Loss Aversion, Survival and Asset Prices

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April, 2014

Abstract

This paper studies the wealth and pricing implications of loss aversion in the presence of arbitrageurs with Epstein-Zin preferences. Our analysis shows that if loss aversion is the only difference in investors’ preferences, then for empirically relevant parameter values, loss-averse investors will be driven out of the market and thus they do not affect long-run prices. The market selection process is slow in terms of wealth shares; but, because of endogenous withdrawal by loss-averse investors from the stock market, it is fast in terms of price impact. We also find that saving behavior is critical in determining survival prospects.

Keywords: loss aversion; Epstein-Zin preferences; market selection; asset pricing

JEL Classifications: G11, G12, D50

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1 Introduction

The behavior of individuals in experiments is sometimes inconsistent with those individuals being well described as expected-utility maximizers with correct expectations. Similarly, aggregate outcomes in asset markets are sometimes inconsistent with the predictions derived from (correct) expected-utility maximizers interacting in a well functioning market.\(^1\)

These observations have motivated the study of various alternative decision theories. One particularly interesting alternative theory studied in the recent behavioral-finance literature is loss aversion which is a salient feature of prospect theory. Researchers have found that loss aversion helps to explain many financial phenomena, including the high mean, excess volatility and predictability of stock returns (e.g., Barberis, Huang and Santos, 2001); the value effect (Barberis and Huang, 2001); and, the GARCH effect in stock returns (McQueen and Vorkink, 2004).

Studies of the impact of loss aversion on markets are typically conducted in representative-agent frameworks in which there is only one investor or equivalently all investors are identical.\(^2\) This research has produced valuable insights into the potential for loss aversion to explain asset-market puzzles, but it has a serious limitation. In particular, there is no trade, as there is no one to trade with in the economies studied in this literature. It is not just the absence of trade that is troubling, rather it is whether the trade that would occur between heterogeneous individuals would dampen or even eliminate the impact of loss aversion on

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\(^1\)See Barberis and Thaler (2003) for a survey of the extensive evidence.

market outcomes, so that financial markets could be approximated by models with traditional investors who are not loss averse. This concern led Barberis and Huang (2009, p. 1567) to caution that one should interpret the equity premium obtained in their representative-agent model as “an upper bound on the equity premium that we would obtain in a more realistic heterogeneous agent economy.”

Formally, the following questions are left unanswered in the literature: In a heterogeneous-agent economy, do loss-averse investors gradually lose their wealth to those investors who are not loss averse? Will loss-averse investors eventually be driven out of the market in the sense that their wealth share converges to zero? If so, how quickly does this occur? Are the implications for wealth and price impact of loss aversion similar in heterogeneous-agent and homogeneous-agent economies? That is, can loss-averse investors have a long-run impact on asset prices even if their wealth share eventually becomes negligible? To answer these questions regarding the long-run and short-run wealth and pricing implications of loss aversion, we need to analyze equilibria in a heterogeneous, dynamic, stochastic economy.

In this paper, we take up this task, and we study the question of whether and how loss-averse investors can survive and influence prices. We analyze a heterogeneous-agent economy with two (classes of) investors and two tradable assets—a risk-free bond and a risky stock. Both investors have recursive preferences. The first investor, labeled the EZ-investor, has Epstein-Zin preferences (Epstein and Zin, 1989) and she represents “rational investors” or “arbitrageurs.” We use Epstein-Zin preferences in order to disentangle the risk-aversion parameter and the elasticity-of-intertemporal-substitution parameter (EIS henceforth) and also to make this rational investor directly comparable with our second, loss-averse investor. The
second investor is called the *LA-investor*, and he has the recursive preference representation proposed by Barberis and Huang (2007, 2009).\(^3\) The LA-investor departs from the EZ-investor in the way he evaluates his investment in the stock market: he derives utility from investing in the market both indirectly, via its contribution to his lifetime consumption, and directly, via its resulting fluctuations in his financial wealth, and he is more sensitive to losses than to gains (loss aversion).

We have two sets of results which, respectively, concern the long-run and short-run wealth and pricing implications of loss aversion, and both sets of results suggest that the market selection mechanism is effective. First, if investors only differ in whether they are loss averse or not, the LA-investor will be driven out of the market and will have no impact on asset prices in the long run for economies with empirically relevant parameter values. This result is driven primarily by the endogenous difference in investors’ equilibrium portfolio choices. Previous studies on portfolio selection among expected-utility maximizers show that the closer an investor’s utility is to log utility, the higher is his/her expected wealth growth rate (De Long, Shleifer, Summers and Waldman, 1991; Blume and Easley, 1992). Our analysis shows that this insight holds for recursive preferences in a general equilibrium setting. Under empirically plausible parameter values, the EZ-investor is more risk averse than a log utility investor, but the nature of loss aversion makes the LA-investor act as if he is even more risk averse than the EZ-investor, and therefore further from the log utility investor. Thus, the LA-investor vanishes.\(^4\)

\(^3\)Throughout this paper, we will use “she”/“her” to refer to the EZ-investor and use “he”/“him” to refer to the LA-investor.

\(^4\)Although the intuition for this result comes from the previous literature, the analysis is nonetheless complex because of the dynamic portfolio choice and savings decisions that our investors face. In particular,
We also find that EIS and time-patience parameters determining intertemporal behavior matter dramatically for survival. Small differences in these parameters can easily offset the negative effects of loss aversion. For instance, in a calibrated economy, a difference in the (annualized) time-patience parameter of two percent can result in the long-run dominance of the LA-investor. This result suggests that what matters for long-run survival is whether investors optimize over savings decisions. So, whether loss-averse investors survive crucially depends on whether they have a high enough saving motive. Tanaka, Camerer and Nguyen (2010) conducted experiments in Vietnamese villages and found that loss-averse investors are also relatively impatient, which according to our analysis, suggests that loss-averse investors are likely to lose out in the long run.

