 6 decomposes the three groups of factors where $\eta_1$ is an indicator function for the 2000s, $\eta_{\text{state}}$ indicates the state fixed effects, and $\tau$ and $x$ are matrices of all of the tax and economic factors respectively. This equation nests the following equations which estimate the volatility separately for the before and the after years denoted by $x|_0$ and $x|_1$ respectively.$^3$ In equation 6 $\delta_1 = \gamma_1$ and $\delta_2 = \gamma_2$. The coefficients on the economic and tax variables interacted with the time group dummy, $\delta_3$ and $\delta_4$, are equal to the difference between the coefficients from the two separate equations, $\gamma_1 - \phi_1$ and $\gamma_2 - \phi_2$ respectively.

\begin{align*}
\log(\sigma^2_{R_i}) &= \delta_0 + \log(x)\delta_1 + \log(\tau)\delta_2 + (\eta_1 \ast \log(x))\delta_3 + (\eta_1 \ast \log(\tau))\delta_4 + \eta_1 + \eta_{\text{state}} + \varepsilon \\
\log(\sigma^2_{R_i|_1}) &= \gamma_0 + \log(x|_1)\gamma_1 + \log(\tau|_1)\gamma_2 + \eta_{\text{state}} + \varepsilon|_1 \\
\log(\sigma^2_{R_i|_0}) &= \phi_0 + \log(x|_0)\phi_1 + \log(\tau|_0)\phi_2 + \eta_{\text{state}} + \varepsilon|_0
\end{align*}

Identifying Assumption: The conditional mean of the error is equal to zero, $E[\varepsilon|x, \tau, \eta_1, \eta_{\text{state}}] = 0$

\textsuperscript{32}Therefore the contribution of the base changes is the increase in volatility unexplained by the observed characteristics, similar to the treatment on the treated (TOT).

\textsuperscript{33}The empirical decomposition differences observable characteristics and the relationship between observable characteristics between the before and after period. The identifying assumption states differences in residuals on average cancel leaving only the observable characteristics and their relationships.

\textsuperscript{34}The before and after years represents the years before and after the structural break found by doing a Quandt likelihood ratio test. For more information on the Quandt likelihood ratio test see the appendix.
This assumption allows the counterfactual volatility to be written as $\phi_0 + E[x|\eta_1 = 1] \phi_1 + E[\tau|\eta_1 = 1] \phi_2$ because the error term conveniently drops out. The plausibility of this assumption depends on the likelihood the estimation suffers from endogenous variables or omitted variables. The panel data allows for three tests of endogenous variables which alleviate some concerns of endogenous variables. The first test includes state neighbor interacted with time fixed effects to control for time varying unobservable characteristics common across groups of states. Different groupings of states are used to control for different types of shocks.\textsuperscript{35} For example, an unobserved shock causing the volatility of both state GDP and tax revenues to increase in the Northeast is controlled for by interacting a year dummy with a geographic neighbor-state dummy.\textsuperscript{36}

The second test controls for spill-over effects by including the tax rates and volatility of economic variables from neighboring states. Finally, tax rates are lagged by two years to test whether tax rates are endogenous. The contemporaneous tax rate is highly correlated with the tax rate from two years prior but the contemporaneous volatility of tax revenue could not be used to influence the tax rate from two years earlier. Intuitively, the volatility of tax revenue is defined as the squared deviations from trend, or transitory shocks, which makes conditioning policies on them difficult.\textsuperscript{37} This dampens typical simultaneity concerns. In all three tests the estimates were similar to the baseline case alleviating some concerns of endogenous variables.\textsuperscript{38}

The estimation of the tax base creates concerns of omitted variable bias because it is a complex combination of economic factors and tax law. Each state has different definitions of what is taxable and these differences can be very nuanced.\textsuperscript{39} Time invariant tax laws are captured by the state fixed effects but all tax law changes cannot be controlled for. To determine the importance of unobserved tax law changes as omitted variables the model is run with and without the personal and corporate income tax brackets. Intuitively and statistically, the number of brackets in a given tax code is an

\textsuperscript{35}Details on the different state groupings used are provided in the appendix.

\textsuperscript{36}Unobserved shocks to an industry affecting all states with the industry are controlled in a similar manner where the neighbor-state grouping is economic. The panel data set is unable to control for a time varying state level unobservable shock.

\textsuperscript{37}As an additional robustness check an autoregressive process is estimated to filter out any time correlation leaving only transitory shocks. The estimates are reasonably robust to this specification.

\textsuperscript{38}The robustness tests are reported in the appendix.

\textsuperscript{39}For example, in Milwaukee County, Wisconsin marshmallows are subject to the local food and beverage tax unless they contain flour and in 2009 Wisconsin changed the law such that ice cream sandwiches sold in grocer’s frozen food section are no longer subject to this tax. (http://www.revenue.wi.gov/faqs/pcs/expo.html. Tax 11.51 Guidelines “Marshmallows unless they contain flour.”)
important factor in the tax base. In addition, the number of brackets is an independent variable highly correlated with other variables, specifically tax rates. The estimates with and without the tax base factors are similar, alleviating some concerns of omitted variable bias.\footnote{The robustness tests are reported in the appendix.}

The estimated difference in volatility is given in equation 7 and decomposed by rearranging terms and adding and subtracting \( \bar{x}_1 \hat{\phi}_1 + \bar{\tau}_1 \hat{\phi}_2 \), where \( \bar{x}_1 \) denotes the average value in the after period. The contribution of tax base changes is captured by the first three terms in equation 7 which encompass the change in intercept and the change in coefficients. The difference attributed to observable differences in business cycle and tax factors are captured by the fourth and fifth terms respectively.\footnote{The formulas in equation 7 are more complicated in the two robustness specifications run and reported in the appendix. First, when state fixed effects are allowed to differ between the two groups the term does not drop out. However, an F-test fails to reject the null all coefficient estimates are the same when the state fixed effects are allowed to differ. Second, when state-neighbor fixed effects are included these terms would not drop out. These additional variables would be included in the unobserved group.}

\[
\hat{\Delta} = \log(\sigma^2_{R,1|1}) - \log(\sigma^2_{R,1|0})
\]

\[
= \hat{\gamma}_0 + \log(\bar{x}_1)\hat{\gamma}_1 + \log(\bar{\tau}_1)\hat{\gamma}_2 - \hat{\phi}_0 - \log(\bar{x}_0)\hat{\phi}_1 - \log(\bar{\tau}_0)\hat{\phi}_2
\]

\[
= \hat{\gamma}_0 - \hat{\phi}_0 + \log(\bar{x}_1)(\hat{\gamma}_1 - \hat{\phi}_1) + \log(\bar{\tau}_1)(\hat{\gamma}_2 - \hat{\phi}_2) + (\log(\bar{x}_1) - \log(\bar{x}_0))\hat{\phi}_1 + (\log(\bar{\tau}_1) - \log(\bar{\tau}_0))\hat{\phi}_2
\]

\[
= \hat{\eta}_1 + \log(\bar{x}_1)\hat{\delta}_3 + \log(\bar{\tau}_1)\hat{\delta}_4 + (\log(\bar{x}_1) - \log(\bar{x}_0))\hat{\delta}_1 + (\log(\bar{\tau}_1) - \log(\bar{\tau}_0))\hat{\delta}_2
\]

\[
\{z\text{Structural Changes} \} + \{z\text{Business Cycle} \} + \{z\text{Tax Factors} \}
\]
5. Results

The first stage given in equation 8 decomposes aggregate tax-revenue volatility into the volatility and covariances of income, sales, and corporate tax revenue. Aggregate tax-revenue volatility increased, on average, by $712 billion in the 2000s. Just over half of this increase is due to an increase in the volatility of income tax revenue. Sales tax-revenue volatility accounts for twenty percent and corporate tax revenue for fourteen percent. The remaining fourteen percent is explained by increases in the covariances of these tax bases. The first stage decomposition demonstrates the importance of income tax revenue in explaining the increase in aggregate tax-revenue volatility. This is consistent with the explanation tax-revenue volatility increased because state governments began to rely more heavily on income taxes resulting in some state portfolios to overweight income tax revenue.

\[
\hat{\Delta}_A = \hat{\Delta}_I + \hat{\Delta}_S (52\%) + \hat{\Delta}_C (14\%) + \hat{\Delta}_{I,S} (7\%) + \hat{\Delta}_{I,C} (4\%) + \hat{\Delta}_{S,C} (3\%)
\]

Table 2 reports the second stage decomposition of tax-revenue volatility into tax factors, business cycle factors, and tax base changes. The first column reports the decomposition for aggregate tax revenue and the second through fourth columns report the decomposition for the income, sales, and corporate tax respectively. Tax factors explain seventy percent of aggregate tax revenue and over fifty percent of each of the three tax bases. The ninety-five percent confidence intervals are calculated by bootstrapping the sample, clustering by state, and reporting the 2.5 and 97.5 percentiles. These estimates are robust to extreme outliers and produce asymmetric confidence intervals. All of the point estimates are statistically significant at the ninety-five percent level except for tax factors in the sales tax. The business cycle variables are a more important factor in explaining the sales tax volatility than they are for explaining the increase for income or corporate tax volatility. This result is consistent with the explanation, state governments began to rely more on income and corporate taxes causing the volatility of the sales tax to be driven more by the business cycle and the increases in volatility of the corporate and income taxes to be explained by tax factors.

