Dynamic Risk Management: Theory and Evidence*

Frank Fehle and **Sergey Tsyplakov**

September 2, 2003

*Frank Fehle is at Barclays Global Investors and the University of South Carolina; E-Mail: `ffehle@moore.sc.edu`; Phone: (803) 777-6980. Sergey Tsyplakov is at the University of South Carolina; E-Mail: `sergey@moore.sc.edu`; Phone: (803) 777-4669. Fax: (803) 777-6876. Mailing address: BFIRE, Moore School of Business, University of South Carolina, Columbia, SC 29208. We would like to thank seminar participants at 2003 Annual Meeting of the Western Finance Association, the University of South Carolina, and, especially, Tim Adam, Shingo Goto, David Haushalter, Ted Moore, Greg Niehaus, Ajay Patel, Sheridan Titman, Stathis Tompaidis, and Donghang Zhang. Authors are grateful to the Caesarea Center and the Western Finance Association for awarding this paper for the 2003 best paper on risk management. The views and opinions expressed in this paper are the authors' and do not represent Barclays Global Investors.

Dynamic Risk Management: Theory and Evidence

Abstract

We present and tests an infinite horizon, continuous time model of a firm that can dynamically adjust the use and maturity of risk management instruments whose purpose is to reduce product price uncertainty thereby mitigating financial distress losses. The dynamic setting relaxes several restricting assumptions common to static models. Specifically, we assume that 1) the firm can adjust its use of risk management instruments over time, 2) risk management instruments expire as time progresses and that the available maturity of the risk management instruments is shorter than the life time of the firm, and 3) there are transaction costs associated with initiation and adjustment of risk management contracts. The model produces a number of new time series and cross-sectional implications on how firms use short-term instruments to hedge long-term cash flow uncertainty. Numerical results describe the optimal timing, adjustment, and rolling-over of risk management instruments, and the choice of contract maturity in response to changes in the firm's product price. The model predicts that firms that are either far from financial distress or deep in financial distress neither initiate nor adjust their risk management instruments, while firms between the two extremes initiate and actively adjust and/or roll-over their risk management instruments. Using quarterly panel data on gold mining firms between 1993 and 1999, the paper finds evidence of a non-monotonic relation between measures of financial distress and risk management activity consistent with the model. We also provide evidence supportive of the model's predictions with respect to the maturity choice of risk management contracts.

JEL Classifications: G30, G32, G33.

Keywords: risk management, dynamic, maturity choice, distress, default, transaction costs.

1 Introduction

Much of our understanding of corporate risk management is based on static models that describe how various capital market imperfections give firms an incentive to reduce risk. While existing models provide rich intuition as to *why* firms should manage risk, they provide fewer predictions about *how* firms translate the incentives to manage risk into actual decisions on the choice of risk management instruments and how these strategies evolve over time.

Our main contribution is to present and test a *dynamic* model of corporate risk management in a continuous-time and infinite-horizon framework.[1] We analyze issues, which are difficult to address in static models, including the optimal timing to initiate risk management contracts, early termination, replacement of expiring and terminated contracts, contract maturity choice, and frequency of adjustment. Many static models assume that firms make one-period decisions to hedge and that these decisions are irreversible and costless.[2] Therefore one-period models also often implicitly assume that the employed risk management instruments have the same duration as the lifetime of the firm. Treating risk management choices as irreversible limits the ability of the static models to recognize the value of dynamic risk management in adapting to changes in market conditions and firm characteristics. The fact that most risk management instruments have shorter maturities than the duration of the firm's operations has important implications for the timing and sequence of risk management decisions and it provides an intuition for the limited effect of risk management on firm exposures observed in empirical studies such as Géczy, Minton, and Schrand (1999), Petersen and Thiagarajan (2000), and Allayannis, Brown, and Klapper (2001).

Following the static model of Smith and Stulz (1985), our model motivates risk management via financial distress costs which are incurred when the firm's product price declines below costs.[3] As a consequence, the model captures the suggestion by Stulz (1996) that firms use risk management not to reduce volatility per se but rather to avoid costly lower-tail outcomes that lead to financial distress. In our model the firm chooses both the timing and maturity of hedging contracts where the

[1] Other papers on hedging that use a continuous-time framework include Stulz (1984), Ho (1984) and Leland (1998).

[2] Throughout the remainder of the paper the terms hedging and risk management are used interchangeably

[3] Although not explicitly modelled, the framework can also accommodate costly external financing as in Froot, Scharfstein, and Stein (1993) as an incentive to manage risk. The model does not directly incorporate other incentives to manage risk which are suggested by existing static theories such as for example taxes (Smith and Stulz (1985)), information asymmetry between managers and investors (DeMarzo and Duffie (1991)), or managerial risk aversion (Smith and Stulz (1985)). Stulz (2002) provides an up-to-date review of risk management rationales.

maximum available maturity is shorter than the duration of the firm's operations and expected cash flows. Hedging contracts are modelled as a portfolio of forward contracts (with an initial contract value of zero) on the firm's product price which is the source of uncertainty in the model evolving as a stochastic process.

In the model the firm faces the question of how to use short-term instruments to hedge long-term operations. Moreover, in a multiperiod dynamic setting, the firm constantly faces the problem that hedging positions entered into earlier may loose their effectiveness as product prices change during the maturity of the contract. Thus, the firm continuously reevaluates and decides whether and how often to adjust its hedging position before expiration, or to wait and keep an existing contract and to enter a new position (if any) upon expiration. We also model the transaction costs associated with both the initiation and early termination of risk management contracts, which complicates the firm's problem further. On one hand, longer term contracts are more favorable since the firm does not have to replace its contracts very frequently. On the other hand, longer-term contracts are less flexible and incur transaction costs should the firm decide to terminate them before they expire. The idea that transaction costs are important determinants of risk management decisions is supported by empirical research such as Nance, Smith and Smithson (1993), Mian (1996), and Géczy, Minton and Schrand (1997) who find evidence consistent with economies of scale of risk management. In practice, the finite life of risk management instruments is most obvious where derivative securities are concerned: there are few liquid derivative security markets (OTC or exchange) which offer maturities beyond ten years. Thus, the theoretical results are most applicable to risk management via derivative securities or other financial risk management contracts. However, arguably even other risk management tools, such as operational hedges, may have a maturity which is shorter than the firm's potentially infinite horizon, and therefore eventually may have to be replaced as their effects expire.

A number of new insights arise from the model with respect to both the time-series and cross-sectional properties of risk management strategies. First, the model implies that there is a non-monotonic relation between risk management activity and product price (and firm leverage). For a firm with fixed debt, the optimal risk management strategy depends on the spot level of the firm's product price, relative to the firm's total costs. At very high prices, firms neither initiate new risk management contracts nor adjust existing contracts as financial distress is not imminent. As prices fall and financial distress becomes more likely, firms enter the active risk management zone in which they are more likely to initiate risk management contracts and actively replace and roll them over in order to avoid financial distress. However, as prices fall further, and firms become deeply distressed, they are again less likely to initiate or adjust risk management contracts. Cross-sectionally, the model suggests a similar non-monotonic relation between factors such as leverage, which affect the likelihood of financial distress, and risk management. While this non-monotonic relation has been suggested previously by Stulz (1996), our model is the first to explicitly generate the relation from a dynamic model.[4]

Secondly, the paper provides new results regarding the maturity choice of risk management contracts. Our model indicates that more deeply distressed firms tend to choose shorter maturities for newly initiated risk management instruments. Also, the model predicts that firms with higher transaction costs tend to change their risk management contracts less often and choose longer maturities. The results also imply that a firm with greater product price volatility chooses risk management contracts with longer maturity.

With respect to the optimal adjustment of risk management instruments, the optimal roll-over strategy of risk management contracts is quite different from mechanical replacement of expiring contracts. Optimal roll-over and replacement decisions depend on the features of risk management contracts already in place, which are more likely to be replaced before they mature if they are out-of-the-money. For portfolio of forwards contracts, this implies that the frequency of risk management contract adjustments should be higher during periods of rising spot prices than during periods of falling spot prices. The features of existing contracts, such as moneyness and remaining maturity, are determined by the historic path of the spot price. Empirically, this implies that firms with identical characteristics which are observed at different points in time but with the same market conditions may have very different risk management contracts in place because the observed firms may have reached a given state via different paths. Similarly, identical firms at the same current spot price may have differing risk management contracts in place, if their histories of firm-level characteristics (e.g. leverage) are different. Therefore, in some cases empirical tests of risk management rationales could be improved by incorporating information about the preceding price and firm history.

To illustrate the applicability of the model, its parameters are calibrated to be consistent with empirical observations of firms in the gold mining industry. Specifically, parameter values are selected that roughly match the time-series properties of gold price returns and the financial ratios and production costs of gold mining firms during the sample period of 1993 to 1999, which is used for the

[4]The non-monotonic relation is also consistent with evidence such as Mian (1996), Tufano (1996), and Géczy, Minton, and Schrand (1997) finding only weak evidence that proxies of distress costs have a positive *monotonic* effect on corporate risk management.

empirical work. Given the calibrated parameters, we show that optimal risk management policies may have little impact on equity volatility, which is consistent with existing empirical findings. The reason is that, at a given time, only the portion of the firm's future cash flows, which occur during the risk management instruments' finite duration, can be hedged.

Several predictions of the model are tested using quarterly derivatives use data for gold mining firms between 1993 and 1999. We employ a similar measure of risk management activity as in Tufano (1996), and Brown, Crabb, and Haushalter (2001). The empirical tests regarding risk management activities do not provide a comprehensive analysis of all existing models, but rather focus on some of the new predictions motivated by our model. The empirical tests indicate that there is indeed evidence of non-monotonicity between the measure of risk management activity and the likelihood of distress as proxied by several measures such as leverage and quick ratio. We also find a significant relation between gold spot prices and risk management activity as suggested by the model.

The tests also extend the existing empirical literature by contributing an analysis of maturity choice of risk management instruments. Based on a measure of the weighted-average maturity of a firm's risk management contracts, we find evidence of a non-monotonic relation between leverage and maturity consistent with the model's predictions.

The remainder of the paper proceeds as follows: the next section introduces the model, and then solutions for the valuation of the firm's equity and the choice of the risk management strategy are provided in section 3; section 4 provides numerical results. Empirical evidence is presented in section 5. Section 6 concludes.

2 Dynamic Risk Management Theory

2.1 Overview

This section develops a continuous time, infinite horizon model of a firm which endogenously and dynamically adjusts its risk management contract which is a function of the firm's exogenous product price.

The model can be described by the following timeline:

<u>At time 0</u>

- The levered firm decides whether to initiate a risk management contract (guaranteeing a set of forward prices for a certain fraction of the firm's output), and chooses its maturity.

Each subsequent time period

◦ The firm produces one unit of product at a fixed cost and realizes cash flows that are determined by the current spot price and the price guaranteed by the risk management contract (if any) and whether or not the firm is in financial distress.

◦ The firm can default, in which case the debtholders recover part of the firm's value and the equityholders get nothing and are obligated to terminate (pay out or cash out) any outstanding risk management contracts, or,

◦ If not in default, the firm meets its periodic debt payments and pays production costs, and then makes a decision with respect to its risk management strategy, i.e.

- the firm can enter a risk management contract and choose its maturity;

- if the firm currently operates with a risk management contract in place, it can choose to terminate the contract early and to cash out (or to pay out) its current position at a fair market value. Both the initiation and the termination of the risk management contract generate transaction costs.

◦ The residual cash flow after debt payments and production costs is paid to the equityholders as dividends.

The firm is assumed to default on its debt optimally, i.e. when the market value of the firm's equity becomes zero. The firm's decisions with respect to the risk management strategy are made from the perspective of the shareholders who maximize the value of their equity stake.[5] Both equity and debt are priced fairly taking into account the risk management strategy of the equityholders. Because of a need to limit the dimensionality of the model, we are forced to make several modeling compromises. First, we do not allow the firm to change the structure of its debt over time. Second, we assume that the firm holds no cash, which implies that it pays all its residual cash flows as dividends. The following subsections present a detailed description of the model.

[5]For comparison reasons we can also consider the choice of the risk management policy from the perspective of all claimholders of the firm to maximize the total value of the firm's debt and equity.