Our second set of results address the timespan of the selection process. The most striking result emerging from this analysis is that the wealth dynamics and the price dynamics are significantly different. The selection process is slow in terms of wealth shares. For example, in calibrated economies, after 50 years, the LA-investor, on average, retains more than 70% of his initial wealth share. However, the selection mechanism is much more effective in terms of price impacts for two reasons. First, introducing non-loss-averse investors immediately reduces the price impact of loss aversion. For instance, the equity premium in a heterogeneous-agent economy with the LA-investor and the EZ-investor each controlling the result does not follow immediately from the previous literature as loss aversion also causes the LA-investor’s saving behavior to be endogenously different from the EZ-investor’s, which might affect the LA-investor’s survival prospects. We nonetheless demonstrate that this loss-aversion-induced difference in savings cannot overcome the LA-investor’s disadvantage from his portfolio choice.

Yan (2008) also makes a similar statement in an economy populated by investors with constant-relative-risk-aversion (CRRA) preferences. Our analysis sharpens Yan’s statement because the recursive preferences in our model separate EIS and risk-aversion parameters.
half the aggregate wealth is much smaller than that in a representative-agent economy populated only with the LA-investor, and this difference can be as large as ten times in calibrated economies. Second, even if the LA-investor starts with a very high wealth share (say, greater than 90%) in the heterogeneous-agent economy, so that the initial equity premium is close to the one generated by the representative loss-averse agent model, the price impact of loss aversion drops quickly in the heterogeneous-agent economy—for example, after 50 years, the equity premium, on average, retains about 30% of its initial value in calibrated economies.

The effectiveness of the selection force in reducing the price impact of loss aversion is due to the first-order-risk-aversion feature of loss aversion. Specifically, loss aversion means that the investor’s utility function has a kink at a reference point, which represents the first-order risk aversion. It is this kink that raises the equity premium in the single representative-agent model, because first-order risk aversion implies that the risk premium is proportional to the standard deviation of returns, as opposed to the expected-utility preference (known as second-order risk aversion) where the risk premium is proportional to the variance of returns (Segal and Spivak, 1990). However, in a heterogeneous-agent economy, this kink also causes the LA-investor to optimally choose not to purchase the risky asset and thus to hold a fully-insured allocation, leaving the stochastic discount factor to be largely determined by the smooth preference of the EZ-investor. So, interestingly and surprisingly, it is the same kink feature of loss aversion that was used to generate a sizable equity premium in the representative-agent economy that now limits the impact of loss aversion on prices in the economies.

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6This result is related to the work of Chapman and Polkovnichenko (2009) who show that the pricing implications of various behavioral rules are sensitive to heterogeneity in a static model. We discuss this related literature in Section 6.
heterogeneous-agent economy.

The result that wealth and price impacts have different selection speeds in our model is in sharp contrast to those obtained in the recent selection models using traditional CRRA preferences (e.g., Yan 2008; Dumas, Kurshev and Uppal, 2009; Fedyk, Heyerdahl-Larsen and Walden, 2012). In those alternative models, wealth and price impacts always move in the same order. For example, Yan (2008) shows that the selection process is excessively slow both in terms of wealth and in terms of price impact in an economy where CRRA investors disagree about the risk of a risky asset; Fedyk, Heyerdahl-Larsen and Walden (2012) argue that the selection speed can become quick once investors disagree about multiple risks. The different wealth and price impact dynamics in our model illustrates that loss aversion is qualitatively distinct from the traditional smooth risk aversion in driving aggregate outcomes in financial markets.

The remainder of this paper is organized as follows. Section 2 outlines the model, and Section 3 characterizes the equilibrium. Section 4 demonstrates the implications for survival and price impact of loss aversion when it is the only difference in investors’ preferences. Section 5 discusses its implications for survival when investors also have different EIS parameters or time-patience parameters. Section 6 discusses related research and the interpretation of our results, and Section 7 concludes. The appendix provides additional computations and proofs as well as the details of the numerical algorithm.

7This result also echoes Kogan, Ross, Wang and Westerfield (2006, 2011) and Cvitanić and Malamud (2011) who underscore that long-run survival and price impact are two distinct concepts. Our result complements theirs by highlighting the distinction in the short run.
2 The Model

We analyze a pure exchange economy with one perishable consumption good, which is the numeraire. The time is discrete and lasts forever: \( t = 0, 1, 2, \ldots \). We follow the market-selection literature and adopt an infinite-horizon framework (e.g., Blume and Easley, 2006; Yan, 2008; Kogan, Ross, Wang and Westerfield, 2011). In reality, it is likely that investors voluntarily enter and exit the economies for some exogenous reasons, such as life cycles. However, in common with almost all analysis in the selection literature, we abstract from this overlapping-generations feature to isolate the force of interest from any ad hoc assumed entry/exit mechanism.\(^8\)

There are two assets—a risk-free bond and a risky stock. The bond is in zero net supply and earns a gross interest rate of \( R_{f,t} \) between time \( t \) and \( t + 1 \). The stock is a claim to a stream of the consumption good represented by the dividend sequence \( \{D_t\}_{t=0}^{\infty} \). It is in limited supply (normalized to 1) and is traded in a competitive market at the (ex-dividend) price \( P_t \). Let \( f_t \equiv \frac{P_t}{D_t} \) and \( R_{t+1} \equiv \frac{P_{t+1}+D_{t+1}}{P_t} \) be the price-dividend ratio at time \( t \) and the gross return on the stock between times \( t \) and \( t + 1 \), respectively.