Table 3 reports estimates of the second stage decomposition for a different method of weighting and a different time trend. The baseline case given in the first column estimates a cubic time trend
Table 2. Results

<table>
<thead>
<tr>
<th>Table 2. Results</th>
<th>Percent Explain</th>
<th>Income</th>
<th>Sales</th>
<th>Corporate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax Portfolio</td>
<td>70.26 %</td>
<td>66.18 %</td>
<td>52.08 %</td>
<td>84.14 %</td>
</tr>
<tr>
<td></td>
<td>[58.42, 88.49]</td>
<td>[50.62, 72.56]</td>
<td>[-40.99, 67.43]</td>
<td>[73.18, 88.78]</td>
</tr>
<tr>
<td>Business Cycle</td>
<td>28.95 %</td>
<td>33.04 %</td>
<td>47.35 %</td>
<td>15.04 %</td>
</tr>
<tr>
<td></td>
<td>[10.69, 40.69]</td>
<td>[18.93, 39.59]</td>
<td>[9.99, 66.77]</td>
<td>[4.66, 19.74]</td>
</tr>
<tr>
<td>Tax Base</td>
<td>0.78 %</td>
<td>0.80 %</td>
<td>0.69 %</td>
<td>0.82 %</td>
</tr>
<tr>
<td></td>
<td>[0.70, 0.87]</td>
<td>[0.70, 0.83]</td>
<td>[0.14, 0.81]</td>
<td>[0.76, 0.84]</td>
</tr>
</tbody>
</table>

State FE: Yes, Yes, Yes, Yes
Observations: 2350, 2350, 2350, 2350

Bootstrapped 95 percentile confidence interval (3000 replications) clustered by state.
Base Case: cubic time trend and kernel matching to produce weights.
Weighted estimates of equation 6.
Volatility of revenue and economic variables calculated as $(x - x_{time\ trend})^2$.

and uses a kernel estimation to produce weights. The second column reports the results with inverse probability weights estimated by a probit. The third column reports the results with a time trend estimated by a Hodrick-Prescott filter. The results are robust to these different weighting and time trend estimations.
### Table 3. Alternative Model Specifications

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>IPW</th>
<th>HP Filter</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Income</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tax Portfolio</td>
<td>66.18 %</td>
<td>64.19 %</td>
<td>80.88 %</td>
</tr>
<tr>
<td></td>
<td>[50.62 , 72.56]</td>
<td>[35.38 , 71.68]</td>
<td>[61.5 , 89.83]</td>
</tr>
<tr>
<td>Business Cycle</td>
<td>33.04 %</td>
<td>35.06 %</td>
<td>18.26 %</td>
</tr>
<tr>
<td></td>
<td>[18.93 , 39.59]</td>
<td>[19.08 , 44.28]</td>
<td>[-7.19 , 26.91]</td>
</tr>
<tr>
<td>Tax Base</td>
<td>0.8 %</td>
<td>0.76 %</td>
<td>0.87 %</td>
</tr>
<tr>
<td></td>
<td>[0.7 , 0.83]</td>
<td>[0.64 , 0.82]</td>
<td>[0.82 , 0.89]</td>
</tr>
<tr>
<td><strong>Sales</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tax Portfolio</td>
<td>52.08 %</td>
<td>50.44 %</td>
<td>49.38 %</td>
</tr>
<tr>
<td></td>
<td>[-40.99 , 67.43]</td>
<td>[-62.15 , 66.75]</td>
<td>[26.81 , 57.58]</td>
</tr>
<tr>
<td>Business Cycle</td>
<td>47.35 %</td>
<td>48.98 %</td>
<td>49.82 %</td>
</tr>
<tr>
<td></td>
<td>[9.99 , 66.77]</td>
<td>[0.23 , 72.25]</td>
<td>[30.58 , 58.07]</td>
</tr>
<tr>
<td>Tax Base</td>
<td>0.69 %</td>
<td>0.63 %</td>
<td>0.79 %</td>
</tr>
<tr>
<td></td>
<td>[0.14 , 0.81]</td>
<td>[-0.04 , 0.78]</td>
<td>[0.66 , 0.84]</td>
</tr>
<tr>
<td><strong>Corporate</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tax Portfolio</td>
<td>84.14 %</td>
<td>83.79 %</td>
<td>73.23 %</td>
</tr>
<tr>
<td></td>
<td>[73.18 , 88.78]</td>
<td>[71.35 , 88.6]</td>
<td>[58.17 , 80.71]</td>
</tr>
<tr>
<td>Business Cycle</td>
<td>15.04 %</td>
<td>15.45 %</td>
<td>25.97 %</td>
</tr>
<tr>
<td></td>
<td>[4.66 , 19.74]</td>
<td>[4.14 , 20.66]</td>
<td>[6.25 , 32.84]</td>
</tr>
<tr>
<td>Tax Base</td>
<td>0.82 %</td>
<td>0.78 %</td>
<td>0.79 %</td>
</tr>
<tr>
<td></td>
<td>[0.76 , 0.84]</td>
<td>[0.71 , 0.82]</td>
<td>[0.72 , 0.82]</td>
</tr>
</tbody>
</table>

- Bootstrapped 95 percentile confidence interval (3000 replications) clustered by state.
- Bootstrap clustered by state.
- Inverse probability weights constructed from probit estimates.
- Weighted estimates of equation 6 with different model specifications.
- Volatility of revenue and economic variables calculated as \((x - x_{time \text{ trend}})^2\).
This section develops a model of optimal taxation given production efficiency uncertainty described by the model in section 3. Each of the government’s tax bases are state-dependent, meaning conventional approaches to evaluating alternative tax structures (e.g. deadweight loss for equal revenue streams) encounter complications because differing tax structures will change the pattern of returns across states of nature. If the government is risk neutral comparing the expected loss of utility for an expected level of revenue will be sufficient. However, if the government is sensitive to both the level and volatility associated with a revenue stream, then comparing expected utility losses will be inadequate. The government’s attitude toward risk depends upon the individual’s preferences and the ability of the government to smooth revenue.

This section decomposes the tradeoffs in the full model into four cases. Cases one and two abstract from costs in deadweight loss by allowing a planner to decide the public-private consumption bundle. Cases one and three abstract from costs in volatility by allowing the government to make its decision after the states of nature are realized. Case four is the full model consisting of costs from deadweight loss and volatility.

<table>
<thead>
<tr>
<th>Certain</th>
<th>Uncertain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planner</td>
<td></td>
</tr>
<tr>
<td>1. Pareto Optimum</td>
<td>2. Volatility Modified</td>
</tr>
<tr>
<td></td>
<td>(DWL)</td>
</tr>
<tr>
<td>Competitive</td>
<td></td>
</tr>
<tr>
<td>3. PF literature</td>
<td>4. Full Model</td>
</tr>
</tbody>
</table>

A. Timing The economy is assumed to be a one period snapshot of a dynamic model where the state of nature within the period is uncertain ex ante. The timing is given below for the certain and uncertain cases; the difference being whether the state of nature has been realized before the government makes its decisions.