2.2 Spot Price and Production Costs

The firm continuously produces a unit of product, which can for example be viewed as a commodity, whose spot price p, continuously evolves through time and is described by the log-normal process:[6]

$$\frac{\mathrm{d}p}{p} = (r - a)\mathrm{d}t + \sigma \mathrm{d}W_p \tag{1}$$

where W_p is a Weiner process under the risk neutral measure Q, σ is the instantaneous volatility coefficient, r is the risk free rate which is assumed to be constant and a, $(a \geq 0)$, is the convenience yield. The cost of production of one unit of product c $(c \geq 0)$ is assumed to be constant. Revenue uncertainty driven by variation in the output spot price is the only source of uncertainty explicitly modelled. In this sense the results will be most applicable to firms facing less cost uncertainty and more revenue uncertainty such as firms in many extraction industries (oil, gold, etc.) which have fairly predictable production costs.

2.3 Risk Management Contracts

At any time the firm can choose to enter (or terminate, if any) a risk management contract that aims to reduce temporarily the risk related to the product's price uncertainty.[7] The risk management contract guarantees a predetermined price for a fixed fraction h, $(0 \leq h \leq 100\%)$, of the firm's product for the chosen maturity τ. When the firm enters a new risk management contract, it chooses the contract's maturity τ, $\tau \leq T$, where T is the maximum maturity available.

Parameter h is constant and can be viewed as the *hedge ratio*, which determines what portion of the cash flow is hedged. While the hedge ratio can potentially be endogenized in the model, we believe that this would add further complexity and distract from the focus of the current analysis which is the optimal timing and maturity choice of risk management contracts.[8] However, we provide comparative statics in section 4.3.2 to assess the effect of different levels of the hedge ratio parameter. While spot price uncertainty for the firm's product is the only source of uncertainty explicitly modelled, choosing

[6] The model can easily be extended for any reasonable price process. For example, we can assume that the price follows a mean reverting process of the Ornstein-Uhlenbeck type, or the two factor process introduced in Gibson and Schwartz (1990).

[7] We assume that the production quantity is non-stochastic. Static models that incorporate quantity risk include Koppenhaver (1985), Morgan, Shome, and Smith (1988), and Brown and Toft (2002).

[8] Static models that incorporate the choice of the hedge ratio include Froot, Sharfstein and Stein (1993), and Kerkvliet and Moffett (1991).

a fixed hedge ratio of less than 100% is also a simple way to assess the effect of additional (non-hedgeable) "background" risk which is not explicitly modelled in the current setting.

The risk management contract consists of a portfolio (continuum) of infinitesimally small (in terms of notional amount) fairly priced forward contracts with continuous maturities between 0 and τ.[9] At the moment of origination, the risk management contract has zero expected value. Given the level of the spot price p_t at origination, the risk management contract guarantees the price of $p_t^* = p_t$ at the current time t, and in the next period $t + \Delta t$ the contract guarantees the price of $p_{t+\Delta t}^* = p_t \cdot e^{(r-a)\Delta t}$, two periods from t the contract guarantees the price of $p_{t+2\Delta t}^* = p_t \cdot e^{(r-a)2\Delta t}$, and so on until maturity. The entire contract guarantees the firm a price schedule for its product $\{p_t^*, p_{t+\Delta t}^*, p_{t+2\Delta t}^*, ..., p_{t+\tau}^*\} = \{p_t, p_t \cdot e^{(r-a)\Delta t}, p_t \cdot e^{(r-a)2\Delta t}, ..., p_t \cdot e^{(r-a)\tau}\}$ during the contract maturity τ, $(\tau \leq T)$. Thus, each risk management contract can be uniquely described by a pair $\{p^*, \tau\}$, where p^* is the contract guaranteed price at which the firm is entitled to sell its product at the current time t, and τ is the time remaining in the contract before it matures. At a given time, p^* is sufficient to uniquely calculate the prices of the remaining maturity predetermined by contract. Although, initially the contract has expected value of zero, its value ("moneyness") will fluctuate as the spot price changes and maturity gradually declines.

The market value of the risk management contract is the present value of the remaining cash flows. Thus the fair value of a risk management contract that has remaining maturity τ is $h \cdot V_t(p^*, p, \tau)$ where:

$$V_t(p^*, p, \tau) = (p^* - p) \int_0^\tau e^{(r-a)s - rs} ds = \frac{(p^* - p)}{a} \cdot [1 - e^{-a\tau}], \quad \tau \leq T \tag{2}$$

where p is the spot price, and p^* is the price at which, according to the contract, the firm is entitled to sell fraction h of its product at the current time t.

If not terminated earlier, the contract expires at its maturity, so that at maturity the firm can either enter a new contract at then prevailing forward prices or go on selling the product at the spot price for some time with a "real option" to enter a new contract at any time in the future. The firm, however, can also terminate an existing contract at any time prior to maturity. In that case the firm either receives the fair value of the remaining cash flows associated with a contract $h \cdot V_t(p^*, p, \tau)$, if the contract is in the money (i.e. if $p^* > p$), or it has to pay the fair value for the contract, if it is

[9] Alternatively we can assume that the firm enters into a continuum of any reasonable derivative contracts including plain vanilla options. We believe that if we consider a richer set of derivative instruments, it would not alter the qualitative results regarding the timing of the risk management contracts and the choice of maturity. Static models incorporating different payoff functions for the hedging contracts are Adam (2002a), and Brown and Toft (2001).

out of the money (i.e. if $p^* < p$).

We assume that the firm has to pay constant transaction costs TC when it initiates a new contract and terminates a contract initiated earlier.[10] Such fixed transaction costs can occur both due to fixed components of trading costs such as brokerage fees as well as due to fixed cost components of administering the risk management contract within the firm (e.g. internal reporting and accounting). The firm adjusts its risk management contracts in a discrete manner since the transaction costs preclude the firm from performing continuous adjustment and roll-over of its risk management position.

One possible risk management strategy for the firm is to adjust and *roll-over* its risk management contracts in the sense that the firm can always replace an existing risk management contract with a new contract of longer maturity by simultaneously terminating the current risk management contract and initiating a new one at the prevailing spot price. Hereby the firm faces a trade-off between incurring transaction costs and operating with a potentially suboptimal contract entered into earlier.

It would be more realistic to allow the firm to have multiple hedging contracts with overlapping maturities. However, that would result in a more complicated hedging structure. Therefore, in order to avoid the need for tracking the multiple contracts and their maturities, we assume that the firm adjusts its hedging position by repurchasing its current hedging contract and entering a new one.

2.4 Debt and Firm Cash Flow

We assume that the firm issues a perpetual non-callable coupon bond. The amount of debt is exogenous and stationary and the equityholders pay a continuous coupon d. The firm uses its income to meet its debt obligation, with any residual being paid out as a dividend to the equityholders. In practice, a firm may retain part of its cash flow and then use it for future debt service, which may affect the risk management strategy and the valuation of equity. Similarly, we ignore the option to store the product and to time the sale of the product. Although feasible, incorporating these features would lead to an increase in the dimensionality of the model and would further complicate the analysis.

The firm's income depends on whether or not the firm has a risk management contract. Thus, the firm's instantaneous dividend at any time t equals either $p - c - d$ if the firm has no risk management

[10] Transaction costs of initiating the portfolio of futures contracts can also be associated with the requirement to post margin at the clearinghouse. Bid-ask spreads and execution costs may also be viewed as a part of the overall transaction costs of a risk management program. (see Ferguson and Mann (2001))

contract outstanding, or $h \cdot p^* + (1-h)p - c - d$ otherwise, where p^* is the price according to the risk management contract originated earlier, c is the constant cost of production of one unit of product.

If there is insufficient cash flow to meet the debt payment, the firm can raise capital by issuing equity. The conditions under which outside capital can be raised, and the costs associated with raising it, will be described in the next section.

By assuming that the firm operates with static debt we ignore potential interactions between capital structure and risk management decisions. In principle, it is possible to extend the model by allowing equityholders to change the level of the debt dynamically as it was done in Mauer and Triantis (1993) or in Titman and Tsyplakov (2002).

2.5 Financial Distress

When the firm's instantaneous cash flow cannot cover the debt payments, the firm experiences *financial distress*. In this situation the firm is required to issue equity. We assume that in financial distress the firm incurs additional cash flow losses because customers, suppliers, or strategic partners may not be willing to deal with financially distressed companies. Unlike default costs that are incurred by debtholders at bankruptcy, distress costs are directly borne by equityholders. These costs are important because they may be incurred long before bankruptcy is imminent and they provide incentives to manage risk. While the requirement of insufficient cash flows is convenient for modelling purposes, in practice, financial distress can occur even when cash flows are low relative to required debt payments but are not yet insufficient.

The magnitude of financial distress costs in the model is determined by how low the firm's cash flow falls relative to the debt payments and production costs. We assume that financial distress costs are proportional to the difference between the firm's required debt payments and its income net of production costs. Specifically, if the firm's dividend rate is negative, i.e.

$$p' - c - d < 0, \tag{3}$$

then the firm incurs distress costs equal to

$$C^P_{Distress} \max[0, \ -p' + c + d], \tag{4}$$

where $C^P_{Distress}$ is constant, and p' equals either the spot price p_t (if the firm does not have an outstanding risk management contract) or otherwise, $p' = hp^* + (1-h)p$, where p^* is the price predetermined by the risk management contract. The firm with a risk management contract outstanding may also be in distress if the price guaranteed by the contract is below the debt payments, while the spot price can actually be above it.

Financial distress, as modeled here, does not create any permanent damage to the firm, but rather causes temporary cash flow loss. In other words, the distress situation does not directly affect the future productivity of the firm. Allowing for permanent damage would require us to keep track of the duration of distress, which would increase the dimensionality of the problem.

2.6 Default and Bankruptcy

The firm defaults optimally (incorporating the value of the risk management contract) when the value of its equity is zero. We assume that in the event of default the equityholders get nothing and the debtholders recover the value U of the unlevered firm minus default costs DC proportional to U, i.e. at default the debt value satisfies $D(p) = (1 - DC) \cdot U(p)$. For simplicity, we assume that the unlevered firm has no access to risk management and that the unlevered firm can permanently shut down its operations when the price drops sufficiently below the costs. The price at which the unlevered firm shuts down its operations is endogenously determined.[11] We assume that at default the equityholders are obligated to terminate (payout or cash out) an outstanding risk management contract $\{p^*, \tau\}$ at a fair market price, $h \cdot V_t(p^*, p, \tau)$, where $V_t(p^*, p, \tau)$ is the value of the contract as described in (2). The last assumption specifies that the counterparties of the risk management contract never default on the their contract payments. Later results show that the firm (behaving optimally) never reaches the default boundary while holding an out-of-the-money contract. Thus the assumption effectively only requires that the other counterparty be without default risk. This is a typical assumption of most theoretical models including Stulz (1984), Smith and Stulz (1985), and Froot, Scharfstein and Stein (1993). An exception is the work by Copper and Mello (1999) who assume that the pricing of the hedging contracts incorporates a premium (spread) that reflects the level of default risk associated with the firm seeking to hedge. As a result, in their model, the terms in the hedging contract affect the choice of the hedging strategy.

[11] In the context of the model, given debt in place, what is assigned as the value of the firm at default, does not affect the risk management strategy. However, this assumption may affect the pricing of the debt.

3 Valuation

The valuation of equity and debt both depend on the firm's risk management strategy. Since we assume complete markets for the firm's product, debt and equity can be regarded as tradeable financial claims for which the usual pricing conditions must hold. Effectively, the model assumes that the information about the product prices and the risk management strategy of the firm is publicly available. The following sections discuss the valuation of equity for the levered firm. The valuation of the unlevered firm is discussed in the appendix.[12]

3.1 Equity Valuation

The equity value $E = E(p, p^*, \tau)$ is the net present value of the cash flows to shareholders that depend on the state variables, which include the spot price p, the price p^* of the firm's current risk management contract, and τ the contract's remaining maturity. The values can be determined by solving stochastic control problems with free boundary conditions, where the control variable is the decision variable $i = i(p, p^*, \tau) \in \{\tau, 0, -1\}$ that describes the firm's decision either 1) to initiate the risk management contract, 2) to keep its risk management position unchanged, or 3) to terminate an existing contract. If $i(p) = \tau$, the firm initiates the contract with maturity τ; if $i(p, p^*, \tau) = -1$, the firm terminates the outstanding contract $\{p^*, \tau\}$; and if $i(p, p^*, \tau) = 0$ the firm keeps its decision unchanged. Note that the decision to terminate the contract depends upon all three state variables, while the decision to initiate the contract depends only on the spot price, since without an existing contract the firm is in the state $(p, p^*, 0)$ for any p^*. Note that in the states where $\tau = 0$ (i.e. no remaining maturity of the contract), the firm does not have a contract outstanding, and the equity value satisfies $E(p, p^*, 0) = E(p, p^{*\prime}, 0)$ for any p^* and $p^{*\prime}$. At the states where $\tau > 0$, the firm has a contract outstanding whose value depends on the current price p^* guaranteed by the contract.