The dividend growth rate \( \theta_{t+1} \equiv \frac{D_{t+1}}{D_t} \) is independently and identically distributed (i.i.d.) over time and follows a distribution given by

\[
\theta_{t+1} = \begin{cases} 
\theta_H, & \text{with probability } \pi_H, \\
\theta_L, & \text{with probability } \pi_L,
\end{cases}
\]

\(^8\)De Long, Shleifer, Summers and Waldman (1990, 1991) may be the best example to illustrate this point. In De Long, Shleifer, Summers and Waldman (1990), they allude to a selection result in an overlapping-generations model; but later on, when they formally conduct an analysis on the market-selection problem, De Long, Shleifer, Summers and Waldman (1991) switch to an infinite-horizon model without any overlapping-generations features.
with \(0 < \theta_L < \theta_H\), \(0 < \pi_H < 1\) and \(\pi_L = 1 - \pi_H\). We use a binomial distribution for the dividend-growth-rate process so that the two tradable assets induce a dynamically-complete financial market. This ensures that our results on survival are driven by the difference in investors’ preferences and not by any assumed incompleteness in the financial-market structure. The market structure is important because whether the market-selection argument is valid depends crucially on the completeness of financial markets (see, among others, Blume and Easley, 2006; Cao, 2011).

The economy is populated by two (classes of) investors, who are distinguished by their preferences. The first investor, labeled the \textit{EZ-investor}, derives utility from intertemporal-consumption plans according to Epstein-Zin preferences (Epstein and Zin, 1989). The second investor, labeled the \textit{LA-investor}, is the investor emphasized in the behavioral-finance literature, see Benartzi and Thaler (1995), Barberis, Huang and Santos (2001), Barberis, Huang and Thaler (2006) and Barberis and Huang (2007, 2009). This investor gets utility not only from consumption but also from fluctuations in the value of his stock holdings, and he is loss averse over these fluctuations. The interactions between the LA-investor and the EZ-investor thus parsimoniously capture how the pricing impact of loss aversion can be affected by the “arbitrageurs” whose preferences are not affected by loss aversion, a topic that has long been of interest in the behavioral-finance literature (see Barberis and Thaler, 2003).\(^9\)

We use the preference specification developed by Barberis and Huang (2007, 2009) to describe the LA-investor’s preferences. According to this specification, the EZ-investor’s preferences are characterized by two parameters, \(\theta_L\) and \(\pi_H\), with \(0 < \theta_L < \theta_H\) and \(0 < \pi_H < 1\). The EZ-investor derives utility from intertemporal-consumption plans according to Epstein-Zin preferences.

\(^9\)It is common in the finance literature to use a two-investor economy to study interactions among traders (e.g., Dumas, 1989; Kogan, Ross, Wang and Westerfield, 2006; Fedyk, Heyerdahl-Larsen and Walden, 2012).
preference is simply a degenerate case of the LA-investor’s preference, where the parameter controlling the term related to loss aversion is set to be zero. Thus, this preference specification allows us to isolate the impact of loss aversion on the LA-investor’s wealth dynamics (survival) and asset prices.

We choose Epstein-Zin preferences to represent arbitrageurs for two additional reasons. First, Epstein-Zin preferences allow us to separate the risk-aversion parameter and the EIS parameter. These two parameters presumably have different roles in determining investors’ survival prospects, as the existing market-selection literature suggests that portfolio decisions, which are more related to risk aversion, and saving behaviors, which are more related to EIS, affect survival in different ways. Second, Epstein-Zin preferences deserve more serious investigation on their own, as the recent literature has shown that Epstein-Zin preferences help to explain many salient features of financial markets.\footnote{See Tallarini (2000), Bansal and Yaron (2004), Uhlig (2007), Gomes and Michaelides (2008), Guvenen (2009) and Campanale, Castro and Clementi (2010), among others.}

Given that the LA-investor’s preference nests the EZ-investor’s preference, we write a uniform preference formulation for both investors as follows. The time $t$ utility of investor $i (\equiv EZ, LA)$ is given by

$$U_{i,t} = H_i [C_{i,t}, \mu_i (U_{i,t+1}|I_t) + b_i E_t [v (G_{i,t+1})]] , \quad (2)$$

where $b_{EZ} = 0$ and $b_{LA} \geq 0$. Here, $H_i (\cdot, \cdot)$ is the aggregator function, which combines current consumption $C_{i,t}$ and the certainty equivalent of future utility (as well as loss aversion utility for the LA-investor, which will be discussed shortly) to generate current utility $U_{i,t}$.
the form
\[ H_i(C, X) = \begin{cases} 
[(1 - \beta_i)C^{\rho_i} + \beta_iX^{\rho_i}]^{1/\rho_i}, & \text{if } 0 \neq \rho_i < 1, \\
C^{1-\beta_i}X^{\beta_i}, & \text{if } \rho_i = 0,
\end{cases} \]
where \( 0 < \beta_i < 1 \) is investor \( i \)'s time-patience parameter. Parameter \( \rho_i \) determines the investor’s elasticity of intertemporal substitution: \( EIS_i \equiv 1/(1 - \rho_i) \).