\footnote{The appendix demonstrates the robustness of this model by extending it to a two-period model.}
C.1 Pareto optimum without uncertainty. In this case the planner chooses $I, C, R,$ and $\beta$ (the input-factor supplied, the level of public and private consumption, and the composition of private consumption) after the state of nature is realized. The planner produces the efficient allocation without deadweight loss or welfare loss from uncertainty.

\[
\max_{C, \beta, R, I} M(C, \sigma_C^2, \beta) + G(R, \sigma_R^2) + L(-I)
\]

subject to

\[
\theta f'(I) = C + R + s
\]

The first order conditions with respect to private and public consumption combined in equation C1.1 dictate the marginal benefit from public and private consumption should be equal.\(^43\) The first order condition with respect to the amount of input to supply states the marginal cost of supplying the input-factor should equal the marginal benefit. Finally, the first order condition with respect to the private consumption composition parameter $\beta$ states the marginal benefit should be zero.

(C1.1) \[G_1(R, \sigma_R^2) = M_1(C, \sigma_C^2, \beta)\]

(C2.1) \[L'(-I) = M_1(C, \sigma_C^2, \beta)\theta f''(I)\]

(C3.1) \[M_3 = 0\]

\(^{43}\)Equal marginal benefits between public and private goods results from the assumptions of a representative individual and the intermediate good is costless to transform into public and private consumption.
C.2 Pareto optimum with uncertainty. In this case the planner chooses $\rho$, $I$, and $\beta$ (the fraction of uncertain production to allocate to the public sector, the input-factor supplied, and the composition of private consumption). The planner allocates resources without deadweight loss but incurs a cost from volatility in private and public consumption.\textsuperscript{44}

$$\max_{\rho,I,\beta} \quad M(C, \sigma_c^2, \beta) + G(R, \sigma_R^2) + L(-I)$$

subject to

\begin{align*}
C &= (1 - \rho)(\theta f(I) - s) \\
\sigma_c^2 &= (1 - \rho)^2 \sigma_y^2 \\
R &= \rho(\theta f'(I)I - s) \\
\sigma_R^2 &= \rho^2 \sigma_y^2
\end{align*}

The optimal level of public consumption when uncertainty exists, characterized by condition C1.2, can be greater or less than the optimal level without uncertainty depending on the relative risk-sharing costs in public and private consumption in equilibrium.\textsuperscript{45} The first order condition with respect to the amount of input supplied, given in equation C2.2, states, the marginal cost of supplying the input-factor should equal the marginal benefit, where the change in volatility is included. Finally, the first order condition with respect to the private consumption composition parameter $\beta$ states, the marginal benefit should be zero, the same as in the first case.

\begin{align*}
(C1.2) \quad G_1 &= M_1 + \rho^2 \sigma_y^2 \frac{\partial \sigma_y}{\partial C} + \rho G_2 \\
(C2.2) \quad ((1 - \rho)M_1 + \rho G_1) \theta f'(I) + ((1 - \rho)^2 M_2 + \rho^2 G_2) \frac{\partial \sigma_y^2}{\partial I} &= L'(-I)
\end{align*}

Table 4 demonstrates the marginal effects on the mean and variance of private and public consumption as production is shifted to the public sector. Because the planner can shift production without deadweight loss the marginal effects cancel for the mean, first column table 4. For the mean the marginal effects are:

$$\sigma_y^2 \equiv (f(I)^2 \sigma_\theta^2 + \sigma_s^2 - 2f(I)\sigma_\theta \sigma_s)$$

\textsuperscript{44} Note if the disutility from public consumption risk weighted by $\rho$ is larger than the disutility of risk from private consumption weighted by $(1 - \rho)$ then the second term on the right hand side of condition C1.2 is positive which implies $G_1 < M_1$.\textsuperscript{45}
marginal effects cancel because the government is able to shift production without deadweight loss, whatever is gained in the private sector must exactly be subtracted from the private sector. The variance of public and private consumption is convex in production meaning shifting production can increase or decrease the sum of the variances.\textsuperscript{46} For example, if the risk preferences for public and private consumption are represented by the same linear function, the best allocation of risk occurs when $\rho = 1/2$ and the cost of risk increases convexly away from this point as demonstrated in figure 8.

\textsuperscript{46} Notice however volatility in the economy does not depend on $\rho$ since volatility in the economy is simply $\sigma^2_y$. This is also apparent if public and private consumption are considered perfect substitutes, in which case, the planner would care about the variance of $c + R$. The variance of $c + R$ is the variance of $c$ plus the variance of $R$ plus 2 times the covariance. In this case: $\sigma_c^2 + \sigma_R^2 + 2\sigma_{c,R} = (1 - \rho)^2\sigma_y^2 + \rho^2\sigma_y^2 + 2(1 - \rho)\rho\sigma_y^2 = \sigma_y^2$. 

\begin{table}
\centering
\caption{Shifting Production Income and Risk Effects}
\begin{tabular}{ccc}
\hline
& $\frac{\partial}{\partial \rho}$ & $\frac{\partial \sigma^2_y}{\partial \rho}$ \\
\hline
$C$ & $-E[\theta f(I) - s]$ & $-2(1 - \rho)\sigma^2_y$ \\
$R$ & $E[\theta f(I) - s]$ & $2\rho\sigma^2_y$ \\
Total & 0 & $-2\sigma_y^2 + 4\rho\sigma^2_y$ \\
\hline
\end{tabular}
\end{table}

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure8.pdf}
\caption{Risk is U-shaped With Respect To $\rho$}
\end{figure}
C.3 Competitive equilibrium without uncertainty. In this case the government and representative individual make their choices after the state of nature is realized. The government chooses the tax rates and the individual chooses both the private consumption composition and the amount of the input-factor to supply. The competitive equilibrium produces deadweight loss.

\[
\max_{\tau_c, \tau_w} M(c, \sigma^2_c, \beta) + G(R, \sigma^2_R) + L(-I)
\]

subject to

\[
c = (1 - \tau_c \beta)(\theta f(I)(1 - \tau_w) - s)
\]

\[
R = \tau_c \beta(\theta f(I)(1 - \tau_w) - s) + \tau_w \theta f(I)
\]

The government will optimally set the marginal benefit of public consumption equal to the marginal benefit of private consumption plus a deadweight loss wedge term, equations C1a.3 and C1b.3.\(^{47}\)

In equilibrium the marginal benefit of public consumption is greater than private consumption implying deadweight loss decreases the optimal level of public consumption relative to the Pareto efficient case (C1.1).\(^{48} \)\(^{49}\)

\[
(C1a.3) \quad G_1 = M_1 + \frac{\omega \beta \varepsilon_{\beta, \tau_c} + \omega f I \varepsilon_{I, \tau_c}}{\varepsilon R, \tau_c}
\]

\[
(C1b.3) \quad G_1 = M_1 + \frac{\omega \beta \varepsilon_{\beta, \tau_w} + \omega f I \varepsilon_{I, \tau_w}}{\varepsilon R, \tau_w}
\]

C.4 Competitive equilibrium with uncertainty. In this case the government chooses the tax rates before the state is realized and the individual chooses the amount of the input-factor to supply and the consumption composition after the state is realized. The competitive equilibrium produces deadweight loss and the uncertainty produces a welfare loss from volatility. The government’s deadweight loss in the competitive equilibrium causes the government’s transformation of private goods into public goods to be costly causing public consumption in the competitive equilibrium to be below the Pareto efficient level.

\(^{47}\)Note $\frac{\beta \varepsilon_{\beta, \tau_c}}{\varepsilon R, \tau_c} < 0$ because $\varepsilon_{\beta, \tau_c} < 0$ and all other terms are positive.