In each state (p, p^*, τ), the shareholders choose their risk management strategy as well as their default policy to maximize the market value of their equity $E(p, p^*, \tau)$. Using standard arbitrage arguments and accounting for the transaction cost of initiating and terminating the risk management contract, the value of the equity is given by the solution to the following stochastic control problem:

$$\max_{i \in \{\tau, 0\},\ 0 < \tau \leq T} [\frac{1}{2}\sigma^2 p^2 E_{pp} + (r - a)pE_p - rE + p - c - d - C^P_{Distress} \max[0,\ -p + c + d]] = 0,$$

$$\tau = 0,\ \text{the firm has no risk management contract in place} \quad (5)$$

[12] An appendix (not included in the paper) describing the debt valuation is available upon request.

$$\max_{i\in\{-1,0\},}[\frac{1}{2}\sigma^2 p^2 E_{pp} + (r-a)pE_p + (r-a)p^* E_{p^*} - rE - E_\tau$$
$$+ hp^* + (1-h)p - c - d - C^P_{Distress}\max[0,\ -hp^* - (1-h)p + c + d]] = 0,$$
$$0 < \tau \leq T,\ \text{the firm has the active risk management contract } \{p^*, \tau\}. \quad (6)$$

where subscripted equity values denote partial derivatives. Note that since we are dealing with an infinite horizon model, the value of the equity is independent of time, i.e., $E_t(p, p^*, \tau) = 0$. The term $-E_\tau$ in (6) represents a linear decrease in the remaining maturity of the outstanding risk management contract as time progresses.

There is a set of free-boundary and smooth pasting conditions that the equity value satisfies. Denote p^1 as the market price at which the firm optimally initiates a risk management contract with maturity τ, i.e. $i(p^1) = \tau$. The free boundary condition when the firm is initiating the contract with maturity τ is the following :

$$E(p^1, p^*, 0) = E(p^1, p^1, \tau) - TC, \qquad \text{for any } p^* \text{ and } \tau,\ 0 < \tau \leq T \quad (7)$$

Denote $p^0(p^{*0}, \tau^0)$ as the price at which the firm terminates its current contract $\{p^{*0}, \tau^0\}$, i.e., $i(p^0, p^{*0}, \tau^0) = -1$. At the boundary at which the firm terminates its current contract $\{p^{*0}, \tau^0\}$, the equity value satisfies:

$$E(p^0, p^{*0}, \tau^0) = E(p^0, p^*, 0) - TC + h \cdot V(p^0, p^{*0}, \tau^0), \quad (8)$$

where TC are the transaction costs introduced earlier and $V(p^0, p^{*0}, \tau^0)$ is the fair value of the outstanding contract $\{p^{*0}, \tau^0\}$ given market price p^0, as calculated in (2).

In the state where an existing contract expires $\{p^*, \varepsilon\}$, i.e., $\varepsilon \xrightarrow{+} 0$, one has the following boundary condition

$$E(p, p^*, \varepsilon) \rightarrow E(p, p^*, 0),\ \text{as } \varepsilon \xrightarrow{+} 0,\ \text{for any } p^*. \quad (9)$$

One also needs to impose the free boundary condition which ensures that the equity value is greater or equal to zero for any firm that has no outstanding contract:

$$E(p, p^*, 0) \geq 0. \quad (10)$$

In the default region one needs to consider two cases: 1) the firm defaults without any risk management contract outstanding, and 2) the firm defaults with an outstanding contract. In the latter case the model assumes that the equityholders are forced to terminate the contract at a fair market price and pay (or receive) the proceeds:

$$E(p^0, p^{*0}, \tau^0) = h \cdot V(p^0, p^{*0}, \tau^0), \qquad \text{for } 0 < \tau^0 < T \quad (11)$$

4 Numerical Results

In the following section we calibrate the model to match the data used in the empirical work. Subsequently, we characterize the firm's optimal risk management dynamics based on the numerical solution and provide comparative statics.

4.1 Calibration and Parameter Values

The model is calibrated to match empirical observations for firms in the gold mining industry, which are described in more detail in section 5. In particular, the calibration seeks to replicate a firm which continuously produces one ounce of gold per year. The parameters of the spot price are calibrated to the gold prices observed during the period between Jan-1992 and Dec-2000. For this period, the daily COMEX gold closing prices obtained from Bloomberg fluctuate between \$242 and \$410 per oz, with an average price of $\$346/oz$. Given that the model assumes that the firm produces one unit of product per year, in the numerical simulations, the price value is varied within a given range with an initial $p = \$345/oz$. Volatility is calculated for daily, monthly and quarterly gold price returns, and equals 11.8%, 10.4%, and 12.1%, respectively. For the base case parameter values, the volatility of the spot price σ is set at 10%.

The 12-month lease rate for gold is used as a proxy for the convenience yield of gold. The average lease rate for gold obtained from Bloomberg for Feb-1995 to Jan-2000 is 2.04%, therefore the value for the convenience yield a is set at 2%.[13] For the same period, the rate of the US three-month treasury bill fluctuates between 6.05% and 2.85%. For the numerical analysis the risk free rate is set to $r = 4.0\%$.

In order to parametrize the level of production costs we use the data reported in Tufano (1996); given his sample of more than 90 US and Canadian gold mining firms, he documents that the average (median) cost is between \$239 and $\$243/oz$ ($\$235$-$\$239/oz$) with a standard deviation across firms of $\$58/oz$. Therefore, for the base case parameter values, production costs of one oz of gold are approximated at $c = \$250/oz$.

The parametrization of the level of the coupon payment d requires an analysis of the level of short-term and long-term debt obligations of the firms in the gold mining industry. Compustat quick ratios in our sample vary between 0.1 and 16.2, with an average of 2.9 and a median of 2.1, while

[13] Schwartz (1996) reports similar numbers in his calibration approach for spot and futures prices as well as for the average convenience yield. See also Fama and French (1988).

the empirically observed mean leverage ratio is 17% (median is 18%) with a standard deviation of 13%. We use the observed quick ratio as a proxy for the ratio of net income to debt payments, which in our model is represented by the ratio of net income to coupon payment, $\frac{p-c}{d}$. Given the assumption in the model that the firm produces one unit (one oz) of gold, we roughly match the observed quick ratios, by setting the level of $d = \$50$ so that the model generates a similar quick ratio, i.e. $\frac{p-c}{d} = \frac{\$345-\$250}{\$50}$ is close to 2. As we shall show later, given the above level of debt payments and other base case parameter values, the leverage generated by the model is 11% which is within the range of the empirically observed leverages for gold mining firms.

The parameters of distress costs are difficult to estimate since one cannot directly observe the cash flow losses that would be attributable to a distress situation. Opler and Titman (1994) show that during industry downturns, more highly leveraged firms experience a drop in equity values which is more than 10% greater than the drop experienced by less highly levered firms. A 10% difference in the drop of equity values implies several hundred percent in temporary cash flow losses in distress situations. For the base case parameter values, the proportional distress costs $C^P_{Distress}$ are initially set at 200% of the difference between required debt payments and the firm's income.

The hedge ratio h is also difficult to parameterize, since in reality it is a choice variable of the firm, which can vary over time, but which is held fixed in the model. Given the data on selected gold mining firms, one observes that a proxy of the hedge ratio varies across firms from 0% to 42% (a detailed description of the empirical estimate is given in section 5). For the base case parameter values, the parameter h equals 50%, which means that the risk management contract covers 50% of the generated cash flow. From the data set of gold mining firms it is observed that the maximum reported maturity of the risk management contracts is 4.6 years. Thus, in the base case, the maximum maturity T is chosen to equal 5 years.

For the calibration of the transaction costs TC of contract initiation and termination, we refer to values documented in the literature. Huang and Masulis (1999), and Ferguson and Mann (2001) report that, for commodity futures, transaction costs associated with the bid-ask spread vary between less than 1 b.p. to up to 15 b.p. as a fraction of the notional value of the contract. Note that transaction costs can also arise at the firm level due to the costs of running the risk management strategy. Therefore, for the base case, TC is set at \$6 which is less than 35 b.p. of the value of the contract to deliver one unit of commodity for 5 years at a typical price of \$345 (nominal value of the contract$= 5 \times \$345 = \$1725$). All base case parameter values are reported in Table 1.

In the subsequent sections we discuss the results for the base case parameter values followed by comparative statics, which also serve as a robustness check for the model results.

4.2 Risk Management Strategy: Base Case

4.2.1 Risk Management Zones

Table 2 reports the firm's decision to enter a new risk management contract (given that the firm does not have a contract in place or that the previous contract just expired) as a function of the spot price. In addition the table also reports the maturity of the new contract. Given the base case parameter values, the firm is in distress for spot prices below \$300. The firm will optimally default if the spot price falls below \$165.

The first result of the model is that the decision to initiate a risk management contract has a non-monotonic relation with the level of the spot price. A firm without a risk management contract in place immediately initiates a risk management contract if the price lies within the range of \$210 to \$315. For prices outside this range the firm does not initiate a new risk management contract. Thus the model predicts three distinct zones: no new contract for high prices (zone 1), new contract initiation (zone 2) for the middle range of prices; and then again no new contract (zone 3) as spot prices decrease further. It is important to stress that these zones refer to the decision to initiate a new risk management contract. However, due to transaction costs firms will not necessarily immediately terminate existing risk management contracts as soon as spot prices move out of the new contract initiation zone.

The intuition behind the non-monotonic relation between initiation and spot price is the following: for very high prices (zone 1), the probability of distress is low so the firm does not initiate the risk management contract because it incurs transaction costs and the contract is likely to expire before the price declines to the distress zone. As the spot price declines, the probability of distress increases, and at some point the firm initiates the risk management contract, since the transaction costs are more than compensated by the smaller expected (distress related) cash losses.[14] As the price declines into the distress zone, the cash losses due to distress increase and so does the probability of bankruptcy; as a result, at prices near the default boundary, the firm has less incentives to initiate the risk management contract because it would reduce (at least temporally) the default option and would

[14]Note that the firm initiates the contract before the price reaches the distress zone. This result can be explained by the fact that the firm that initiates the risk management contract can lock the price for only 50% of its cash flows while the other 50% is still vulnerable to price fluctuations.

benefit the debtholders at the expense of the equityholders. This last result is in line with the asset substitution problem identified by Jensen and Meckling (1976) that suggests that a firm near bankruptcy has an incentive to increase the volatility of its assets.

The non-monotonic relation between spot price and risk management also has important cross-sectional implications via a change in parameter d, the level of debt payments: an increase in the level of debt payments leads to a parallel shift of the zones up and down along the price scale. This result is straightforward because changes in the debt level imply parallel shifts of the distress zone and the default boundary as well. Therefore, given the same spot price, firms with different debt payments (as determined by leverage) would make different decisions with respect to the initiation of new risk management contracts.

4.2.2 Maturity Choice of Risk Management Contracts

Table 2 indicates that given the base case parameter values, the firm chooses risk management contracts with the maximum available maturity of 5 years in most parts of zone 2. However, as the price approaches the lower end of the zone, the firm chooses shorter maturity contracts. This result is driven by the default option, which becomes increasingly important at lower prices: at a price which puts the firm into distress (but not yet default), there is a high probability of early termination for a contract with long maturity either because the price could drop further and default occurs which triggers early contract termination or because the price could increase and the firm terminates the existing risk management contract early to replace it with a new one which locks in higher forward prices. In both cases the firm will incur transaction costs which can be avoided by using a shorter maturity contract which may expire (without transaction costs) before early termination becomes necessary.

The fact that, with the exception of very distressed firms, the maturity of new contracts is chosen near or at the maximum available maturity implies that one should observe demand for long-term risk management contracts. However, as mentioned in the introduction, many derivatives markets either do not exist or are highly illiquid for long maturities. This is most likely due to limited supply for such contracts. The idea that derivatives markets become less liquid as maturity increases could be incorporated into the model by allowing the transaction costs to vary with maturity.

4.2.3 Adjustment of Risk Management Contracts / Roll-Over Strategy

The previous results establish that a firm will always initiate a new risk management contract in price zone 2 given that no contract is in place at the time. Thus expiring contracts will always be replaced immediately in zone 2. However, the preceding discussion of maturity choice already alluded to the fact that the firm's risk management strategy is by no means static within price zone 2. The firm's decision to incur transaction costs for early termination and replacement of an existing contract depends on the characteristics (forward prices and remaining maturity) of the contract in place and the current spot price, which determines the forward prices available from new contracts. Ceteris paribus, if the current spot price is high relative to the spot price at which the existing contract was initiated, the firm is more willing to replace. In these cases the firm is also more willing to replace the longer the remaining maturity of the existing contract, while very short-term contracts are not replaced early as they will expire costlessly in the near future.