The function \( \mu_i(U_{i,t+1}|I_t) \) is the certainty equivalent of random future utility \( U_{i,t+1} \) conditional on time \( t \) information \( I_t \), and it has the form
\[ \mu_i(U|I_t) = \begin{cases} 
[E_t(U^{\zeta_i})]^{1/\zeta_i}, & \text{if } 0 \neq \zeta_i < 1, \\
\exp[E_t(\log(U))], & \text{if } \zeta_i = 0,
\end{cases} \]
where \( E_t(\cdot) \equiv E(\cdot|I_t) \) is the expectation operator conditional on time \( t \) information \( I_t \) and where parameter \( \zeta_i \) determines the investor’s risk attitude toward aggregate future utility, as the implied parameter \( RA_i \equiv 1 - \zeta_i \) is the investor’s relative-risk-aversion coefficient. We assume that the investors have correct beliefs so that we can focus on the effects of differences in loss aversion.

Up to this point, the elements of the investor’s preference that we have discussed are entirely standard. What is non-standard in (2) is that a new term, \( b_iE_t[v(G_{i,t+1})] \), is added to the second argument of \( H_i(\cdot, \cdot) \), allowing the investor to get utility directly from the performance of investing in the stock. This term captures the non-consumption utility that the agent derives directly from the specific gamble of investing in the stock rather than just indirectly via this gamble’s contribution to next period’s wealth and the resulting consumption; the latter effect has already been captured by the certainty-equivalent function, \( \mu_i(U_{i,t+1}|I_t) \). To ease exposition, we refer to this new term as loss-aversion utility, and its components—parameter \( b_i \), argument \( G_{i,t+1} \) and function \( v(\cdot) \)—are further specified as
follows.

First, parameter $b_i$ determines the relative importance of the loss-aversion utility term in the investor’s preference. For the LA-investor, $b_{LA} > 0$, meaning that, to a certain extent, his utility depends on the outcome of his stock investment over and above what that outcome implies for total wealth. For the EZ-investor, $b_{EZ} = 0$, meaning that she derives no direct utility from financial-wealth fluctuations.

Second, the variable $G_{i,t+1}$ defines the gamble that investor $i$ is taking by investing in the risky stock. Specifically, let $W_{i,t}$ be investor $i$’s wealth at the beginning of time $t$, and let $s_{i,t}$ be the fraction of post-consumption wealth allocated to the stock. Then this investment portfolio provides the investor with a gamble represented by

$$G_{i,t+1} = s_{i,t} (W_{i,t} - C_{i,t}) (R_{t+1} - R_{f,t}) ,$$

that is, the amount invested in the stock, $s_{i,t} (W_{i,t} - C_{i,t})$, multiplied by its return in excess of the risk-free rate, $R_{t+1} - R_{f,t}$. As is standard in the literature (e.g., Barberis and Huang, 2001, 2007, 2008, 2009; Gomes, 2005; Barberis and Xiong, 2009), the risk-free rate, $R_{f,t}$, is assumed to be the “reference point” determining whether a particular outcome is treated as a gain or a loss: as long as $s_{i,t} > 0$, the stock’s return is only counted as a gain (loss) if it is larger (smaller) than the risk-free rate.

Finally, function $v(\cdot)$ determines how the investor evaluates gains and losses. We follow Barberis and Huang (2007, 2009) in assuming that $v(\cdot)$ is a piecewise-linear function:

$$v(G) = \begin{cases} 
G, & \text{if } G \geq 0, \\
\lambda G, & \text{if } G < 0, 
\end{cases}$$

with $\lambda > 1$. This function assigns positive utility to gains and negative utility to losses.
More importantly, it assigns greater negative utility to losses than positive utilities to gains of the same magnitude. This feature is known as “loss aversion” in the literature, and it is the behavioral bias that the LA-investor exhibits. Parameter $\lambda$ controls the degree of loss aversion: a one-dollar loss brings the investor $\lambda > 1$ units of negative non-consumption utility, while a one-dollar gain brings him only one unit of positive non-consumption utility.

To summarize, the economy is characterized by the following two groups of exogenous parameters: (i) technology parameters: $\theta_H$, $\theta_L$, $\pi_H$ and $\pi_L$; and (ii) preference parameters: $b_{LA}$, $\lambda$, $\{\beta_i, \rho_i, \zeta_i\}_{i=AZ,LA}$. The technology is defined by equation (1), and the preferences are defined by equations (2)-(6).

3 Equilibrium

We consider Markov equilibriums in which price-dividend ratios, the risk-free rate, and the optimal consumption and portfolio decisions are all functions of a state variable and in which the state variable evolves according to a Markov process. The Markov state variable $\omega_t$ is the LA-investor’s wealth as a fraction of aggregate wealth:

$$\omega_t \equiv \frac{W_{LA,t}}{W_{LA,t} + W_{EZ,t}}. \quad (7)$$

Intuitively, $\omega_t$ captures the state of the economy, because it determines the strength of the pricing impact of the LA-investor’s trading behavior. The reason that we can summarize the state with a single variable is that the preferences of investors are homogeneous in wealth.

We follow the literature in assuming that in an equilibrium aggregate consumption and
aggregate dividends are equal.\textsuperscript{11} Under this assumption, even a representative-agent econ-
omy with loss-aversion preferences cannot match the historical equity premium,\textsuperscript{12} as the
equilibrium stock returns are not volatile enough to induce the loss-averse investor to aban-
don the stock market.

A Markov equilibrium is formally defined as follows.