\(^{48}\)Intuitively, deadweight loss in the competitive equilibrium causes the government’s transformation of private goods into public goods to be costly causing public consumption in the competitive equilibrium to be below the Pareto efficient level.
The optimization problem is given below.\textsuperscript{50}

\[
\max_{c, \tau_c, \tau_w} M(c, \sigma_c^2, \beta) + G(R, \sigma_R^2) + L(-I)
\]

subject to

\[
c = (1 - \tau_c \beta)(\theta f(I)(1 - \tau_w) - s) \quad \sigma_c^2 = (1 - \tau_c \beta)^2 \sigma_y^2
\]

\[
R = \tau_c \beta(\theta f(I)(1 - \tau_w) - s) + \tau_w \theta f(I) \quad \sigma_R^2 = \Theta^2 \sigma_\theta^2 + \tau_c^2 \beta^2 \sigma_s^2 - 2\Theta \tau_c \beta \sigma_{s, \theta}
\]

Conditions C1a.4 and C1b.4 encompass the full tradeoff between deadweight loss and volatility involved with consumption and income taxation. The deadweight loss wedge is always nonnegative but the cost to volatility can be positive or negative depending upon the relative values of other tax rates. The marginal cost from volatility with respect to a tax rate is positive if it is relatively larger than the other tax rates. For example, when \(\tau_w < \tau_c \beta(\sigma_{\theta, s} - f(I)\sigma_\theta^2)/(f'(I)I(1 - \tau_c \beta)\sigma_\theta^2)\) the uncertainty term in condition C1b.4 is negative, implying increasing the income tax rate would decrease the cost of volatility in public and private consumption. Figure 9 demonstrates this point, showing the variance of tax revenue is U-shaped with respect to an individual tax rate.\textsuperscript{51}

\begin{align*}
(C1a.4) \quad G_1 &= M_1 + \frac{\omega_M \varepsilon_{\Delta, \tau_c} + \omega_G \varepsilon_{\Delta, \tau_c}}{\varepsilon_{R, \tau_c}} \text{Deadweight loss} + \frac{\omega_M \varepsilon_{\Delta, \tau_w} + \omega_G \varepsilon_{\Delta, \tau_w}}{\varepsilon_{R, \tau_w}} \text{volatility} \\
(C1b.4) \quad G_1 &= M_1 + \frac{\omega_M \varepsilon_{\Delta, \tau_w} + \omega_G \varepsilon_{\Delta, \tau_w}}{\varepsilon_{R, \tau_w}}
\end{align*}

The optimal tax rates depend on the optimal level of risk sharing between public and private consumption. This tradeoff depends on the risk attitudes which encompass both the risk preferences and the ability of individuals and governments to smooth volatile streams, functionally captured by \(\omega_M\) and \(\omega_G\). The optimal tax rates also depend on the ability of governments to hedge idiosyncratic

\textsuperscript{50}\sigma_y^2 = \frac{1}{\sigma^2}[(1 - \tau_w)^2 W^2]\sigma_\theta^2 + \sigma_s^2 - \frac{2}{\theta}[(1 - \tau_w)W\sigma_{\theta, s} \text{ and } \Theta = f(I)\tau_c \beta + f'(I)I[(1 - \tau_c \beta)\tau_w].
\textsuperscript{51}\omega_M = \sigma_c^2 M_2 / R \text{ and } \omega_G = -\sigma_R^2 G_2 / R.
risk associated with consumption and income taxes. The fact governments can hedge some risk away is captured in figure 9 which demonstrates increasing a tax rate can actually decrease the risk in aggregate tax revenue. In equilibrium the government balances the volatility concerns (risk sharing between public and private consumption and hedging idiosyncratic risk) with deadweight loss.
7. Ramsey Results

1. Volatility-Adjusted Samuelson Rule Condition C1 in each of the four cases is a Samuelson condition. In the first case C1.1 is the representative agent version of the original Samuelson rule, stating the marginal benefit of the public good should equal the marginal benefit of the private good times the cost of transforming private consumption to public consumption, in this case the cost is one. When deadweight loss is added in the third case condition C1a.3 demonstrates the modified Samuelson rule stating if the government creates deadweight loss by transforming private consumption to public consumption then there will be a wedge in the second best condition.

In the second case condition C1.2 contains a wedge created by the cost of volatility. Similar to case three, the marginal benefit of public consumption no longer equals the marginal benefit of private consumption. In contrast to case three, the wedge created by the disutility of volatility can cause more or less public consumption in the second best solution. The volatility-adjusted Samuelson rules are given by conditions C1a.4 and C1b.4 which trade off deadweight loss and volatility. In this condition the government could produce the same level of public good with less deadweight loss by changing the tax rates on income and consumption. However, the increased cost from volatility caused by this change outweighs the benefit from the decrease in deadweight loss.

2. Volatility-Adjusted Ramsey Rule The volatility-adjusted Ramsey rule is given by condition 9 which combines conditions C1a.4 and C1b.4 from case four.\textsuperscript{52,53} This condition states the marginal deadweight loss plus marginal volatility cost should be equal across tax bases. This condition nests the Ramsey rule with certainty, which combines conditions C1a.3 and C1b.3 from case three, which consists of only the base elasticities (the first term on each side).\textsuperscript{54}

\begin{equation}
\omega_B \varepsilon_{B, \tau_c} + \omega_\sigma \varepsilon_{\sigma, \tau_c} = \omega_B \varepsilon_{B, \tau_w} + \omega_\sigma \varepsilon_{\sigma, \tau_w}
\end{equation}

The Ramsey rule defines the optimal mix of taxation and is derived from a government maximizing a representative individual’s utility. The Ramsey rule in equation 9 provides a condition determining
if a state government’s portfolio is balanced between tax bases. As a corollary of this result it is
sufficient to show the elasticity of the base and the variance with respect to a given tax rate are both
larger than the elasticities with respect to a different tax rate to demonstrate a state government’s
portfolio is imbalanced. This corollary does not depend on the utility weights $\omega_i$ and therefore can
be determined without choosing a functional form for utility. I estimate the elasticities in equation
9 for each state in the following section to determine which states have imbalanced portfolios.\textsuperscript{55}

8. IMBALANCED STATE GOVERNMENT PORTFOLIOS

This section estimates the elasticity of the tax base and the variance of income and sales tax
revenue with respect to its tax rate for each state.\textsuperscript{56} The sufficient condition for a state’s portfolio
to be imbalanced is both the tax base and variance elasticities for a given tax base being larger
than another tax bases. Intuitively, this means deadweight loss and variance of tax revenue can
be decreased without sacrificing expected levels of revenue by shifting the weights on different tax
bases.\textsuperscript{57}

\begin{equation}
\log(R_i) = \log(\tau_i) + \log(B_i)
\end{equation}

\begin{equation}
\log(B_i) = \varphi_0 + \log(\tau_i) \varphi_1 + \log(\tau) \varphi_2 + \log(x) \varphi_3 + \epsilon
\end{equation}

\begin{equation}
\log(R_i) = \pi_0 + \log(\tau_i) \pi_1 + \log(\tau) \pi_2 + \log(x) \pi_3
\end{equation}

The elasticity of the tax base with respect to the tax rate is $\varphi_1$ which equals $1 - \pi_1$, where $\pi_1$ is the
elasticity of tax revenue with respect to the tax rate estimated from equation 10.2. The statistic
of interest is $\varepsilon_{B,\tau_i} = \varepsilon_{base,\tau_i}/\varepsilon_{R_i,\tau_i}$ which equals $(\pi_1 - 1)/\pi_1$. From equation 11 the elasticity of the
variance of tax revenue with respect to the tax rate can be estimated in a similar manner where

\textsuperscript{55}I estimate four elasticities for each state for a total of 200 elasticities. I compare the four elasticities estimated for
each state and run an F-test to determine if the difference is statistically significant.

\textsuperscript{56}This section focuses on the income and sales tax because they are the two major sources of tax revenue for most
states.

\textsuperscript{57}This analysis makes the assumption the ratio of the elasticity of the variance of private consumption with respect
to a tax rate and the elasticity of tax revenue with respect to a tax rate is the same across tax rates. This assumes
the variance of private consumption depends on the amount of revenue collected but not how it is collected.
\[ \varepsilon_{\sigma_{R_i}, \tau_i} = (\xi_1 - 1)/\xi_1. \]

\[ \log(\sigma_{R_i}^2) = \zeta_0 + \log(\tau_i)\xi_1 + \log(\tau)\xi_2 + \log(x)\xi_3 \]

I estimate equations 10.2 and 11 using a two step process. The first step estimates the similarity between states in their observable characteristics and assigns a weight to the state. The second step estimates a weighted regression of equation 10.2 and 11 for each state. These equations could be estimated for each state using only data from the state for the years 1963-2010 but other state’s experiences are informative and are used to supplement the state’s data by weighting other states based on how informative its experience is. For example, Wisconsin’s data has a high weight in Minnesota’s estimation but a low weight in California’s.

The results of the estimation of 10.2 and 11, mapped in figure 10, determine whether a state’s portfolio is balanced, overweights the income tax, or overweights the sales tax. I find 12 state tax portfolios overweight the sales tax, 16 overweight the income tax, and 20 are balanced. The state tax portfolios I identify as balanced are unable to lower both tax-revenue volatility and deadweight loss but may be able to improve welfare by adjusting their tax portfolio. The states with imbalanced tax portfolios can lower both tax-revenue volatility and deadweight loss by adjusting their tax portfolio.