Table 3 analyzes the firm's adjustment / roll-over strategy of risk management contracts as a function of the current spot price p, the spot price p^* at initiation of the existing contract, and the remaining maturity τ of the existing contract. Specifically, for various combinations of p and p^* (measuring the moneyness of the existing contract), the table shows the range of remaining maturity within which an existing contract will be replaced; empty inputs in the table indicate that the existing contract is not replaced. For example, as the table shows, the firm terminates contracts issued at a price of \$270, if 1) the spot price increases to the level of \$285 and the contract remaining maturity is greater than 0.55 years, or 2) if the price increases to \$300 or above and the remaining maturity is longer than 0.25 years.

As argued above, the table indicates that within price zone 2 the firm often terminates and replaces out-of-the-money risk management contracts. The propensity to replace as proxied by τ increases as contracts are further out of the money. Intuitively, within zone 2, the firm wants to lock in at higher prices and get rid of lower price contracts to lessen the additional cash losses due to distress, even though the contract termination incurs transaction costs. But, if the price either declines or does not increase enough, the firm keeps the contract until maturity and then immediately enters a new contract if the spot price is still within zone 2. Thus, the model predicts that the firm keeps its risk management contract until it matures if it is in the money and tends to terminate the risk management contract before maturity if it is out of the money. Empirically, these results imply that, conditionally on observing the firm in zone 2, periods of price increases should lead to more frequent adjustments to risk management contracts, while these adjustments should be less frequent during periods of price decreases.

4.2.4 Evolution of Risk Management Contracts and Spot Price History

The previous sections establish that the firm will always hold a risk management contract in price zone 2, and that the firm will frequently terminate and replace risk management contracts within zone 2. However, even outside zone 2, one may still observe the firm with a risk management contract initiated earlier while the price was in zone 2, if the spot price subsequently drifts away from zone 2 during the maturity of the contract. Therefore, this section considers the evolution of the firm's risk management position. In particular, we analyze 1) the probability that the firm is observed with a risk management contract and 2) the remaining average maturity of the observed contract. Note that the remaining maturity of an existing risk management contract is not the same as the maturity choice of a newly initiated contract discussed previously.

Generally, one expects the probability of observing the firm with a risk management contract to decline as the price moves away from zone 2 either above or below. To verify the above statement we simulate ten-year spot price paths, while recording the characteristics of the risk management contract (if any) for each price.[15] Specifically, for each price level on the simulated path, we calculate the probability of observing the firm with an unexpired risk management contract, and the average maturity of the observed contract conditional on the contract being in place. Since the price level at which the path starts may affect the observed probability, the simulations are repeated for three different starting spot price levels, \$195, \$270 and \$345, which represent price levels in zones 3, 2, and 1 respectively.

The simulation results, reported in Table 4, confirm our previous intuition that for a given price outside zone 2, one can observe otherwise identical firms at different points in time that at the same price level have different risk management contracts in place. The reason for this is that the firms may have reached the same price level via different paths and some of them may still have remaining contracts initiated earlier, while for others all contracts initiated earlier may have expired. In Table

[15] Specifically, 500,000 simulated paths are run. For the simulations the drift of the spot price is adjusted from the risk neutral to the real measure in order to match the empirically observed drift of the gold price of 9.9% during 1970-2000. While simulating the adjusted stochastic process we keep track of the hedging contract of the firm and incorporate its hedging strategy calculated in (5)-(6) . If at any time the simulated price reaches the default boundary, the path is terminated and a new path is started.

4 this is evidenced by the fact that the calculated probability and the average remaining maturities of the risk management contracts vary with the starting price level of the simulations given the same current spot price level. These results imply that information about the current spot price is not always sufficient to uniquely predict whether a firm will have risk management contracts in place, and to predict the observed characteristics of the contract. For example, when the current spot price is \$345, the probability of observing a firm with a risk management contract in place is 28% when the starting spot price is also \$345. For the same current spot price of \$345, but a starting spot price of \$270, the probability of observing a firm with a risk management contract is much higher at 97%. Similarly, the average observed maturity is one year in the former case and approximately three years in the latter case. In order to adequately predict the optimal risk management strategy of the firm, one may need to have information about the past time series of the price. The results also suggest that even if two firms are exposed to identical price paths and currently have identical leverage, they might still exhibit different risk management strategies if their leverage histories, and hence their distress and default boundary histories differ.

The simulation results also have an interesting cross-sectional implication with respect to observed maturity. As shown in Table 4, risk management contracts observed outside zone 2 tend to have shorter remaining maturities than contracts inside zone 2. This is intuitive since contracts outside zone 2 are "old" contracts, which the firm does not terminate due to transaction costs. Similar to the argument given in the previous section, one can relate observed maturity to cross-sectional variation in leverage: at a given spot price, firms with low, medium, and high leverage may be observed in zones 3, 2, and 1, and therefore should exhibit short, long, and short observed remaining maturity of their risk management contracts, respectively. The results in Table 4 also reveal that the average remaining maturity of the risk management contract is the longest, if the firm is close to the distress boundary. This implies that near the distress boundary the firm more frequently replaces its risk management contracts before they mature, and thus is observed with "fresh" contracts having long remaining maturities.

4.2.5 Value Creation, Equity Exposure, and Risk Management

In order to measure the impact of risk management on firm value and on the volatility of equity, this section analyzes a firm that has no access to risk management contracts, for example, due to prohibitively high transaction costs. Table 5 reports that the difference in values between firms with and without access to risk management increases as the spot price declines. This result is straightforward because the impact of risk management becomes more important for lower spot prices.

One can also measure the extent to which the availability of risk management can affect the spot price exposure and reduce the volatility of equity returns. We compare the instantaneous equity volatility of the firm with and without access to risk management.[16] As expected, the results show that the reduction of equity volatility is greater for lower prices. However, the economic contribution appears to be relatively small. For example, at a price of $240 (distress zone), risk management can reduce equity volatility only by 2.4%. The reasons for such a small decrease in volatility are twofold: the firm can only reduce the uncertainty of its cash flows for a limited maturity, while the uncertainty of the income to be received after the maximum maturity cannot be reduced at all, and the unhedged cash flows after the maximum maturity are more uncertain and contribute a bigger portion to the overall volatility of equity, even though the later cash flows are discounted more.

4.3 Comparative Statics

The following sections examine how changing the initial parameter values affects the firm's risk management strategy. In each case, one parameter is varied while all other parameters are held at the level in the base case.

4.3.1 Transaction Costs

Table 6 provides information similar to Table 2 (decision to initiate and maturity of new contracts given that no contract is in place) for different transaction cost levels. Firms with higher transaction costs tend to have a narrower price zone in which they initiate risk management contracts and tend to choose longer maturities for the risk management contracts. This result is in line with empirical studies such as Dolde (1993), Nance, Smith and Smithson (1993), Mian (1996), and Géczy, Minton and Schrand (1997) showing that smaller firms, which arguably incur higher costs associated with maintaining risk management programs, are more often observed without any risk management contracts in place compared to bigger firms in the same industry.

[16] Based on Ito's lemma, an instantaneous volatility of the equity is given by $\sigma p \frac{E_p}{E}$, where the subscripted equity values denote partial derivatives.

4.3.2 Maximum Hedge Ratio

An increase in the value of parameter h implies that the firm can hedge a greater fraction of its cash flow with a given risk management contract. As shown in Table 7, an increase in h results in a narrower price range within which the firm initiates risk management contracts because the firm can wait longer before initiating risk management contracts. Moreover, a higher h parameter implies that the firm tends to choose shorter maturities.

One empirical implication of this result is that smaller firms and more homogeneous firms that tend to have less variety of products, are exposed to fewer risks. Such firms are able to hedge a greater fraction of their cash flows using a particular risk management contract. Thus, the empirical interpretation of the model is the following: the probability of observing a firm with risk management contracts is lower for firm types that can hedge a greater fraction of cash flows with a single risk management contract. However, note that the model does not consider multiple sources of uncertainty which in reality may be less than perfectly correlated giving less homogeneous firms a natural diversification benefit thereby reducing the need of such firms to use risk management contracts.

4.3.3 Distress Costs

Table 8 shows that an increase in the distress costs $C^P_{Distress}$ implies a narrower zone 3, which is the lower-price part of the distress zone in which the firm does not initiate risk management contracts. Also, firms tend to pick shorter maturity contracts. Intuitively, firms that loose a greater portion of cash flows in distress tend to use their risk management contracts more often within the distress zone. In addition, such firms tend to replace and roll-over their contracts more frequently which results in a selection of shorter-term contracts. Thus industries with high distress costs should exhibit shorter maturity hedging programs.

4.3.4 Spot Price Volatility

As reported in Table 9, an increase in the volatility of the spot price implies that the firm employs risk management contracts over a wider range of spot prices. Also, due to an increase in the value of the default option, the critical price level at which the firm defaults decreases as the volatility increases. For the same reason, zone 3 (the distressed risk management zone without new contract initiation) tends to widen with volatility implying that for higher volatility, the value of the default option exceeds the value of the option to reduce risk. This prediction can potentially be tested by checking whether distressed firms tend to intensify their risk management programs during periods of higher expected uncertainty.

The results also imply that a firm with greater volatility of its product price chooses risk management contracts with longer maturity. Intuition for this result can be gained by analyzing the roll-over strategy of risk management contracts. Additional results (not shown) indicate that the firm with greater price volatility tends to wait for a greater price change before terminating an existing contract. Specifically, such a firm terminates an existing contract and immediately enters a new contract, only if the spot price increases by a significantly higher amount than in the cases with lower volatility. Therefore, a firm exposed to higher volatility tends to prefer longer maturity contracts.

5 Empirical Evidence

In this section we test several predictions derived from the model with a particular focus on the time-series properties and the predicted non-monotonicity of risk management activities, and on risk management maturity choice.[17]

5.1 Data

The empirical analysis employs survey data from the gold mining industry which were used in several previous studies such as Tufano (1996), Brown, Crabb, and Haushalter (2001), and Adam (2002b).[18] The 100 companies that are included in the data set are publicly traded gold producers based in the United States and Canada. The data contain quarterly information from the first quarter of 1993 through the third quarter of 1999 on the risk management instruments held by these firms, including the amount of the firms expected future production and specific information regarding each of the firms hedging positions, for example the strike price and approximate maturity of each option. A detailed description of this data set is provided in Tufano (1996).

Following Tufano (1996), deltas (Δ) are computed for each option position, and deltas of negative one are assigned to all other short positions, and deltas of positive one are assigned to all other long

[17]We also perform an analysis along the lines of Petersen and Thiagarajan (2000) confirming the limited impact of risk management on equity exposure to variations in gold prices. The results are not shown in the paper, but are available from the authors upon request.

[18]The raw data are provided by Ted Reeve, a financial analyst for Scotia Capital Markets.

positions.[19] The hedged volume HV of each hedging position is then computed as the product of delta and notional volume NV. Each quarter hedged volume is summed over all hedge positions, and divided by the sum of forecasted production $PROD$ to arrive at the hedging delta-percentage DP. The hedging positions are categorized into annual maturity buckets of up to five years. Note that the model views risk management as a set of discrete choices of initiation and termination of risk management contracts. In the data one does not necessarily expect to observe such discrete changes in the risk management strategy. Rather one expects the intensity of risk management as measured by DP to vary with market conditions and firm characteristics as predicted by the model.

To test the model's predictions with respect to the firm's hedge maturity choice, the average maturity MAT of the firm's hedge position is computed using hedged volume as weights. To assign maturities it is assumed that each hedge position matures on the final day of the period in which it is classified in the survey. Note that both the measure of risk management activity DP and the measure of risk management maturity MAT describe the characteristics of observed risk management contracts rather than the initiation and cancellation of risk management contracts.

For each firm quarter in the survey the following data items are obtained from Compustat: market value of equity, total assets, quick ratio, total debt, and the Z-score for the likelihood of financial distress as introduced by Altman (1968). All data items are quarterly with the exception of the Z-score which has annual frequency. Leverage is computed as total debt divided by total assets.

Compustat data are available for 51 of the 100 companies which appear at least once in the survey. Compustat firms appear on average in 17 of 27 survey quarters, while companies not on Compustat appear on average in 12 of 27 survey quarters. Since we intend to study the time-series properties of risk management activities, 15 of the 51 Compustat firms are excluded, because they appear in less than 12 survey quarters, which may bias our sample to larger, more successful firms. The remaining 36 firms constitute the data set for the empirical work.