\textbf{Definition 1} A Markov equilibrium consists of (i) a stationary price-dividend ratio function,
\( f : [0, 1] \rightarrow \mathbb{R}_{++} \), (ii) a risk-free rate function, \( R_f : [0, 1] \rightarrow \mathbb{R}_{++} \), (iii) a pair of consumption
propensity functions, \( \alpha_{LA} : [0, 1] \rightarrow [0, 1] \) and \( \alpha_{EZ} : [0, 1] \rightarrow [0, 1] \), (iv) a pair of stock
investment policies, \( s_{LA} : [0, 1] \rightarrow \mathbb{R} \) and \( s_{EZ} : [0, 1] \rightarrow \mathbb{R} \), and (v) a transition function for the state variable, \( \omega : [0, 1] \times \{ \theta_H, \theta_L \} \rightarrow [0, 1] \), such that
(i) the consumption policy functions and the portfolio policy functions maximize investors’
preferences given the distribution of the equilibrium return processes;
(ii) good and securities markets clear; and
(iii) the transition function of the state variable is generated by investors’ optimal decisions
and the exogenous consumption-growth-rate process.

We next solve investors’ decision problems and the market clearing conditions to construct
such an equilibrium.

\textsuperscript{11}For consumption-based models, see, among others, Lucas (1978) and Mehra and Prescott (1985); for
models studying loss aversion, see, among others, Gomes (2005) and Berkelaar and Kouwenberg (2009).
\textsuperscript{12}See the first economy studied by Baberis, Huang and Santos (2001), and Subsection 4.1 below.
3.1 Investors’ Decisions

Investor $i$ chooses consumption $C_{i,t}$ and the fraction of post-consumption wealth allocated to the stock $s_{i,t}$ to maximize

$$U_{i,t} = H_i [C_{i,t}, \mu_i (U_{i,t+1} | I_t) + b_i E_t [v (G_{i,t+1})]]$$

subject to the definition of capital gains/losses in stock investment

$$G_{i,t+1} = s_{i,t} (W_{i,t} - C_{i,t}) (R_{t+1} - R_{f,t})$$

and the standard budget constraint

$$W_{i,t+1} = (W_{i,t} - C_{i,t}) M_{i,t+1},$$

where

$$M_{i,t+1} = R_{f,t} + s_{i,t} (R_{t+1} - R_{f,t})$$

(8)

is the gross return on the investor’s portfolio, and functions $H_i (\cdot, \cdot)$, $\mu_i (\cdot)$, and $v (\cdot)$ are given by equations (3), (4) and (6), respectively.

For brevity, we only derive the first-order conditions characterizing the investor’s optimal decisions for the case of a non-unitary EIS (i.e., for the case of $\rho_i \neq 0$ in the aggregator function $H_i (\cdot, \cdot)$). The first-order conditions for the case of a unitary EIS are relegated to Appendix A.

The Bellman equation for the investor’s problem is

$$U_{i,t} \equiv J_i (W_{i,t}, I_t)$$

$$= \max_{C_{i,t}, s_{i,t}} [(1 - \beta_i) C_{i,t}^{\rho_i} + \beta_i [J_i (W_{i,t+1}, I_{t+1}) | I_t] + b_i E_t [v (G_{i,t+1})]^{\rho_i}]^{1/\rho_i}.$$ 

Because functions $H_i (\cdot, \cdot)$, $\mu_i (\cdot)$, and $v (\cdot)$ are all homogeneous of degree one, the indirect
value function $J_i(W_{i,t}, I_t)$ is also homogeneous of degree one:

$$J_i(W_{i,t}, I_t) = A_i(I_t) W_{i,t} \equiv A_{i,t} W_{i,t}.$$ 

Therefore,

$$A_{i,t} W_{i,t} = \max_{C_{i,t}, s_{i,t}} \left[ (1 - \beta_i) C_{i,t}^{\rho_i} + \beta_i (W_{i,t} - C_{i,t})^{\rho_i} \right] \left[ \mu_i (A_{i,t+1} M_{i,t+1} | I_t) + b_t E_t [v (s_{i,t} (R_{t+1} - R_{f,t}))] \right]^{1/\rho_i},$$

which implies that the consumption and portfolio decisions are separable.

In particular, the portfolio decision is determined by

$$B_{i,t}^* \equiv \max_{s_{i,t}} [\mu_i (A_{i,t+1} M_{i,t+1} | I_t) + b_t E_t [v (s_{i,t} (R_{t+1} - R_{f,t}))]] , \tag{9}$$

and after defining the consumption propensity as

$$\alpha_{i,t} \equiv \frac{C_{i,t}}{W_{i,t}},$$

the consumption decision is made based on

$$A_{i,t} = \max_{\alpha_{i,t}} \left[ (1 - \beta_i) \alpha_{i,t}^{\rho_i} + \beta_i (1 - \alpha_{i,t})^{\rho_i} (B_{i,t}^*)^{\rho_i} \right]^{1/\rho_i}. \tag{10}$$

The first-order condition for optimal consumption propensity $\alpha_{i,t}^*$ is

$$B_{i,t}^* = \left( \frac{1 - \beta_i}{\beta_i} \right)^{1/\rho_i} \left( \frac{\alpha_{i,t}^*}{1 - \alpha_{i,t}^*} \right)^{1 - 1/\rho_i}. \tag{11}$$

Combining equations (10) and (11) delivers

$$A_{i,t} = (1 - \beta_i)^{1/\rho_i} (\alpha_{i,t}^*)^{1-1/\rho_i},$$

which, by the recursive structure, in turn implies

$$A_{i,t+1} = (1 - \beta_i)^{1/\rho_i} (\alpha_{i,t+1}^*)^{1-1/\rho_i}. \tag{12}$$