State governments expose their tax revenues to unnecessary levels of risk by holding imbalanced tax portfolios. In decades with little economic volatility tax revenue from states with imbalanced portfolios look similar to those with balanced portfolios. However, in decades with increased economic volatility, such as the 2000s, states with imbalanced portfolios experience elevated levels of tax-revenue volatility. I find a positive correlation between states holding imbalanced tax portfolios and states with the largest increases in volatility in the 2000s. The correlation is positive for both overweight income tax portfolios and overweight sales tax portfolios demonstrating the importance of balance and not just stable tax bases.

---

58 The weights can be calculated parametrically using a probit or semi-parametrically using a kernel estimation. The baseline results reported use a probit and the results are robust to using a kernel estimation.

59 The estimates of the elasticities are reported in the online appendix.

60 A utility function would need to be assumed to determine how states with balanced portfolios could increase welfare by adjusting their tax portfolio.
9. Conclusion

This paper undertakes an empirical explanation of the increase in tax-revenue volatility U.S. states experienced in the 2000s, creates a normative theory of optimal taxation in the context of uncertainty, and applies these empirical and theoretical results to estimate the balance of state tax portfolios. The main contribution of this paper is to provide evidence, theoretically and empirically, of the importance of tax-revenue volatility. Theoretically, this paper demonstrates mechanisms for fiscal policy to limit future volatility in tax revenue. Empirically, this analysis provides strong evidence tax policy changes leading up to the 2000s are important in explaining the increase in volatility of state tax revenues in the 2000s. In contrast, economic conditions are less important in
explaining the increase in tax-revenue volatility leaving the question open whether volatility in tax revenue slows recoveries from recessions.

Recessions in the the sample years 1963 through 2010 can be grouped into recessions that recovered quickly and those that did not, estimated by whether a bear market occurred or not.\textsuperscript{61} The resulting groups are the recessions of 1969, 1973, 2001, and 2007 as the ‘sluggish’ recessions and the recessions of 1980, 1981, and 1990 together as the ‘passing’ recessions. Interestingly, state tax-revenue volatility had two periods of exceptional volatility corresponding to the early 1970s and the 2000s. Suggesting state tax-revenue volatility might be important in explaining whether a recession will recover sluggishly or not.

State tax-revenue volatility adds uncertainty to the economy negatively impacting economic growth. One of the striking facts about the economic conditions in 2011 is consumption in the economy remains low. This fact is consistent with individuals believing the uncertainty in the economy is large. This belief whether correct or not may be directly affected by the uncertainty in state expenditures due to the extreme swings in state tax revenues. Therefore future research is need to better understand the causes and effects of tax-revenue volatility.

\textsuperscript{61}A bear market for these purposes is defined as a twenty percent decline in the nominal value of the S&P 500 index that lasted at least six months.
References


Appendix

10. APPENDIX: STATISTICAL TESTS

Figure 11 depicts the Quandt Likelihood Ratio which determines the structural break for the model. This figure demonstrates the break occurred in the early 2000s and is statistically significant. Figure 12 demonstrates that the regressions contain variables that are stationary. The volatility measures are stationary because they are measures that have filtered out the time trend and the Adjusted Dickey-Fuller test formally shows this. Finally, figure 13 is a scatter plot of the corporate tax rate by year for all states. This figure demonstrates the data before and after 2000 look similar and formally have enough overlap to run the weighted regressions.
Figure 12. Adjusted Dickey-Fuller Test Statistics: Stationarity

Figure 13. Scatter Corporate Tax Rate by Year
11. APPENDIX: WEIGHTING DECOMPOSITION

The decomposition method introduced by DiNardo, Fortin, and Lemieux (1996) (DFL) provides a method for estimating counterfactual distributions without assuming linearity (assumption 1). Similarly to the regression decomposition the estimated counterfactual distributions of the volatility are used to decompose the contribution of each of the factors. The actual and counterfactual distributions, given in equation 12, differ by the densities they are integrated over.\(^{62}\)

\[
\text{Actual Distribution} \quad f_1^1(\log(\text{Revenue}_{i,t})) \equiv \int f(\log(\text{Revenue}_{i,t})|z)h(z|D = 1)dz
\]

\[
(12)
\]

\[
\text{Counterfactual Distribution} \quad f_0^0(\log(\text{Revenue}_{i,t})) \equiv \int f(\log(\text{Revenue}_{i,t})|z)h(z|D = 0)dz
\]

The important insight of DFL is that the counterfactual distribution can be written as a weighted function of the actual distribution. The weight is the ratio of the conditional density functions which by Bayes’ rule can be rewritten as the ratio of propensity scores normalized by the number of observations in each group, \(\omega = P(D = 1|z)/P(D = 0|z))(P(D = 1)/P(D = 0)).\(^{63}\) This realization by DFL transforms a possibly impossible problem of integration over many variables into a simple reweighting problem where the weights can be estimated by a logit or probit model.

\[
\text{Counterfactual Distribution} \quad f_0^1(\log(\text{Revenue}_{i,t})) \equiv \int \omega f(\log(\text{Revenue}_{i,t})|z)h(z|D = 1)dz
\]

The increase in volatility of tax revenue can be decomposed using different counterfactual distributions. The increase that cannot be explained by differences in observable characteristics is again attributed to the structural change, which captures the second hypothesis. Formally, this is given by the difference between the mean of the actual distribution of the years after the structural break and the mean of the counterfactual distribution that would have occurred if all of the observable characteristics had been similar to those after the structural break. This is similar to the effect of the treatment on the treated (TOT).

\(^{62}\)In equation 12 \(z\) represents all observable characteristics, tax and economic.

\(^{63}\)The weight is \(h(z|D = 0)/h(z|D = 1)\) where \(h(z|D = 1) = h(z_j = z_0)P(D = 0|z_j = z_0)/P(D = 0)\) by Bayes’ rule.
The rest of the increase in volatility is what can be explained by observable characteristics. The marginal effect that can be explained by economic factors is given by the difference in the means of the counterfactual distribution that would have occurred if all observable variables would have been similar to the characteristics in the years after the structural break and the counterfactual that would have occurred if only the tax variables would have been similar to the characteristics of the states after the structural break. Similarly, the marginal tax effect can be found by the difference of the means of the two counterfactual distributions formally given in equation 13. The conditional weights \( \omega_x = \frac{P(D = 1|\tau)}{P(D = 0|\tau)}(P(D = 1)/P(D = 0)) \) and \( \omega_\tau = \frac{P(D = 1|x)}{P(D = 0|x)}(P(D = 1)/P(D = 0)) \) are used to calculate the other two counterfactual distributions.

\[
\text{Structural Factors} \quad \int \log(\text{Revenue}_{i,t}) f(\log(\text{Revenue}_{i,t}) | z) h(z | D = 1) dz \\
- \int \omega \log(\text{Revenue}_{i,t}) f(\log(\text{Revenue}_{i,t}) | z) h(z | D = 1) dz
\]

\[
\text{Economic Factors} \quad \int \omega \log(\text{Revenue}_{i,t}) f(\log(\text{Revenue}_{i,t}) | z) h(z | D = 1) dz \\
- \int \omega_x \log(\text{Revenue}_{i,t}) f(\log(\text{Revenue}_{i,t}) | z) h(z | D = 1) dz
\]

\[
\text{Tax Factors} \quad \int \omega \log(\text{Revenue}_{i,t}) f(\log(\text{Revenue}_{i,t}) | z) h(z | D = 1) dz \\
- \int \omega_\tau \log(\text{Revenue}_{i,t}) f(\log(\text{Revenue}_{i,t}) | z) h(z | D = 1) dz
\]

This method controls for nonlinearities and is asymptotically more efficient than matching or regression models, Hirano et al. (2000). In this context controlling for nonlinearities will decrease the upward bias in the structural factor estimates from the regression analysis. The typical concern with this method is a selection bias, for example, when individuals choose their group based on unobservable characteristics. This selection bias is a violation of the second assumption above, \( E[\epsilon|x, \tau, D, I.state] = 0 \). While the selection bias is not an issue in this context because states cannot choose their groups, the second assumption may still be violated if endogenous variables are included. Finally, this method depends on the occurrence of observations that “look similar” in both groups of years, formally that there is sufficient overlap of independent variables. Overlap would be a problem if the set of state tax rates in the early years were disjoint from the set of tax
rates in the later years. Figure 4 is a scatter plot of the corporate tax rates for all states for the year 1963 to 2010 and demonstrates graphically sufficient overlap.