Summary statistics are shown in Table 10. The mean and median hedging delta-percentage are 15% and 12%, respectively. Both values are comparable to the results of Brown, Crabb, and Haushalter (2001), who use a sample similar to ours. Mean and median maturity of the hedging instruments is 1.6 years. Both hedging delta-percentage and maturity exhibit considerable time-

[19]We assume that all options mature on the final day of the period in which they are classified in the survey. The delta for an option contact is calculated using the Black-Scholes model, volatility is based on the annualized standard deviation of gold prices for the previous 90 trading days. The price of gold is the closing price on COMEX from Bloomberg. Risk-free rates are from Bloomberg. Gold lease rates are from Bloomberg and Kitco.

series variation as measured by the cross-sectional average of each variable's within-firm time-series standard deviation. Mean and median leverage are 18% and 17%, respectively, with considerable cross-sectional variation, which is also present for quick ratios and Z-scores.

5.2 Univariate Results

5.2.1 Risk Management and Financial Distress

One testable implication of the model is the non-monotonic relation between risk management activities and the likelihood of financial distress. The model predicts that firms which are deep in financial distress and firms which are far away from financial distress have less incentives to reduce risk than firms which are between the two extremes.

To test this prediction of non-monotonicity cross-sectionally we use leverage, the quick ratio, and the Compustat Z-score, which is based on Altman (1968), as measures for the likelihood of financial distress. For leverage and the quick ratio, we divide all observed firm quarters into three equal-sized groups (proxying low, medium, and high likelihood of distress). For the Z-score, firm quarters are assigned to the three groups based on the cut-offs of 1.81 and 3.00 used by Altman (1968). As shown in Table 11, these sorts generate dispersion in the measures across the groups.

We then compare risk management activity as proxied by average hedging delta-percentages for the three groups based on each of the measures. The average hedging delta-percentage increases when moving from the low likelihood of distress group to the medium likelihood of distress group regardless of the measure used. Based on t-tests, the difference between the first two subsets is significant at the 1% level for the leverage and quick ratio sorts, and significant at the 10% level for the Z-score sorts. More importantly, there is also evidence of the non-monotonic relation predicted by the model: for the leverage and quick ratio sorts, hedging delta-percentages decrease as one moves from the medium likelihood of distress group to the high likelihood of distress group. The magnitudes of the decreases are 5% and 3%, respectively. The former decrease is significant at the 1% level, while the latter is significant at the 5% level. For the Z-score sorts, there is a statistically insignificant increase in the hedging delta-percentage.

The observed evidence of a non-monotonic relation between leverage and risk management activity sheds light on results in previous studies such as, for example, Nance, Smith, and Smithson (1993), and Tufano (1996), which do not find supportive evidence when testing for the positive monotonic relation predicted by a static view of risk management.

5.2.2 Risk Management and Spot Prices

The model suggests that, holding a firm's costs constant, there should be a non-monotonic relation between hedging delta-percentages and spot prices. As the spot price decreases towards the firm's costs, one expects firms to intensify (initiate) risk management suggesting a higher hedging delta-percentage. However, as the spot price falls significantly below the firm's costs, one expects the relation to reverse as firms use less risk management again.

Observing this non-monotonic relation between spot prices and hedging delta-percentages in the data is complicated by the following two issues. First, firms may not maintain constant costs during the sample period (e.g. changes in leverage may cause changes in interest expense), which would shift the spot price at which the relation reverses. Secondly, even if costs are relatively stable over time, one may not have enough variation in spot prices during the sample period to observe the same firm in the three different zones. In other words, firms with very low costs (in zone 1) may exhibit no relation between spot price and hedging delta-percentage as their risk management activities are not affected by financial distress considerations. Firms with medium costs may exhibit a purely negative relation between spot price and hedging delta-percentage as a spot price increase (decrease) moves them from zone 2 (1) to zone 1 (2). Firms with high costs may exhibit a purely positive relation between spot price and hedging delta-percentage as a spot price increase (decrease) moves them from zone 3 (2) to zone 2 (3).

To investigate the above issues the firms in the sample are split into subsets based on a proxy of costs. In Table 12, firms are split based on their average leverage (proxying for interest expense). We hypothesize that firms with extremely low leverage exhibit only a weak relation between spot prices and hedging delta-percentages, and that firms with extremely high leverage exhibit a positive relation between spot prices and hedging delta-percentages. The remaining firms with medium leverage should exhibit a negative relation between spot prices and hedging delta-percentages. Cut-offs for leverage are chosen such that approximately 10-15% of firms are in either extreme group. The sample quarters are then split evenly based on the average spot price during the quarter. Supporting the suggested non-monotonicity, the results indicate no relation between hedging delta-percentages and spot prices for firms with extremely low leverage, and a negative relation between hedging delta-percentages and spot prices in the middle group. Firms with extremely high leverage do not exhibit a significant relation which may be due to the small sample size.[20]

[20]As a robustness check we also estimate a time-series regression (results not shown) for each firm in which we regress the delta-percentage on the average spot price during the quarter, the square of the average spot price, and

5.3 Panel Results for Risk Management Activity

In this section we report regression tests for the model's predictions regarding risk management activity as proxied by hedging delta-percentages. Since the focus of the analysis is on the time-series properties of hedging delta-percentages and the non-monotonic relation between measures of the likelihood of financial distress and hedging delta-percentages, the regressions do not include other variables previously suggested in the literature which may explain cross-sectional variation in hedging delta-percentages.[21] Thus we estimate a fixed effects panel regression model allowing the intercepts to pick up potentially unexplained firm-specific variation. Specifically, the following specification is estimated for firm i and quarter t:

$$DP_{it} = c_i + \beta_1 Spot_t + \beta_2 Vol_t + \beta_3 MV_{it} + \beta_4 Lev_{it} + \beta_5 Lev_{it}^2 + \varepsilon_{it}, \tag{12}$$

where DP is the hedging delta-percentage, $Spot$ is the average spot gold price (in \$/ounce) during the quarter, Vol is the annualized standard deviation of daily gold price returns during the quarter, MV is the market value (in \$1,000,000) of the firm's equity at the end of the quarter, Lev is the firm's leverage measured as the ratio of debt to assets, Lev^2 is squared leverage, and ε is an error term. The inclusion of squared leverage is intended to pick up the non-monotonic relation between hedging and leverage. A squared term for the spot price is not included, since the univariate results in section 5.2.1 indicate that there might be insufficient variation in the spot price during the sample period.

Based on the theoretical predictions and the results in the previous sections, one expects the coefficient for the average spot price to be negative, and the coefficients for leverage and squared leverage to be positive and negative, respectively. The comparative statics in section 4.3.4 suggest a positive relation between spot price volatility and hedging delta-percentages. The comparative statics in section 4.3.1 suggest a positive relation between market value (proxying for the relative importance of transaction costs) and hedging delta-percentages. The latter two hypotheses are

the volatility of the spot price (calculated as the annualized standard deviation of daily gold returns over the 90 days preceding the end of the quarter). For most regressions we find a negative relation between average spot prices and delta-percentages.

[21] We reestimate the regressions described below (results available upon request) as pooled OLS adding several accounting variables previously used in the literature to explain cross-sectional variation in risk management. Data availability for the new variables leads to a significant reduction in sample size. However, the main results for average spot price, leverage and leverage squared are unaffected.

based on the observation that the risk management contract initiation zone widens with an increase in volatility and tightens with an increase in transaction costs.

The regression results in Table 14 show evidence of the non-monotonic relation between leverage and hedging delta-percentages: both leverage coefficients have the predicted signs and are significant at the 1% level. The negative relation between average spot prices and hedging delta-percentages also bears out in the regression results with a significance level of 7%. The coefficients for volatility and market value have the predicted positive signs but are not significant. The relation between volatility and hedging delta-percentages may not appear in the data because the volatility measure exhibits relatively low variation during the sample period. The adjusted R-Square for the regression is 50%.[22]

5.4 Risk Management Maturity

The model also provides several predictions with respect to the maturity of the risk management instruments used. To test these predictions, the following regression model is estimated:[23]

$$Mat_{it} = c + \beta_1 Spot_t + \beta_2 MV_{it} + \beta_3 Lev_{it} + \beta_4 Lev_{it}^2 + \varepsilon_{it}, \tag{13}$$

where Mat is the average remaining maturity (in years) of hedging instruments weighted by delta-adjusted hedged volume, and all other variables are as defined previously.

The simulation results in section 4.2.4 suggest that, within risk management price zone 2, maturity declines with the spot price. Similar to the results in the previous sections, the model generates a prediction of cross-sectional non-monotonicity: for a given spot price, firms that are far away from financial distress, close to but not in financial distress, and firms in financial distress should exhibit

[22] The univariate results suggest that firms with extremely low or extremely high leverage ratios may not exhibit the negative relation between spot prices and delta-percentages which is observed for the majority of firms with medium leverage. Thus the estimate of the spot price coefficient might be improved by accounting for leverage differences. We generate the following series of *adjusted* average spot prices as the product of spot price and a leverage dummy, which takes the values 0, 1, -1 for $AvgLev_i \leq Low$, $AvgLev_i > Low$ and $AvgLev_i < High$, and $AvgLev_i \geq High$, respectively. $AvgLev$ is the firm's average leverage across all firm quarters, and Low and $High$ are cut-offs which vary from 2.5% to 7.5%, and from 37.5% to 42.5%, respectively. We then replace $Spot$ in specification (12) with the adjusted series. The coefficient estimates (not shown) for the adjusted average spot price indeed tend to increase in magnitude and significance compared to the average spot price used in the base case.

[23] Firm fixed-effects are not included in this regression model since we are not aware of existing theoretical work suggesting variation in risk management maturity which is not already contained in our set of independent variables.

low, high, and low observed remaining maturity, respectively. As before, leverage is used as a proxy for the likelihood of financial distress. The comparative statics on transaction costs in section 4.3.1 show that, given a spot price, the maturity of newly initiated contracts increases with transaction costs. However, the active risk management zone (zone 2) is wider for lower transaction costs. Thus, it could also be the case that larger firms (with lower transaction costs) adjust their contracts more frequently, and thereby are more often observed with a "fresh" maturity, and consequently exhibit longer observed maturity. Therefore, the sign of the coefficient for market value in the above regression indicates which effect might dominate.

Table 14 provides the results of the above regression. The coefficients for both leverage measures are highly significant and consistent with the predicted non-monotonic cross-sectional relation between leverage and observed maturity. The coefficient for market value has a negative sign, and is significant at the 10% level indicating that the effect of a wider active risk management zone for firms with lower transaction costs dominates their choice of shorter initial maturities. The coefficient for the average spot price is insignificant. This is most likely due to insufficient variation in the observed spot price. The adjusted R-Square is 18%.

6 Conclusion

In this paper we provide a dynamic model of corporate risk management. The model incorporates the initiation, early termination, replacement, maturity choice, and frequency of adjustment of risk management instruments. While static models provide valuable intuition as to why firms manage risk, we believe that this model further develops our understanding of the dynamic aspects of risk management, which may help us better understand observed risk management practices. The model generates many new and sometimes quite different implications as compared to related static models. Moreover, the empirical evidence suggests that the model does help explain observed risk management among gold mining companies.

We believe that the model might serve as a basis for developing normative decision tools that can assist practitioners in developing the risk management strategies. For example, the model could help to develop a strategy of the optimal hedging of risks associated with predetermined long-term delivery contracts that have longer maturity than the maturity of the available risk management instruments.[24] In order to move closer toward developing realistic and implementable models, the

[24]For example, we can explore the case of hedging the oil-based delivery contracts entered by Metallgesellschaft

current model setting could be extended along a number of dimensions. First, the current model's assumption that the firm holds no cash, which implies that it cannot use cash reserves to off-set the negative impact of distress costs, could be relaxed. Also, it is assumed that the firm has to terminate its existing risk management contract before entering into a new one. This assumption can be relaxed, if one can consider a portfolio of risk management instruments of different types. Second, one could extend the model by allowing the firm to change its debt level over time. With a dynamic capital structure, the model can be used to explore the interaction between the choices of risk management contracts and capital structure policy. The model can also incorporate potential agency problems that may arise either between managers and equityholders or between equityholders and debtholders. It appears reasonably straight-forward to modify the model to capture the extent to which the risk management dynamics can be affected by managerial compensation contracts which arguably have shorter maturity than the firm's horizon. The model could also be extended to explore the idea of selective hedging whereby managers incorporate superior information relative to other market participants into the risk management strategy. Finally, one could incorporate an endogenous choice of optimal hedge ratios and/or the choice of optimal hedge payoff functions. While these potential extensions could provide additional insights, they would increase the complexity of the model beyond the focus of our analysis in this paper.