Substituting equations (11) and (12) into equation (9) gives the following single program,

\[\text{All of the first-order conditions of the investor’s problem are both necessary and sufficient, as the objective functions are concave.}\]
which summarizes the investor’s consumption and portfolio decisions:

\[
\left( 1 - \beta_i \right)^{1/\rho_i} \left( \frac{\alpha_{i,t}^*}{1 - \alpha_{i,t}^*} \right)^{1-1/\rho_i} = \max_{s_{i,t}} \left\{ \mu_i \left[ (1 - \beta_i)^{1/\rho_i} \left( \alpha_{i,t+1}^* \right)^{1-1/\rho_i} M_{i,t+1} I_t \right] + b_i E_t [v(s_{i,t} (R_{t+1} - R_{f,t}))] \right\}.
\]

As a consequence, solving the investor’s partial-equilibrium problem boils down to solving a fixed-point problem defined by the first-order condition and the value function of the above maximization problem. In the Markov equilibrium, the investor’s consumption policy and investment policy are both functions of the state variable \( \omega_t \), i.e., \( s_{i,t}^* = s_i (\omega_t) \) and \( \alpha_{i,t}^* = \alpha_i (\omega_t) \); the first-order condition and the value function of program (13) thus form a system of two equations with these two unknown functions; given the equilibrium asset return processes \( R_{t+1} \) and \( R_{f,t} \), these partial-equilibrium optimal policies can be computed from this system.

Deriving the first-order conditions for the portfolio choice is not simple, as the utility function, \( v(\cdot) \), the function that the investor uses to evaluate gains/losses, is not differentiable everywhere but instead has a kink at the origin. As will become clear in the subsequent analysis, it is this non-differentiability at the origin that is responsible for the non-participation of the LA-investor in the stock market. Formally, the optimal stock investment \( s_{i,t}^* \) is characterized by the following conditions:

\[
FOC_{i,+} \equiv (1 - \beta_i)^{1/\rho_i} \left[ E_t \left( \alpha_{i,t+1}^{(1-1/\rho_i)\zeta_i} M_{i,t+1}^{\zeta_i} \right) \right]^{1/\zeta_i} E_t \left[ \alpha_{i,t+1}^{(1-1/\rho_i)\zeta_i} M_{i,t+1}^{\zeta_i - 1} (R_{t+1} - R_{f,t}) \right] + b_i E_t [v(R_{t+1} - R_{f,t})] = 0, \text{ for } s_{i,t}^* > 0,
\]

\[ \text{(14)} \]

\[ \text{To be precise, these conditions apply to the case of a non-unitary risk aversion, i.e., they are true when } RA_i \neq 1 \text{ or } \zeta_i \neq 0 \text{ in the certainty-equivalent function } \mu(\cdot). \text{ For the case of a unitary risk aversion, simply replace the first terms with } (1 - \beta_i)^{1/\rho_i} e^{(1-1/\rho_i)E_t[\log(M_{i,t+1})]} E_t \left( \frac{R_{t+1} - R_{f,t}}{M_{i,t+1}} \right), \text{ which can be obtained from the limiting formula, } \lim_{\zeta_i \to 0} E_t \left( x^{\zeta_i} \right) \bigg|^{1/\zeta_i} = e^{E_t[\log(x)]}. \]
\[ FOC_{i,-} \equiv (1 - \beta_i)^{1/\rho_i} \left[ E_t \left( \alpha_{i,t+1}^{(1-1/\rho_i)\kappa} M_{i,t+1}^{\kappa} \right) \right]^{1/\kappa_i} E_t \left[ \alpha_{i,t+1}^{(1-1/\rho_i)\kappa} M_{i,t+1}^{\kappa - 1} (R_{t+1} - R_{f,t}) \right] \]
\[ - b_i E_t [v (R_{f,t} - R_{t+1})] = 0, \text{ for } s^*_{i,t} < 0, \] (15)

\[ FOC_{i,+} \leq 0 \text{ and } FOC_{i,-} \geq 0, \text{ for } s^*_{i,t} = 0. \] (16)

For the EZ-investor, the expressions of \( FOC_{i,+} \) and \( FOC_{i,-} \) are the same because \( b_{EZ} = 0 \).

Therefore, her first-order conditions reduce to the following equation:

\[ E_t \left[ \alpha_{EZ,t+1}^{(1-1/\rho_{EZ})\kappa_{EZ}} M_{EZ,t+1}^{\kappa_{EZ} - 1} (R_{t+1} - R_{f,t}) \right] = 0. \] (17)

### 3.2 Stock Prices and Wealth Dynamics

In this subsection, we rely on market-clearing conditions to derive the expression of price-dividend ratios \( f_t \equiv \frac{P_t}{D_t} \) and the evolution of the state variable \( \omega_t \).

The consumption-good-market-clearing condition is

\[ C_{EZ,t} + C_{LA,t} = D_t. \] (18)

Using the definition of consumption propensity, we can express the consumption levels as products of consumption-propensity functions and individual wealth levels:

\[ C_{EZ,t} = \alpha_{EZ} (\omega_t) W_{EZ,t} \text{ and } C_{LA,t} = \alpha_{LA} (\omega_t) W_{LA,t}. \]

Then, substituting the above expressions into the consumption-good-market-clearing condition gives

\[ \alpha_{EZ} (\omega_t) W_{EZ,t} + \alpha_{LA} (\omega_t) W_{LA,t} = D_t. \] (19)