12. APPENDIX: HIGHER MOMENT DECOMPOSITION

In the text the expected utility is assumed to be fully characterized by the first two moments of the public and private good, which is sufficient when the goods are jointly normally distributed or when the utility function is quadratic. Generally, the expected utility can be written as in equation 14 below where Ω consists of the second moment and higher that is necessary to fully characterize the joint distribution between public and private goods. This composition of the expected utility is much more general than the case where the joint distribution is normal but is not fully general because not every distribution can be uniquely characterized by its moments. However, in the case that the joint distribution is normal the distribution can be fully characterized by the first two moments and Ω consists solely of the second moments of the private and public good. If the utility function is additive, such that \( U_{1,2} = 0 \), then the level of social welfare can be written as the second line in the equation below.

\[
\int U(c(\theta), G) f(\bar{c}, \bar{R}, \Omega) \equiv M((\bar{c}, \bar{R}, \Omega) = M((\bar{c}, \Omega_c) + G((\bar{R}, \Omega_R) \quad \text{When } U_{1,2} = 0
\]

(14)

(15)

To generalize the formulas in the text to the case where higher moments are needed to characterize the expected utility replace all of the partial derivatives of the second moment with the partial derivative of Ω.

To demonstrate this transformation consider a Cobb Douglas utility where total consumption, which is a combination of technology and savings shocks, is assumed to be distributed uniformly with mean \( \mu \) and standard deviation \( \sigma \). Writing the utility function in terms of total consumption \( c \) and the shift parameter \( \beta \) gives the following form where the density function is \( \frac{1}{2\pi\sqrt{3}} \) for \( c \in [-\sqrt{3}\sigma, \sqrt{3}\sigma] \).
and zero everywhere else.

\[ E[U(c, \beta)] = E[\log c + \alpha \log \beta + (1 - \alpha) \log (1 - \beta)] \]

\[ = E[\log c] + \alpha \log \beta + (1 - \alpha) \log (1 - \beta) \]

\[ = \int_{-\sqrt{3}\sigma}^{\sqrt{3}\sigma} \log \frac{1}{2\sigma \sqrt{3}} dc + \alpha \log \beta + (1 - \alpha) \log (1 - \beta) \]

\[ M(\mu, \sigma^2, \beta) = \frac{(\sigma \sqrt{3} + \mu)(\log(\mu + \sigma \sqrt{3}) - 1) + (\sigma \sqrt{3} - \mu)(\log(\mu - \sigma \sqrt{3}) - 1)}{2\sigma \sqrt{3}} + \alpha \log \beta + (1 - \alpha) \log (1 - \beta) \]

The preceding line is a function of the mean, standard deviation, and \( \beta \) alone.

13. **Appendix: Consumption Base Decomposition**

In the text private consumption of the representative agent is decomposed into consumption that is taxed and consumption that is untaxed such that the fraction \( \beta \) of total consumption is taxed and \((1 - \beta)\) is untaxed. This decomposition changes the variables from two consumption goods into total consumption and the fraction spent on taxable items. This section demonstrates the change of variables and its benefits.

First, start with two goods \( B, N \) such that the consumption of \( B \) is taxed and the consumption of \( N \) is not taxed and the representative agent has utility \( U(B, N) \) over the two goods. By definition \( B = \beta c \) and \( N = (1 - \beta)c \). The utility function can be written as a function of \( \beta \) and \( c \) by substituting these equations in for \( B \) and \( N \). If the utility function is homothetic then the utility function can be written as \( U(B, N) = v(\beta)U(c) \) otherwise \( U(B, N) = U(c, \beta) \). The budget constraint is given
below written both as a function of $B$ and $N$ and $\beta$ and $c$.

$$W = (1 + \tau_c)B + N$$

$$= (1 + \tau_c)\beta c + (1 - \beta)c$$

$$= c(1 + \beta\tau_c)$$

Now we want to know the welfare impact of a tax change. We can separate the impact into the wealth effect and the substitution effect where the substitution effect is the deadweight loss from the behavioral responses.

$$\frac{\partial U(B, N)}{\partial \tau_c} = U_1 \frac{\partial B}{\partial \tau_c} + U_2 \frac{\partial N}{\partial \tau_c}$$

$$= U_1(S_{B,\tau_c} - \frac{\partial B}{\partial M} B) + U_2(S_{N,\tau_c} - \frac{\partial N}{\partial M} B) \quad \text{Slutsky Decomposition}$$

$$= U_1 S_{B,\tau_c} + U_2 S_{N,\tau_c} - 
\underbrace{U_1 \frac{\partial B}{\partial W} B + U_2 \frac{\partial N}{\partial W} B}_{\text{Substitution Effect}} - 
\underbrace{U_1 \frac{\partial B}{\partial W} B + U_2 \frac{\partial N}{\partial W} B}_{\text{Income Effect}}$$

The benefit of writing the utility in terms of $\beta$ and $c$ is that $U_1 \frac{\partial c}{\partial \tau_c}$ captures the income effect and $U_2 \frac{\partial \beta}{\partial \tau_c}$ captures the behavioral response and deadweight loss.

$$- \left( U_1 \frac{\partial B}{\partial W} B + U_2 \frac{\partial N}{\partial W} B \right) = U_1 \frac{B}{c} \frac{\partial c}{\partial \tau_c} + U_2 \frac{B(1 - \beta)}{c\beta} \frac{\partial c}{\partial \tau_c}$$

$$= U_1 \beta \frac{\partial c}{\partial \tau_c} + U_2(1 - \beta) \frac{\partial c}{\partial \tau_c}$$

$$= M_1 \frac{c}{\tau_c}$$
The first equality holds because of the following.

\[
\frac{\partial B}{\partial W} = \frac{\partial \beta c}{\partial W}
\]

\[
= \beta \frac{\partial c}{\partial W}
\]

\[
= \frac{\beta}{1 + \tau_c \beta}
\]

where \( c = \frac{W}{1 + \tau_c \beta} \)

\[
= -\frac{\partial c}{\partial \tau_c} \frac{1}{c}
\]

where \( \frac{\partial c}{\partial \tau_c} = -\frac{W \beta}{(1 + \tau_c \beta)^2} = -\frac{c \beta}{(1 + \tau_c \beta)} \)

The last equality holds because of the following.

\[
U_1 = \Psi_1 \frac{\partial B}{\partial c} + \Psi_2 \frac{\partial N}{\partial c} = \Psi_1 \beta + \Psi_2 (1 - \beta)
\]
Now show the deadweight loss calculation.

\[ U_1 S_{B,\tau_c} + U_2 S_{N,\tau_c} = 0 \]

Substitution Effect

\[
U_2 \frac{\partial \beta}{\partial \tau_c} = \frac{\partial \beta}{\partial \tau_c} \left( \frac{U_1 \partial B}{\partial \beta} + \frac{U_2 \partial N}{\partial \beta} \right)
\]

\[
= \frac{\partial \beta}{\partial \tau_c} \left( \frac{c}{1 + \beta \tau_c} - \frac{U_2 c(1 + \tau_c)}{1 + \beta \tau_c} \right)
\]

\[
= \frac{\partial \beta}{\partial \tau_c} \left( \frac{U_1 c}{1 + \beta \tau_c} - \frac{U_2 c(1 + \tau_c)}{1 + \beta \tau_c} \right)
\]

\[
= 0
\]

where

\[
\frac{\partial B}{\partial \beta} = c + \frac{\beta c}{\beta}
\]

\[
= c - \frac{\beta \tau_c}{1 + \beta \tau_c}
\]

\[
= \frac{c}{1 + \beta \tau_c}
\]

\[
\frac{\partial N}{\partial \beta} = -c + \frac{(1 - \beta) c}{\beta}
\]

\[
= -c - \frac{(1 - \beta) W \tau_c}{1 + \beta \tau_c}
\]

\[
= -\frac{c(1 + \tau_c)}{1 + \beta \tau_c}
\]
The consumption tax base is defined as a proportion of total consumption, $\beta c$, where $\beta$ captures the individual’s behavioral response.\(^{64}\) An increase in consumption tax revenue associated with an increase in consumption tax rate is $\beta c(1 + \varepsilon_{\beta,\tau_c})$ which is negative when $\varepsilon_{\beta,\tau_c} < -1$.\(^{66}\) Consumption tax revenue is directly affected by the wage income tax rate, highlighting the importance of accounting for all tax rates when estimating tax revenue from a specific source.\(^{67}\)