Although not explicitly explored in the paper, the model implies that other features of optimal risk management design, such as the risk management contract's payoff function, may be less important than previously thought once viewed in a dynamic setting; this is due to the fact that the firm can adjust its risk management contracts and thus is not permanently locked into a potentially suboptimal design.[25] Recognizing firms' ability to adjust risk management contracts increases the importance of the transaction costs of risk management adjustments and suggests a *trade-off between optimal design and transaction costs*. In such a setting it is conceivable that a firm would choose a contract whose payoff function is suboptimal (given the current state variables), but whose transaction costs of early termination are low compared to a contract with a preferable payoff function. For example, a firm might prefer an exchange-traded derivatives contract (without customization to the firm's specific situation) to an over-the-counter contract with high transaction costs of initiation and termination.

described in Miller and Cobb (1995), and Mello and Parsons (1995).

[25]Brown and Toft (2001) provide results as to how quantity risk and variation in the risk management horizon affect the optimal hedge ratios and payoff functions, which are also studied by Adam (2002a). The effect of quantity risk on optimal hedging is also addressed by Banerjee and Noe (2001).

References

[1] Adam, T., 2002a, Risk management and the credit risk premium, *Journal of Banking and Finance* 26, 243-269.

[2] Adam, T., 2002b, Why firms use non-linear hedging strategies, Working Paper, Hong Kong University of Science & Technology.

[3] Allayannis, G., G. W. Brown, and L. F. Klapper, 2001, Exchange Rate Risk Management: Evidence from East Asia. University of Virginia Working Paper.

[4] Altman, E., 1968, Financial ratios, discriminant analysis and the prediction of corporate bankruptcy, *Journal of Finance* 23, 589-609.

[5] Banerjee, S., and T. H. Noe, 2001, Exotics and electrons: Electric power crises and financial risk management, Working Paper, Tulane University.

[6] Barraquand, J., and D. Martineau, 1995, Numerical Valuation of High Dimensional Multivariate American Securities, *Journal of Financial and Quantitative Analysis* 30, 383-405.

[7] Brown, G. W., P. R. Crabb, and D. Haushalter, 2001, Are firms successful at selective hedging?, Working Paper, UNC Chapel Hill.

[8] Brown, G. W., and K. B. Toft, 2001, How firms should hedge, Working Paper, UNC Chapel Hill.

[9] Cooper, I. and A. Mello, 1999, Corporate Hedging: The Relevance of Contract Specifications and Banking Relationships, *European Finance Review*, 195-223.

[10] DeMarzo, P., and D. Duffie, 1995, Corporate Incentives for Hedging and Hedge Accounting, *The Review of Financial Studies*, 743-771.

[11] Dolde, W., 1993, The trajectory of corporate financial risk management, *Journal of Applied Corporate Finance*, 33-41.

[12] Fama, E., and K. French, 1988, Business Cycles and the Behavior of Metals Prices, *Journal of Finance* 43, 1075-1093.

[13] Ferguson, M. F., and S. C. Mann, 2001, Execution Costs and Their Intraday Variation in Futures Markets, *Journal of Business*, Vol. 74, No. 1. (Jan.,), 125-160.

[14] Flam, S., and R. J-B. Wets, 1987, Existence Results and Finite Horizon Approximates for Infinite Horizon Optimization Problems, *Econometrica*, 55, 1187-1209.

[15] Froot, K., D. Scharfstein, and J. Stein , 1993, Risk management: Coordinating corporate investment and financing policies, *Journal of Finance* 48, 1629–1658.

[16] Ho, Thomas S. Y., 1984, Intertemporal Commodity Futures Hedging and the Production Decision, *Journal of Finance* 39, 351-376.

[17] Huang, R, and R. Masulis, 1999, FX Spreads and Dealer Competition Across, *Review of Financial Studies* 12, 61-93.

[18] Géczy, C., B.A. Minton, and C. Schrand, 1997, Why Firms Use Currency Derivatives, *Journal of Finance* 52, 1323-1354.

[19] Géczy, C., B.A. Minton, and C. Schrand, 1999, Choices Among Alternative Risk Management Strategies: Evidence from the Natural Gas Industry, Wharton School Working Paper.

[20] Gibson, R., and E. Schwartz, 1990, Stochastic Convenience Yield and the Pricing of Oil Contingent Claims, *Journal of Finance*. 45, 959-976.

[21] Jensen, M., and W. Meckling, 1976, Theory of the Firm: Managerial Behavior, agency Costs, and Ownership structure, *Journal of Financial Economics* 4, 305-360.

[22] Kerkvliet, J., and M. Moffett, 1991, The Hedging of an Uncertain Future Foreign Currency Cash Flow, *Journal of Financial and Quantitative Analysis* 26, 565-578.

[23] Koppenhaver, G., 1985, Bank Funding Risks, Risk Aversion, and the Choice of Futures Hedging Instrument, *Journal of Finance* 40, 241-255.

[24] Kushner, H., and P. Dupuis, 1992, Numerical Methods for Stochastic Control Problems in Continuous Time, *Springer Verlag.*

[25] Langetieg, T., 1986, Stochastic Control of Corporate Investment when Output Affects Future Prices, *Journal of Financial and Quantitative Analysis* 21, 239-263.

[26] Leland, H., 1998, Agency Costs, Risk Management, and Capital Structure, *Journal of Finance* 53, 1213-1243.

[27] Mauer, D., and A. Triantis, 1994, Interactions of Corporate Financing and Investment Decisions: A Dynamic Framework, *Journal of Finance* 49, 1253-1277.

[28] Mercenier, J., and P. Michel, 1994, Discrete-Time Finite Horizon Approximation of Infinite Horizon Optimization Problems with Steady-State Invariance, *Econometrica* 62, 635-656.

[29] Mello A. and J. E. Parsons, 1995, Maturity Structure of a Hedge Matters: Lessons from the Metallgesellschaft Debacle, *Journal of Applied Corporate Finance*, 106-120.

[30] Mian, S. L., 1996, Evidence on Corporate Hedging Policy, *Journal of Financial and Quantitative Analysis* 31, 419-439.

[31] Miller, M., and C. L. Cobb, 1995, Metallgesellschaft and the economics of the synthetic storage, *Journal of Applied Corporate Finance*, 62-75.

[32] Morgan, G. E., D. Shome, S. D. Smith, 1988, Optimal Futures Positions for Large Banking Firms, *Journal of Finance* 43, 175-195.

[33] Nance, D., C. W. Smith Jr., and C. Smithson, 1993, On the determinants of corporate hedging, *Journal of Finance* 48, 267–284.

[34] Opler, T., and S. Titman, 1994, Financial Distress and Corporate Performance, *Journal of Finance* 49, 1015-1040.

[35] Petersen, M.A., and S.R. Thiagarajan, 2000, Risk Measurement and Hedging: With and Without Derivatives, *Financial Management* 29:4: 5-30.

[36] Schwartz, E., 1997, The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging, *Journal of Finance* 52, 923-973.

[37] Smith, C.W. and Stulz, R. M., 1985, The determinants of firms 'Hedging Policies', *Journal of Financial and Quantitative Analysis* 20, 391–405.

[38] Stulz, R. M., 1984, Optimal hedging policies, *Journal of Financial and Quantitative Analysis* 19, 127–139.

[39] Stulz, R., 1996, Rethinking risk management, *Journal of Applied Corporate Finance* 9, 8-24.

[40] Stulz, R. M., 2002, Derivatives, Financial Engineering & Risk Management (South-Western College Publishing, Mason, Ohio).

[41] Tufano, P., 1996, Who Manages Risk? An Empirical Examination of Risk Management Practices in the Gold Mining Industry, *Journal of Finance* 51, 1097-1137.

[42] Titman, S., and S. Tsyplakov, 2002, A Dynamic Model of Optimal Capital Structure, working paper.

7 Appendix

A Valuation of the Unlevered Firm

The value of the unlevered firm $U(p)$ satisfies the following equation

$$\frac{1}{2}\sigma^2 p^2 U_{pp} + (r-a)pU_p - rU + p - c - C^P_{Distress}\max[0,\ -p+c] = 0 \quad (14)$$

with additional boundary condition $U(p) > 0$ corresponding to the case in which the firm uses its option to permanently shut down its operations, if the spot price drops far below its production costs c.

B Numerical Algorithm

This appendix describes the numerical algorithm that is applied to solve stochastic control problems (5)- (6). For each case one needs to find a solution that satisfies simultaneously the maximization problems and partial differential equations. The algorithm is based on the finite-difference method augmented by a "policy iteration".[26]

The calculations are complicated by the fact these are infinite horizon stochastic optimization problems, where the values of the equity and debt are time independent. Therefore, numerical solutions require reformulating the model into a finite horizon approximation.[27] The procedure is initialized by approximating (guessing) values for the functions in each node of the terminal time period. This reformulation effectively implies that a derivative with respect to time is added to the equations of each optimal stochastic control problem. For example, in the valuation problem for the all-equity firm, a new term E_t is added to the left hand side in equation (5). The errors that result from the approximation of functions at the terminal time can be reduced by increasing the length of the horizon of the problem and iterating until the derivative E_t is indistinguishable from zero for each node on the grid.

For each problem we use a discrete grid and a discrete time step $\Delta t = \Delta\tau$. The state space (p, p^*, τ) is discretized using a three-dimensional grid $N_p \times N_{p^*} \times N_\tau$ with corresponding spacing

[26] See, for example, Kushner and Dupuis (1992), Barraquand and Martineau (1995) and Langetieg (1986) for the theory and applications of numerical methods for solving stochastic control problems.

[27] Flam and Wets (1987), and Mercenier and Michel (1994) discuss the approximation of infinite horizon problems in deterministic dynamic programming models.

between nodes in each dimension of $\Delta p, \Delta p^*$, and $\Delta\tau$ and where $\Delta X = \frac{X_{\max}-X_{\min}}{N_x}$ and $X \in \{p, p^*, \tau\}$; $X_{\max}$ and $X_{\min}$ are the upper and low boundaries. The grid step in each dimension is chosen to achieve stability of the algorithm. In each node on the grid (p, p^*, τ) the partial derivatives are computed according to the Euler method. For example, the first and second derivatives of the equity value with respect to p and p^* are

$E_p(p, p^*, \tau) = \frac{E(p+\Delta p, p^*, \tau) - E(p-\Delta p, p^*, \tau)}{2\Delta p}$,

$E_{pp}(p, p^*, \tau) = \frac{E(p+\Delta p, p^*, \tau) - 2E(p, p^*, \tau) + E(p-\Delta p, p^*, \tau)}{\Delta p \Delta p}$, with appropriate modifications at the grid boundaries.

B.1 Calculation of Unlevered Firm Value and Equity Value

First of all, one needs to calculate the value of the unlevered firm $U(p)$ which depends upon the price level only. Its value satisfies PDE in (14) which can be numerically solved using standard explicit finite-difference scheme taking into account boundary condition, $U(p) \geq 0$.

The procedure for the calculation of the equity and debt value is more complex since it incorporates the decision to initiate the risk management contract. The procedure starts with the approximation of the values of the equity and debt for the "terminal" time t. In each node (p, p^*, τ) the terminal values are set $E_{(t)}(p, p^*, \tau)$=max$(0,\, U(p) - \frac{d}{r}) + h \cdot V(p, p^*, \tau)$ and debt $D_{(t)}(p, p^*, \tau) = (1 - DC)U(p)$ if $E_{(t)}(p, p^*, \tau) = 0$, otherwise $D_{(t)}(p, p^*, \tau) = \frac{d}{r}$. This approximation undervalues the equity and debt since default option is ignored. However, by running backward recursion for a relatively large number of Δt steps, the values on the grid converge to the steady state values since the initial mispecifications of the terminal values are "smoothed" away due to discounting. Thus, working backward in time for each node on the grid according to the explicit finite-difference scheme and taking into account the risk management decision, the value of equity $E_{(t-\Delta t)}$ at each node (p, p^*, τ) at time $t - \Delta t$ is determined as follows:

$$E_{(t-\Delta t)}(p, p^*, \tau) = \max_{i\in\{\tau,0\}} \Big[(p' - c - d - C^P_{Distress}\max[0,\ -p' + c + d])\Delta t + e^{-r\Delta t}\mathbb{E}_Q[E_{(t)}]\Big] =$$
$$= \max_{i\geq 0}\Big[[p - i]\Delta t + E^U_{(t)}(p, p^*, \tau) + \Delta t \mathcal{L}[E_{(t)}(p, p^*, \max(0, \tau - \Delta t))]\Big] \quad (15)$$

where $\mathcal{L}[E^U_{(t)}(p, p^*, \max(0, \tau - \Delta t))]$ is the differential operator applied to $E_{(t)}$ in the node $(p, p^*, \max(0, \tau - \Delta t))$

$$\mathcal{L}[Z] = \frac{1}{2}\sigma^2 p^2 Z_{pp} + (r - a)pZ_p + (r - a)p^* Z_{p^*} - Z_\tau - rZ \quad (16)$$

in which all partial derivatives are computed according to the Euler method, where $\max(0, \tau - \Delta t)$ is the remaining maturity of the contract (if any) at time t given the maturity τ at time $t - \Delta t$. In the (15) $p' = p$, if $\tau = 0$, or $p' = hp^* + (1-h)p$ otherwise, where p^* is the predetermined price of the risk management contract originated earlier. The second equality in (15) comes from Euler decomposition of the equation in (5)-(6) in which a new term E_t^U is added, where $E_t^U = \frac{E^U_{(t+\Delta t)} - E^U_{(t)}}{\Delta t}$.