Let \( W_t \equiv W_{EZ,t} + W_{LA,t} \) be the aggregate wealth in the economy at time \( t \). Recall that the definition of \( \omega_t \) in equation (7) implies that \( W_{EZ,t} = (1 - \omega_t) W_t \) and \( W_{LA,t} = \omega_t W_t \).
Therefore, equation (19) becomes

\[ [\alpha_{EZ}(\omega_t)(1 - \omega_t) + \alpha_{LA}(\omega_t)\omega_t]W_t = D_t, \]

which implies

\[ W_t = \frac{D_t}{\alpha_{EZ}(\omega_t)(1 - \omega_t) + \alpha_{LA}(\omega_t)\omega_t}. \]  

(20)

Because the bond is in zero net supply, and the stock has a net supply of one share, the aggregate economy wealth is also equal to the stock price plus its dividend:

\[ W_t = P_t + D_t. \]  

(21)

Combining equations (20) and (21) gives the price-dividend ratio function:

\[
f(\omega_t) = \frac{(1 - \omega_t)\alpha_{EZ}(\omega_t)}{(1 - \omega_t)\alpha_{EZ}(\omega_t) + \omega_t\alpha_{LA}(\omega_t)} \frac{1 - \alpha_{EZ}(\omega_t)}{\alpha_{EZ}(\omega_t)} + \frac{\omega_t\alpha_{LA}(\omega_t)}{(1 - \omega_t)\alpha_{EZ}(\omega_t) + \omega_t\alpha_{LA}(\omega_t)} \frac{1 - \alpha_{LA}(\omega_t)}{\alpha_{LA}(\omega_t)}.
\]

(22)

Equation (22) says that the price-dividend ratios in the heterogeneous-agent economy are equal to a weighted average of two terms: \( \frac{1 - \alpha_{EZ}(\omega_t)}{\alpha_{EZ}(\omega_t)} \) and \( \frac{1 - \alpha_{LA}(\omega_t)}{\alpha_{LA}(\omega_t)} \). The expressions of these two terms correspond to the price-dividend ratios in the representative-agent economies populated only by the EZ-investor and by the LA-investor, respectively. So, roughly speaking, the price-dividend ratio in a heterogeneous-economy is the weighted average of the price-dividend ratios in representative-agent economies, although the weight is not simply the wealth share but is instead a rather complex expression related to the wealth share and investors’ optimal consumption policies.

Given the price-dividend ratio function \( f_t = f(\omega_t) \) and the Markov structure of the state-variable evolution \( \omega_{t+1} = \omega(\omega_t, \theta_{t+1}) \), the distribution of stock returns \( R_{t+1} \) also has a

\[ \text{To see this, note that, in a representative-agent economy, the agent holds the whole share of the stock and consumes the entire dividend, which means that } \alpha_{i,t}W_{i,t} = \alpha_{i,t}(P_t + D_t) = D_t \text{ and thus } P_t/D_t = (1 - \alpha_{i,t})/\alpha_{i,t}. \]
Markov structure and is determined by
\[ R(\omega_t, \theta_{t+1}) \equiv R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{P_{t+1}/D_{t+1} + 1}{P_t/D_t} D_{t+1} = \frac{f(\omega_t, \theta_{t+1})}{f(\omega_t)} + 1. \] (23)

We now turn to examine how the state variable, \( \omega_t \), evolves over time. The gross return to the LA-investor’s optimal portfolio is
\[ M_{LA}(\omega_t, \theta_{t+1}) \equiv M_{LA,t+1} = R_{f,t} + s_{LA,t} (R_{t+1} - R_{f,t}) \]
\[ = R_f(\omega_t) + s_{LA}(\omega_t) [R(\omega_t, \theta_{t+1}) - R_f(\omega_t)]. \] (24)

Therefore, the LA-investor’s next period wealth is
\[ W_{LA,t+1} = [1 - \alpha_{LA}(\omega_t)] W_{LA,t} M_{LA}(\omega_t, \theta_{t+1}) \]
\[ = [1 - \alpha_{LA}(\omega_t)] \omega_t M_{LA}(\omega_t, \theta_{t+1}) \frac{\alpha_{EZ}(\omega_t) (1 - \omega_t) + \alpha_{LA}(\omega_t) \omega_t}{\alpha_{EZ}(\omega_t) (1 - \omega_t) + \alpha_{LA}(\omega_t) \omega_t} D_t, \] (25)
where the second equation follows from \( W_{LA,t} = \omega_t W_t \) and equation (20).

Applying equation (20) one period forward gives
\[ W_{t+1} = \frac{D_{t+1}}{\alpha_{EZ}(\omega_{t+1}) (1 - \omega_{t+1}) + \alpha_{LA}(\omega_{t+1}) \omega_{t+1}}. \] (26)

Combining equations (25) and (26) and recalling the definition of \( \omega_{t+1} \equiv \frac{W_{LA,t+1}}{W_{t+1}} \) and \( \theta_{t+1} \equiv \frac{D_{t+1}}{D_t} \), we have
\[ \omega_{t+1} = \frac{[1 - \alpha_{LA}(\omega_t)] \omega_t M_{LA}(\omega_t, \theta_{t+1}) [\alpha_{EZ}(\omega_{t+1}) (1 - \omega_{t+1}) + \alpha_{LA}(\omega_{t+1}) \omega_{t+1}]}{[\alpha_{EZ}(\omega_t) (1 - \omega_t) + \alpha_{LA}(\omega_t) \omega_t] \theta_{t+1}}, \] (27)
which implicitly determines the evolution of \( \omega_t \): \( \omega_{t+1} = \omega(\omega_t, \theta_{t+1}) \).