14. Appendix: Two Period Model

The savings shock in the static model presented in the paper can be formulated as a shock in beliefs to whether the technological shock in the first period is permanent or temporary in a two period model. In the first period the government sets tax rates $\tau_c, \tau_w, \tau_\pi$ for the first period before the state of nature $(\theta_1, A, \mu)$ is realized. The state of nature has three components; $\theta$, the technological efficiency in period 1, $A$ the bargaining power of labor, and $\mu$ the belief that the technological efficiency is period 2 will be the same as in period 1 as opposed to the mean level $\bar{\theta}$. As $\mu$ represents the belief that the technological shock in the first period is permanent. After the state of nature is determined the individual chooses $(I, c, \beta, s)$ the amount of input to supply, the level of consumption, the proportion of consumption in the consumption tax base, and the amount of first period production to save. Savings in the first period become input in the second period. The rate of return on savings depends on the efficiency of production in the second period which is uncertain. However, the individual believes with probability $\mu$ that the technology will be the same in period 2 as it was in period 1 and with probability $(1 - \mu)$ the technology will be the mean level of technology. This belief over the technology in the second period determines the expected return of savings. Finally, production occurs where the single-output good $X(\theta, I)$ can be costlessly transformed into private consumption, public consumption, and second period input and utility is

\(^{64}\)The consumption tax base is characterized by $\beta(\tau_c)$ where the dependence on the consumption tax rate is suppressed for notational ease.

\(^{65}\)The consumption tax rate is defined as $c_{after-tax} = (1 - \tau_c \beta)c_{pre-tax}$ which is equivalent to $(1 + t_c \beta)c_{after-tax} = c_{pre-tax}$ where $\beta \tau_c = \beta t_c/(1 + \beta t_c)$.

\(^{66}\)The wage income tax is assumed to be a linear tax on all wage income. The change in wage income tax revenue associated with an increase in the wage income tax rate is $\theta f'(I)I[1 + \varepsilon_{\theta,\tau_c}(1 + f''(I)I/f'(I))]$ where the equilibrium level of input supplied by the individual is a function of the wage income tax rate.

\(^{67}\)This point informs the empirical analysis estimation of tax base.
realized.

\( X(\theta, I) = \theta f(I) \)  

(16)  

\( X(\theta, I) = c + g + s \)  

(17)  

In the second period savings from the first period become input in the second period production. The government adjusts tax rates before the state of nature \( \theta \) is realized. After the state of nature is realized the individual decides how to split consumption between taxable and tax-free consumption, \( B \) and \( N \) respectfully. Note that the individual supplies all of their savings in the second period and can not supply any more. Therefore income in the second period is determined by saving in the first period and second period state of nature. Finally, production occurs where the single-output is costlessly transformed into private and public consumption and utility is realized.

The individual maximizes expected two-period utility taking as given wages and profits. The individual knows the state of nature in the first period and has beliefs \( \mu \) over the second period state of nature.

\[
\max_{B_1, B_2, N_1, N_2, I, s} \quad u^1(B_1, N_1) + L(-I) + \rho u^2(B_2, N_2)
\]  

(18)  

Subject to first and second period budget constraints

\[
w_1(1 - \tau_w)I_1 + (1 - \tau_\pi)\pi_1 = (1 + t_c)B_1 + N_1 + s
\]  

(19)  

\[
w_2(1 - \tau_w')I_2 + (1 - \tau_\pi')\pi_2 = (1 + t_c')B_2 + N_2
\]  

(20)  

where

\[
w_1 = h(\theta_1)f'(I) + A \quad \pi_1 = \theta[f(I) - (f'(I) + A)I]
\]  

(21)  

\[
E[w_2] = \mu \theta_1 f'(I_2) + (1 - \mu) \bar{\theta} f'(I_2).
\]  

(22)
The first-order conditions

$$\partial I_1 : L'(-I) = \lambda w_1 (1 - \tau_w)$$

$$\partial B_1 : u_1^1 = \lambda (1 + t_c)$$

$$\partial B_2 : \rho u_1^2 = \phi (1 + t'_c)$$

$$\partial N_1 : u_2^1 = \lambda$$

$$\partial N_2 : \rho u_2^2 = \phi$$

$$\partial s : \lambda = \phi E[w_2]$$

(23) 

$$u_2^1 = \rho u_2^2 E[w_2] \quad \frac{u_1^1}{u_2^1} = \rho E[w_2]$$

As $\mu$ increases, assuming $\theta_1 > \bar{\theta}$, the expectation of the return to savings increases which implies that savings increases, consumption in period 1 decreases and consumption in period 2 increases.

15. Appendix: Mathematical Derivations

15.1. C.1 Pareto optimum without uncertainty or deadweight loss.

$$\max_{c,\beta,R,I} \quad M(c, \sigma_c^2, \beta) + G(R, \sigma_R^2) + L(-I)$$

subject to

$$\theta f(I) = c + R + s$$
Lagrangian

\[ \mathbb{L} = M(c, \sigma_c^2, \beta) + G(R, \sigma_R^2) + L(-I) + \lambda(\theta f(I) - c - R - s) \]

First-order conditions

\[ \frac{\partial \mathbb{L}}{\partial c} : M_1 = \lambda \]

\[ \frac{\partial \mathbb{L}}{\partial \beta} : M_3 = 0 \]

\[ \frac{\partial \mathbb{L}}{\partial R} : G_1 = \lambda \]

\[ \frac{\partial \mathbb{L}}{\partial I} : L'(-I) = \lambda \theta f'(I) \]

Combining the first and third first-order condition gives the first condition in the text C1.1 and combining the first and fourth gives the second condition in the text C2.1.

15.2. **C.2 Pareto optimum with uncertainty.**

\[ \max_{\rho, \beta, I} \quad M(c, \sigma_c^2, \beta) + G(R, \sigma_R^2) + L(-I) \]

subject to

\[ c = (1 - \rho)(\theta f(I) - s) \]

\[ R = \rho(\theta f(I) - s) \]

\[ \sigma_c^2 = (1 - \rho)^2 \sigma_y^2 \]

\[ \sigma_R^2 = \rho^2 \sigma_y^2 \]

\[ \sigma_y^2 \equiv (f(I)^2 \sigma_y^2 + \sigma_s^2 - 2f(I)\sigma_{\theta,s}) \]
First-order conditions
\[
\frac{\partial L}{\partial \rho} : \quad M_1 \frac{\partial c}{\partial \rho} + G_1 \frac{\partial R}{\partial \rho} + M_2 \frac{\partial \sigma_c^2}{\partial \rho} + G_2 \frac{\partial \sigma_R^2}{\partial \rho} = 0
\]
\[
\frac{\partial L}{\partial \beta} : \quad M_3 = 0
\]
\[
\frac{\partial L}{\partial I} : \quad L'(-I) = M_1 \frac{\partial c}{\partial I} + G_1 \frac{\partial R}{\partial I} + M_2 \frac{\partial \sigma_c^2}{\partial I} + G_2 \frac{\partial \sigma_R^2}{\partial I}
\]
Rearranging the first and third first-order condition and substituting in the definitions of \( c, R, \sigma_c^2 \), and \( \sigma_R^2 \) produces the first and second conditions in the text C1.2 C2.2 respectively.

15.3. **C.3a Constrained Competitive Equilibrium.** In this economy the input-factor is inelastically supplied and the government is constrained to only taxing consumption. In this case the government chooses the tax rate on consumption and the representative individual chooses the composition of private consumption both after the state is realized. The individual’s optimization problem is given below

\[
(25) \quad max_{\beta} \quad M(c, \sigma_c^2, \beta) + G(R, \sigma_R^2)
\]
subject to
\[
c = (1 - \tau_c \beta)(\theta f(I) - s)
\]

The first-order condition demonstrates that when the consumption tax rate is positive the Pareto optimal condition that \( M_3(c, \sigma_c^2, \beta) = 0 \) no longer holds.

\[
M_1 \tau_c(\theta f(I) - s) = M_3
\]

The government’s optimization problem is given below.

\[
max_{\tau_c} \quad M(c, \sigma_c^2, \beta) + G(R, \sigma_R^2)
\]
subject to

\[ R = \tau_c \beta (\theta f(I) - s) \]

The first-order condition

\[ M_1 \frac{\partial c}{\partial \tau_c} + G_1 \frac{\partial R}{\partial \tau_c} + M_3 \frac{\partial \beta}{\partial \tau_c} = 0 \]

The condition in the text C1a.3 comes from rearranging the first-order condition above noting that

\[ \frac{\partial R}{\partial \tau_c} = -\frac{\partial c}{\partial \tau_c} \]

from the budget constraints of \( R \) and \( c \).