If $\tau = 0$, we check whether or not it is optimal for the equityholders to initiate a risk management contract. The equityholders initiate the contract, if for some τ

$$E_{(t)}(p, p^*, 0) < \max_{\tau, \tau \leq T} [E_{(t)}(p, p, \tau) - TC] \tag{17}$$

where TC is the transaction costs. If inequality (17) is satisfied then the equity value is set equal to the maximum over all admissible τ in the right-hand side of (17).

Also at each node we check whether or not the equity value (without value of the contract) $E_{(t)}(p, p^*, \tau)$ is non-negative. If the equity value becomes negative, default occurs i.e.

$$\text{if } E_{(t)}(p, p^*, \tau) < h \cdot V(p^0, p^*, \tau), \text{ then the equityholders default.} \tag{18}$$

We repeat this backward induction procedure for $t - 2\Delta t$, $t - 3\Delta t, .., t - N\Delta t$ until the value function for $E_{(t)}(p, p^*, \tau)$ reaches a steady state in each node on the grid, i.e., until $\max_{(p, p^*, \tau)} |E_{(t)}(p, p^*, \tau)$ $- E_{(t-\Delta t)}(p, p^*, \tau)| < \varepsilon$, where ε is the predetermined accuracy level. We have found this procedure to be robust to the choice of the values at the terminal time. The procedure for the computation of the debt value is similar.

Table 1: Base Case Parameter Values

Base case parameters for the numerical solution of the model.

Parameter	Value
σ, volatility of the product price	10%
T, maximum maturity of risk management contract	5 years
α, convenience yield	2% per year
r, risk-free rate	4% per year
$C^{P}_{Distress}$, proportional costs of financial distress	200%
TC, transaction costs of entering/canceling risk management contract	\$6
h, hedge ratio of the contract	0.5
C, production costs	\$250 per year
d, debt payment	\$50 per year

Table 2: Risk Management Strategy: Base Case

Column 4 reports the firm's decision to initiate a new risk management contract (given that there is no contract in place) as a function of the current spot price p (column 1). Column 5 reports the choice of contract maturity in years. The table also shows interest coverage ratios (column 2), price zones for initiation / no initiation of new contracts (column 6), and the price zone for financial distress and default (column 3). The distress zone is the price range where the product price is below the sum of production costs and debt payments. The interest coverage ratio is the ratio of net income to debt payments $\frac{p-c}{d}$. The parameter values are as in the base case reported in Table 1.

(1)	(2)	(3)	(4)	(5)	(6)
Spot Price	Interest Coverage Ratio	Default/ Distress/ No Distress	Initiate New RM Contract?	Maturity of New RM Contract	Risk Management Zones
165	-1.7	Default	-	-	
180	-1.4	Distress	No	-	Zone 3
195	-1.1	Distress	No	-	
210	-0.8	Distress	Yes	4.7	
225	-0.5	Distress	Yes	5.0	
240	-0.2	Distress	Yes	5.0	
255	0.1	Distress	Yes	5.0	Zone 2
270	0.4	Distress	Yes	5.0	
285	0.7	Distress	Yes	5.0	
300	1.0	No Distress	Yes	5.0	
315	1.3	No Distress	Yes	5.0	
330	1.6	No Distress	No	-	
345	1.9	No Distress	No	-	Zone 1
360	2.2	No Distress	No	-	

Table 3: Adjustment of Risk Management Contracts / Roll-Over Strategy

The table shows the firm's adjustment / roll-over strategy of risk management contracts. Inputs of the table are the range of remaining maturity τ of the existing contracts for which the firm replaces the contract given that the contract was originated at price p^* and the current spot price is p. Various combinations of p and p^* measure the moneyness of the existing contract. Empty inputs indicate that the existing contract is not replaced. For example, the firm replaces contracts initiated at a price of \$270, if 1) the spot price increases to the level of \$285 or above and the contract's remaining maturity is greater than .55 years, or 2) if the price increases to \$300 or above and the remaining maturity is longer than .25 years. The parameter values are as in the base case reported in Table 1.

Current Spot Price, p	Spot Price at Initiation of Existing Contract, p^*							
	210	225	240	255	270	285	300	315
210	-	-	-	-	-	-	-	-
225	$0.70<\tau$	-	-	-	-	-	-	-
240	$0.25<\tau$	$0.55<\tau$	-	-	-	-	-	-
255	$0.15<\tau$	$0.25<\tau$	$0.55<\tau$	-	-	-	-	-
270	$0.10<\tau$	$0.15<\tau$	$0.25<\tau$	$0.55<\tau$	-	-	-	-
285	-	$0.10<\tau$	$0.15<\tau$	$0.25<\tau$	$0.55<\tau$	-	-	-
300	-	-	$0.10<\tau$	$0.15<\tau$	$0.25<\tau$	$0.30<\tau$	$0.35<\tau<1.35$	$0.35<\tau<1.15$
315	-	-	-	-	-	-	$0.55<\tau<2.85$	$0.55<\tau<2.2$
330	-	-	-	-	-	-	-	-

Table 4: Evolution of Risk Management Contracts and Spot Price History

The table reports the results of spot price simulations and the observed risk management contracts along the simulated price paths. For each spot price level on the path (column 1), we report the probability (*prob*) of observing the firm with a risk management contract, and the average remaining maturity $\overline{\tau}$ (in years) of the observed contract conditional on the contract being in place (see columns 3-8). The simulation results are reported for three different starting spot price levels: \$345, \$270 and \$195 (see columns 3-4, 5-6 and 7-8 respectively). The results are based on 500,000 simulated paths for each starting point. The drift of the spot price is adjusted from the risk neutral to the real measure in order to match the empirically observed drift of the gold prices of 9.9% during 1970-2000. If at any time the simulated price reaches the default boundary, the path is terminated and a new path is started. The table also shows the initiation decision and maturity choice for new contracts (given that no contract is in place). The parameter values are as in the base case reported in Table 1.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Maturity of	Starting Spot Price for Simulation					
Spot	New Risk management	345		270		195	
Price	Contract	*prob*	$\overline{\tau}$	*prob*	$\overline{\tau}$	*prob*	$\overline{\tau}$
165	Default	0%	0.0	0%	0.0	0%	0.0
180	No New Contract	84%	1.7	84%	1.7	76%	2.3
195	No New Contract	91%	2.0	93%	2.1	78%	2.7
210	4.7	100%	2.4	100%	2.5	100%	3.9
225	5.0	100%	2.4	100%	2.7	100%	4.0
240	5.0	100%	2.4	100%	3.0	100%	4.0
255	5.0	100%	2.6	100%	3.4	100%	4.1
270	5.0	100%	2.9	100%	3.9	100%	4.1
285	5.0	100%	3.3	100%	4.1	100%	4.1
300	5.0	100%	3.8	100%	4.1	100%	4.2
315	5.0	100%	4.2	100%	4.1	100%	4.1
330	No New Contract	55%	2.1	99%	3.6	100%	3.7
345	No New Contract	28%	1.0	97%	3.1	99%	3.3
360	No New Contract	19%	0.6	94%	2.7	97%	2.9
375	No New Contract	18%	0.5	90%	2.3	95%	2.5
390	No New Contract	17%	0.4	85%	1.9	93%	2.2
405	No New Contract	16%	0.3	80%	1.6	90%	1.9
420	No New Contract	14%	0.3	75%	1.4	87%	1.7
435	No New Contract	13%	0.2	70%	1.2	83%	1.5
450	No New Contract	12%	0.2	65%	1.0	80%	1.3
465	No New Contract	12%	0.2	61%	0.9	77%	1.2
480	No New Contract	11%	0.2	59%	0.8	74%	1.1

Table 5: Value Creation, Equity Exposure, and Risk Management

Columns 4-7 report firm value, and equity volatility as a function of the spot price (column 1) for two firm types: for a firm that has access to risk management contracts, and for a firm that does not have access to risk management contracts. The table also reports interest coverage ratio (column 2), initiation decision and maturity choice (in years) for new risk management contracts for the firm that has access to risk management contracts (column 3). "No New Contract" indicates that the firm does not initiate a new contract for the given price level. "Default" indicates that the firm defaults for the given price level. The interest coverage ratio is the ratio of net income to debt payment $\frac{p-c}{d}$. The parameter values are as in the base case reported in Table 1

(1)	(2)	(3)	(4)	(5)	(6)	(7)
		Firm with Access to Risk Management			Firm without Access to Risk Management	
Spot Price	Interest Coverage Ratio	Maturity of New Risk management Contract	Firm Value	Equity Volatility	Firm Value	Equity Volatility
165	-1.7	Default	188	-	188	-
180	-1.4	No New Contract	813	201.9%	744	235.4%
195	-1.1	No New Contract	1667	105.2%	1545	113.6%
210	-0.8	4.7	2634	70.4%	2468	74.7%
225	-0.5	5.0	3637	52.7%	3444	55.6%
240	-0.2	5.0	4642	42.1%	4436	44.4%
255	0.1	5.0	5628	35.1%	5421	36.9%
270	0.4	5.0	6584	30.1%	6386	31.7%
285	0.7	5.0	7504	26.4%	7322	27.7%
300	1	5.0	8383	23.4%	8223	24.7%
315	1.3	5.0	9219	21.3%	9088	22.4%
330	1.6	No New Contract	10029	19.8%	9926	20.7%
345	1.9	No New Contract	10826	18.8%	10744	19.4%
360	2.2	No New Contract	11613	18.0%	11548	18.4%

Table 6: Comparative Statics: Transaction Costs

Columns 3-7 report choices of maturity in years of new hedging contracts for firms with different levels of transaction costs, TC, as a function of the current spot price (column 1). "No New Contract" indicates that the firm does not initiate a new contract for the given price level. "Default" indicates that the firm defaults for the given price level. Interest coverage ratio (column 2) is the ratio of net income to debt payment $\frac{p-c}{d}$. The other parameter values are as in the base case reported in Table 1.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
		TC=2	TC=4	TC=6	TC=10	TC=20
Spot Price	Interest Coverage Ratio	Maturity of New RM Contract	Maturity of New RM Contract	Maturity of New RM Contract	Maturity of New RM Contract	Maturity of New RM Contract
165	-1.7	Default	Default	Default	Default	Default
180	-1.4	No New Contract	No New Contract	No New Contract	No New Contract	No New Contract
195	-1.1	3.3	4.0	No New Contract	No New Contract	No New Contract
210	-0.8	3.6	4.25	4.7	5.0	No New Contract
225	-0.5	3.9	4.55	5.0	5.0	5.0
240	-0.2	4.2	4.8	5.0	5.0	5.0
255	0.1	4.45	5.0	5.0	5.0	5.0
270	0.4	4.6	5.0	5.0	5.0	5.0
285	0.7	4.75	5.0	5.0	5.0	5.0
300	1	5.0	5.0	5.0	5.0	5.0
315	1.3	5.0	5.0	5.0	5.0	5.0
330	1.6	5.0	No New Contract	No New Contract	No New Contract	No New Contract
345	1.9	No New Contract	No New Contract	No New Contract	No New Contract	No New Contract
360	2.2	No New Contract	No New Contract	No New Contract	No New Contract	No New Contract