15.4. C.3 Competitive equilibrium without uncertainty. In this case the government chooses the tax rates and the individual chooses the private consumption composition and the amount of the input-factor to supply all after the state is realized. The representative agent's optimization problem is given below.

\[ \max_{\beta, I} M(c, \sigma^2_c, \beta) + L(-I) \]

subject to

\[ c = (1 - \tau_c \beta)(\theta f(I)(1 - \tau_w) - s) \]

first-order conditions

\[ \partial I : \quad M_1(c, \sigma^2_c, \beta)(1 - \tau_c \beta)\theta f'(I)(1 - \tau_w) - L'(-I) = 0 \]

\[ \partial \beta : \quad -M_1(c, \sigma^2_c, \beta)\tau_c \theta f(I)(1 - \tau_w) + M_3(c, \sigma^2_c, \beta) = 0 \]

The government’s optimization problem is given below.

\[ \max_{\tau_c, \tau_w} M(c, \sigma^2_c, \beta) + G(R, \sigma^2_R) + L(-I) \]
Subject to
\[ c = (1 - \tau_c \beta)(\theta f(I)(1 - \tau_w) - s) \]
\[ R = \tau_c \beta(\theta f(I)(1 - \tau_w) - s) + \tau_w \theta f(I) \]

First-order conditions
\[
\frac{\partial R}{\partial \tau_c} : \quad M_1 \frac{\partial c}{\partial \tau_c} + M_3 \frac{\partial \beta}{\partial \tau_c} + G_1 \frac{\partial R}{\partial \tau_c} - L'(I) \frac{\partial I}{\partial \tau_c} = 0
\]
\[
\frac{\partial R}{\partial \tau_w} : \quad M_1 \frac{\partial c}{\partial \tau_w} + G_1 \frac{\partial R}{\partial \tau_w} - L'(I) \frac{\partial I}{\partial \tau_w} = 0
\]

Note that previously \( \frac{\partial R}{\partial \tau_c} = -\frac{\partial c}{\partial \tau_c} \) but that is no longer true. In this case \( \frac{\partial R}{\partial \tau_c} = -\frac{\partial c}{\partial \tau_c} + \theta f'(I) \frac{\partial I}{\partial \tau_c} \) to see this note the two conditions below.

\[
\frac{\partial c}{\partial \tau_c} = -\beta(\theta f(I)(1 - \tau_w) - s) - \tau_c(\theta f(I)(1 - \tau_w) - s) \frac{\partial \beta}{\partial \tau_c} + (1 - \tau_c \beta)((1 - \tau_w)\theta f'(I) \frac{\partial I}{\partial \tau_c})
\]
\[
\frac{\partial R}{\partial \tau_c} = \beta(\theta f(I)(1 - \tau_w) - s) + \tau_c(\theta f(I)(1 - \tau_w) - s) \frac{\partial \beta}{\partial \tau_c} + \tau_c \beta(1 - \tau_w)\theta f'(I) \frac{\partial I}{\partial \tau_c} + \tau_w \theta f'(I) \frac{\partial I}{\partial \tau_c}
\]
\[
= \beta(\theta f(I)(1 - \tau_w) - s) + \tau_c(\theta f(I)(1 - \tau_w) - s) \frac{\partial \beta}{\partial \tau_c} - (1 - \tau_c \beta)((1 - \tau_w)\theta f'(I) \frac{\partial I}{\partial \tau_c}) + \theta f'(I) \frac{\partial I}{\partial \tau_c}
\]
\[
= -\frac{\partial c}{\partial \tau_c} + \theta f'(I) \frac{\partial I}{\partial \tau_c} \quad \text{Deadweight loss}
\]

The conditions in the text C1a.4 and C1b.4 come from rearranging the first-order conditions above.

15.5. **C.4 Competitive equilibrium with uncertainty.** In this case the government chooses the tax rates before the state is realized and the individual chooses the private consumption composition and the amount of the input-factor to supply after the state is realized. The representative agent’s
optimization problem is given below.

\[ \max_{\beta, I} \quad M(c, \sigma_c^2, \beta) + L(-I) \]

subject to

\[ c = (1 - \tau_c \beta)(\theta f(I)(1 - \tau_w) - s) \]

first-order conditions

\[ \partial I : \quad M_1(1 - \tau_c \beta) \theta f'(I)(1 - \tau_w) - L'(-I) = 0 \]
\[ \partial \beta : \quad -M_1 \tau_c \theta f(I)(1 - \tau_w) + M_3 = 0 \]

The government’s optimization problem is given below.

\[ \max_{\tau_c, \tau_w} \quad M(c, \sigma_c^2, \beta) + G(R, \sigma_R^2) + L(-I) \]

Subject to

\[ c = (1 - \tau_c \beta)(\theta f(I)(1 - \tau_w) - s) \]
\[ R = \tau_c \beta \theta f(I)(1 - \tau_w) - s + \tau_w \theta f(I) \]
\[ \sigma_c^2 = (1 - \tau_c \beta)^2 \sigma_y^2 \]
\[ \sigma_R^2 = (\tau_c \beta)^2 \sigma_y^2 \]
\[ \sigma_y^2 \equiv \left( f(I)^2(1 - \tau_w)^2 \sigma_y^2 + \sigma_s^2 - 2 f(I)(1 - \tau_w) \sigma_{\theta,s} \right) \]
First-order conditions

\[
\frac{\partial \tau_w}{\partial \tau_w} : \quad M_1 \frac{\partial c}{\partial \tau_w} + M_2 \frac{\partial \sigma_c^2}{\partial \tau_w} + M_3 \frac{\partial \beta}{\partial \tau_w} + G_1 \frac{\partial R}{\partial \tau_w} + G_2 \frac{\partial \sigma_R^2}{\partial \tau_w} - L'(-I) \frac{\partial I}{\partial \tau_w} = 0
\]

\[
\frac{\partial \tau_c}{\partial \tau_c} : \quad M_1 \frac{\partial c}{\partial \tau_c} + M_2 \frac{\partial \sigma_c^2}{\partial \tau_c} + M_3 \frac{\partial \beta}{\partial \tau_c} + G_1 \frac{\partial R}{\partial \tau_c} + G_2 \frac{\partial \sigma_R^2}{\partial \tau_c} - L'(-I) \frac{\partial I}{\partial \tau_c} = 0
\]

Note that previously \( \frac{\partial R}{\partial \tau_c} = - \frac{\partial c}{\partial \tau_c} \) but that is no longer true. In this case \( \frac{\partial R}{\partial \tau_c} = \frac{\partial c}{\partial \tau_c} + \theta f'(I) \frac{\partial I}{\partial \tau_c} \) to see this note the two conditions below.

\[
\frac{\partial c}{\partial \tau_c} = -\beta(\theta f(I)(1 - \tau_w) - s) - \tau_c(\theta f(I)(1 - \tau_w) - s) \frac{\partial \beta}{\partial \tau_c} + (1 - \tau_c \beta)((1 - \tau_w)\theta f'(I) \frac{\partial I}{\partial \tau_c})
\]

\[
\frac{\partial R}{\partial \tau_c} = \beta(\theta f(I)(1 - \tau_w) - s) + \tau_c(\theta f(I)(1 - \tau_w) - s) \frac{\partial \beta}{\partial \tau_c} + \tau_c \beta(1 - \tau_w)\theta f'(I) \frac{\partial I}{\partial \tau_c} + \tau_w \theta f'(I) \frac{\partial I}{\partial \tau_c}
\]

\[
= \beta(\theta f(I)(1 - \tau_w) - s) + \tau_c(\theta f(I)(1 - \tau_w) - s) \frac{\partial \beta}{\partial \tau_c} - (1 - \tau_c \beta)((1 - \tau_w)\theta f'(I) \frac{\partial I}{\partial \tau_c}) + \theta f'(I) \frac{\partial I}{\partial \tau_c}
\]

\[
= - \frac{\partial c}{\partial \tau_c} + \theta f'(I) \frac{\partial I}{\partial \tau_c}
\]

Deadweight loss

The conditions in the text C1a.4 and C1b.4 come from rearranging the first-order conditions above.

E-mail address: nathan.seegert@gmail.com