Table 7: Comparative Statics: Maximum Hedge Ratio

Columns 3-7 report choices of maturity in years of new hedging contracts for firms with different levels of hedge ratio parameter, HR, as a function of the current spot price (column 1). "No New Contract" indicates that the firm does not initiate a new contract for the given price level. "Default" indicates that the firm defaults for the given price level. Interest coverage ratio (column 2) is the ratio of net income to debt payment $\frac{p-c}{d}$. The other parameter values are as in the base case reported in Table 1.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
		HR=0.1	HR=0.25	HR=0.5	HR=0.75	HR=1.00
Spot Price	Interest Coverage Ratio	Maturity of New RM Contract	Maturity of New RM Contract	Maturity of New RM Contract	Maturity of New RM Contract	Maturity of New RM Contract
165	-1.7	Default	Default	Default	Default	Default
180	-1.4	No New Contract	No New Contract	No New Contract	No New Contract	No New Contract
195	-1.1	No New Contract	No New Contract	No New Contract	4	3.7
210	-0.8	No New Contract	5.0	4.7	4.25	4.05
225	-0.5	No New Contract	5.0	5.0	4.55	4.45
240	-0.2	No New Contract	5.0	5.0	4.9	4.75
255	0.1	5.0	5.0	5.0	5.0	4.95
270	0.4	5.0	5.0	5.0	5.0	5.0
285	0.7	5.0	5.0	5.0	5.0	5.0
300	1	5.0	5.0	5.0	5.0	5.0
315	1.3	5.0	5.0	5.0	No New Contract	No New Contract
330	1.6	5.0	5.0	No New Contract	No New Contract	No New Contract
345	1.9	No New Contract	No New Contract	No New Contract	No New Contract	No New Contract
360	2.2	No New Contract	No New Contract	No New Contract	No New Contract	No New Contract

Table 8: Comparative Statics: Distress Costs

Columns 3-7 report choices of maturity in years of new hedging contracts for firms with different levels of proportional distress costs, $C^P_{Distress}$, as a function of the current spot price (column 1). "No New Contract" indicates that the firm does not initiate a new contract for the given price level. "Default" indicates that the firm defaults for the given price level. Interest coverage ratio (column 2) is the ratio of net income to debt payment $\frac{p-c}{d}$. The other parameter values are as in the base case reported in Table 1.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
		$C^P_{Distress}$=0.5	$C^P_{Distress}$=0.75	$C^P_{Distress}$=2	$C^P_{Distress}$=3	$C^P_{Distress}$=4
Spot Price	Interest Coverage Ratio	Maturity of New RM Contract	Maturity of New RM Contract	Maturity of New RM Contract	Maturity of New RM Contract	Maturity of New RM Contract
135	-2.3	Default	-	-	-	-
150	-2	No New Contract	Default	-	-	-
165	-1.7	No New Contract	No New Contract	Default	-	-
180	-1.4	No New Contract	No New Contract	No New Contract	Default	Default
195	-1.1	No New Contract	No New Contract	No New Contract	No New Contract	No New Contract
210	-0.8	5.0	5.0	4.7	4.1	3.8
225	-0.5	5.0	5.0	5.0	4.4	4.0
240	-0.2	5.0	5.0	5.0	4.7	4.3
255	0.1	5.0	5.0	5.0	5.0	4.5
270	0.4	5.0	5.0	5.0	5.0	4.8
285	0.7	5.0	5.0	5.0	5.0	4.9
300	1	5.0	5.0	5.0	5.0	5.0
315	1.3	No New Contract	5.0	5.0	5.0	5.0
330	1.6	No New Contract	No New Contract	No New Contract	No New Contract	No New Contract
345	1.9	No New Contract	No New Contract	No New Contract	No New Contract	No New Contract
360	2.2	No New Contract	No New Contract	No New Contract	No New Contract	No New Contract

Table 9: Comparative Statics: Spot Price Volatility

Columns 3-7 report choices of maturity in years of new hedging contracts for firms with different levels of spot price volatility, σ, as a function of the current spot price (column 1). "No New Contract" indicates that the firm does not initiate a new contract for the given price level. "Default" indicates that the firm defaults for the given price level. Interest coverage ratio (column 2) is the ratio of net income to debt payment $\frac{p-c}{d}$. The other parameter values are as in the base case reported in Table 1.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
		σ=6%	σ=8%	σ=10%	σ=12%	σ=14%
Spot Price	Interest Coverage Ratio	Maturity of New RM Contract	Maturity of New RM Contract	Maturity of New RM Contract	Maturity of New RM Contract	Maturity of New RM Contract
150	-2	-	-	-	Default	Default
165	-1.7	-	Default	Default	No New Contract	No New Contract
180	-1.4	Default	No New Contract	No New Contract	No New Contract	No New Contract
195	-1.1	No New Contract	No New Contract	No New Contract	4.75	4.8
210	-0.8	4.35	4.7	4.7	5.0	5.0
225	-0.5	4.55	4.95	5.0	5.0	5.0
240	-0.2	4.7	5.0	5.0	5.0	5.0
255	0.1	4.75	5.0	5.0	5.0	5.0
270	0.4	4.8	5.0	5.0	5.0	5.0
285	0.7	4.85	5.0	5.0	5.0	5.0
300	1	5.0	5.0	5.0	5.0	5.0
315	1.3	No New Contract	5.0	5.0	5.0	5.0
330	1.6	No New Contract	No New Contract	No New Contract	5.0	5.0
345	1.9	No New Contract	No New Contract	No New Contract	No New Contract	No New Contract
360	2.2	No New Contract	No New Contract	No New Contract	No New Contract	No New Contract

Table 10: Descriptive Statistics: Gold Mining Firms

The table shows sample mean, median, standard deviation, minimum, and maximum for 36 U.S. and Canadian gold mining firms. Data is quarterly (except Z-Score) from 1/93 to 3/99. For each firm, quarters are averaged; the resulting averages are used to calculate the statistics shown across firms. Hedging delta-percentage is the total delta-adjusted volume of risk management contracts divided by forecasted production. Standard Deviation of delta-percentage is the standard deviation of the hedging delta-percentage across firm quarters. Change in hedging delta-percentage is calculated as the quarter-to-quarter absolute value of the change in the hedging delta-percentage which is not due to changes in production forecasts or changes in option deltas. Maturity (in years) is the average remaining maturity of hedging contracts weighted by delta-adjusted volume. Market value is the value of the firm's equity (in \$M). Production (in 1,000 ounces) is the firm's forecasted gold production over the next four to five years. Assets (in \$M) is the firm's total assets. Debt (in \$M) is the firm's total debt. Leverage is the ratio of total debt to total assets. Z-Score is a measure of bankruptcy prediction as in Altman (1968) observed at the end of the previous year. Hedging and production data are from the Global Gold Hedge Survey. All other data are from Compustat.

	Mean	Median	St. Dev.	Minimum	Maximum
Quarters in Survey	21	23	6	12	27
Delta-Percentage	15%	12%	12%	0%	42%
St. Dev. Delta-Percentage	11%	10%	7%	0%	30%
Change in Delta-Percentage	5%	4%	4%	0%	20%
Maturity	1.6	1.6	0.6	0.5	2.6
St. Dev. Maturity	.5	.5	.3	.2	1.2
Market Value	804	343	1,264	9	4,784
Production	477	215	692	12	2,931
Assets	669	263	995	7	4,011
Debt	185	26	349	0	1,414
Leverage	18%	17%	13%	1%	48%
Quick Ratio	2.9	2.1	3.0	0.1	16.2
Z-Score	4.3	2.8	5.5	-0.4	29.2

Table 11: Univariate Statistics: Risk Management and Financial Distress

The table shows average hedging delta-percentages, t-Tests, and number of observations for subsets of firm quarters in the gold mining firm sample. Subsets are formed by ranking firm quarters on three measures for the likelihood of financial distress: leverage, quick ratio, and Z-score. Averages and maxima for the measures are also shown. For leverage and quick ratio, subsets are formed by dividing all quarters with valid observations for the measure into three equal-sized groups. For the Z-score firm quarters are assigned based on the cut-offs of 1.81 and 3.00 used by Altman (1968). Hedging data are from the Global Gold Hedge Survey. All other data are from Compustat. Data are quarterly (except Z-Score) from 1/93 to 3/99. *** Significant at 1%; ** Significant at 5%; * Significant at 10%.

		Likelihood of Distress		
		Low	Medium	High
Leverage	Mean	2%	18%	39%
	Max	11%	25%	112%
Delta-Percentage	Mean	9%	21%	16%
	t-Test	***7.97		***2.68
	Observations	215	240	223
Quick Ratio	Mean	6.8	1.8	0.3
	Max	54.6	2.7	1.0
Delta-Percentage	Mean	11%	19%	16%
	t-Test	***5.38		**1.76
	Observations	233	242	233
Z-score	Mean	9.03	2.45	0.48
	Max	68.30	2.98	1.78
Delta-Percentage	Mean	14%	15%	18%
	t-Test	*1.30		(1.28)
	Observations	267	198	260

Table 12: Univariate Statistics: Risk Management and Spot Prices

The table shows average hedging delta-percentages, t-Tests, and number of observations for subsets of firm quarters in the gold mining firm sample. Subsets are formed by independent sorts on average firm leverage (across all firm quarters) and average spot price of gold during the quarter. Observations are split into two approximately equal-sized groups based on the spot price. Observations are split into three groups based on average firm leverage allocating approximately 10-15% of all observations into the top and bottom leverage category. Hedging data are from the Global Gold Hedge Survey. Spot gold prices are from Datastream. All other data are from Compustat. All other data are from Compustat. Data are quarterly from 1/93 to 3/99. *** Significant at 1%.

Leverage		Price < \$358/oz.	Price > 358/oz.
0-5%	Mean Delta-Percentage	10.5%	12.5%
	t-Test	-.97	
	Observations	68	42
5-40%	Mean Delta-Percentage	19.3%	13.7%
	t-Test	***3.71	
	Observations	285	309
> 40%	Mean Delta-Percentage	10.6%	9.6%
	t-Test	.42	
	Observations	24	42

Table 13: Panel Results for Risk Management Activity

The table shows coefficient estimate (intercepts not shown), standard error, t-tests, p-values, number of observations, and adjusted fit for the following fixed-effects panel regression of firm quarters in the gold mining firm sample:

$$DP_{it} = c_i + \beta_1 Spot_t + \beta_2 Vol_t + \beta_3 MV_{it} + \beta_4 Lev_{it} + \beta_5 Lev_{it}^2 + \varepsilon_{it}, \tag{19}$$

where DP is the hedging delta-percentage, $Spot$ is the average spot gold price (in \$/ounce) during the quarter, Vol is the volatility of daily gold price returns during the quarter, MV is the market value (in \$1,000,000) of the firm's equity at the end of the quarter, Lev is the firm's leverage measured as the ratio of debt to assets, Lev^2 is squared leverage, and ε is an error term. Hedging data are from the Global Gold Hedge Survey. Spot gold prices are from Datastream. All other data are from Compustat. Data are quarterly from 1/93 to 3/99. Standard errors are robust to heteroskedasticity and autocorrelation.

Variable	Estimate	Standard Error	t-Statistic	p-Value
Spot Price	-.00036	.000197	-1.81	7.0%
Volatility	.24	.360	0.69	49.3%
Market Value$\times 10^{-6}$	9.4	9.04	1.03	30.1%
Leverage	.70	.160	4.39	.002%
Leverage Squared	-.69	.230	-3.01	.3%
Observations	642			
Adjusted R-Square	.50			

Table 14: Risk Management Maturity

The table shows coefficient estimate (intercepts not shown), standard error, t-tests, p-values, number of observations, and adjusted fit for the following regression of firm quarters in the gold mining firm sample:

$$Mat_{it} = c + \beta_1 Spot_t + \beta_2 MV_{it} + \beta_3 Lev_{it} + \beta_4 Lev_{it}^2 + \varepsilon_{it}, \tag{20}$$

where Mat is the average maturity (in years) of hedging instruments weighted by delta-adjusted hedged volume, *Spot* is the average spot gold price (in \$/ounce) during the quarter, MV is the market value (in \$1,000,000) of the firm's equity at the end of the quarter, *Lev* is the firm's leverage measured as the ratio of debt to assets, and ε is an error term. Hedging data are from the Global Gold Hedge Survey. Spot gold prices are from Datastream. All other data are from Compustat. Data are quarterly from 1/93 to 3/99. Standard errors are robust to heteroskedasticity and autocorrelation.

Parameter	Estimate	Standard Error	t-Statistic	p-Value
Intercept	1.30	.33	4.1	.0001%
Spot Price	-.00050	.00091	-.54	58.7%
Market Value$\times 10^{-6}$	0.65	0.39	1.7	9.6%
Leverage	4.30	.70	6.2	.0001%
Leverage Squared	-5.10	.14	-3.8	.0001%
Observations	528			
Adjusted R-Square	.18			