Finally, substituting \( W_{EZ,t} = (1 - \omega_t) W_t \), \( W_{LA,t} = \omega_t W_t \) and equation (20) into the stock-market-clearing condition,
\[ P_t = s_{EZ,t} (1 - \alpha_{EZ,t}^*) W_{EZ,t} + s_{LA,t}^* (1 - \alpha_{LA,t}^*) W_{LA,t}, \]
we link investors’ policy functions to the price-dividend ratio function as follows:
\[ f(\omega_t) = \frac{s_{EZ}(\omega_t) [1 - \alpha_{EZ}(\omega_t)] (1 - \omega_t) + s_{LA}(\omega_t) [1 - \alpha_{LA}(\omega_t)] \omega_t}{\alpha_{EZ}(\omega_t) (1 - \omega_t) + \alpha_{LA}(\omega_t) \omega_t}. \] (28)

To summarize, computing the equilibrium requires solving for the seven unknown func-
tions, $f(\cdot)$, $R_f(\cdot)$, $\alpha_{LA}(\cdot)$, $\alpha_{EZ}(\cdot)$, $s_{LA}(\cdot)$, $s_{EZ}(\cdot)$ and $\omega(\cdot, \cdot)$ from the system formed by equations (13)-(16), (22)-(24), (27) and (28). This system consists of seven independent equations: two value functions (equation (13) for $i = EZ, LA$), two first-order conditions (one of equations (14)-(16) and equation (17)), two market-clearing conditions (equations (22) and (28)), and a state-variable evolution function (equation (27)). Equations (23) and (24) are intermediate steps for calculating the wealth dynamics.

Two remarks are in order. First, although the market is complete, the standard Pareto-efficiency technique commonly used in the market-selection literature (e.g., Blume and Easley, 2006; Yan, 2008; Kogan, Ross, Wang and Westerfield, 2011; Borovička, 2012) cannot be applied here, because the LA-investor’s preference is non-differentiable and depends not only on the intertemporal-consumption plans but also on the endogenous stock-return process per se, thereby making it necessary to explicitly solve the equilibrium. We therefore develop an algorithm based on Kubler and Schmedders (2003) to compute the Markov equilibrium and use simulations to analyze the survival and price impact of the LA-investor. The details of the algorithm are delegated to Appendix C.

Second, our analysis ignores the issue of the existence and uniqueness of the equilibrium. As is well-known in the literature, it is difficult to establish general results on the existence and uniqueness of the equilibrium in heterogeneous-agent models. Therefore, in the present paper, we simply start the analysis under the assumption that an equilibrium exists and use numerical methods to find this equilibrium. Rigorously speaking, a numerical method can never find the exact equilibrium; what it finds, if anything, is the “$\epsilon$-equilibrium” defined by Kubler and Schmedders (2003), who interpret the computed $\epsilon$-equilibrium as an approximate
equilibrium of some other economy with endowments and preferences that are very close to those in the original economy.

4 Loss Aversion and Market Selection

In this section, we first analyze the representative-agent economies, that is, economies populated by homogeneous investors. This analysis (presented in Section 4.1) serves two purposes. First, it verifies the result that loss aversion raises equity premiums, which is well-known in the literature (e.g., Benartzi and Thaler, 1995; Barberis, Huang and Santos, 2001). Second, it provides a useful springboard for our analysis of the heterogeneous-agent economy, because it provides intuitions for how loss aversion changes an investor’s investment and saving behaviors.

We then move to the more realistic economy populated by both the EZ-investor and the LA-investor and apply the algorithm in Appendix C to numerically compute the equilibrium price functions \( f(\cdot) \) and \( R_f(\cdot) \), policy functions \( \alpha_{LA}(\cdot) \), \( \alpha_{EZ}(\cdot) \), \( s_{LA}(\cdot) \) and \( s_{EZ}(\cdot) \), and the state-variable transition function \( \omega(\cdot, \cdot) \). We use simulations to show how loss aversion affects the investor’s wealth accumulation and pricing impacts both in the short run and in the long run via portfolio decisions in Section 4.2 and via saving behaviors in Section 4.3. To isolate the role of loss aversion, in these two subsections, we assume that both investors have otherwise identical preferences except that the LA-investor derives loss-aversion utility and the EZ-investor does not.

[INSERT TABLE 1 HERE]
Before solving the models, we need to calibrate the parameter values. Because we are interested in the implications of preferences, we allow the preference parameters to vary while fixing the four technology parameters in equation (1) for all computations and simulations. We interpret one period as one month and follow Mehra and Prescott (1985) in setting \( \pi_H = \pi_L = \frac{1}{2} \) so that the economy is in a boom or a recession with equal probability.\(^{16}\) Based on the data spanning the 20th century, the historical mean and volatility of the log annual consumption growth process are 1.84% and 3.79%, respectively (see Barberis and Huang (2009)). To match these two moments, we set \( \theta_H = 1.0126 \) and \( \theta_L = 0.9906 \). Table 1 summarizes our choice of technology parameters.

### 4.1 Representative-Agent Economy

In this subsection, we assume that the EZ-investor and the LA-investor have identical preferences; that is, \( \beta_{EZ} = \beta_{LA} \equiv \beta \), \( \rho_{EZ} = \rho_{LA} \equiv \rho \), \( \zeta_{EZ} = \zeta_{LA} \equiv \zeta \) and \( b_{EZ} = b_{LA} \equiv b \). As a result, the economy is the well-studied representative-agent economy.

In this case, the representative agent has to hold the stock in equilibrium, so that the first-order condition given by equation (14) with \( M_{i,t+1} = R_{t+1} \) defines the optimality of the investor’s investment decision. As mentioned in the discussions in footnote 15, the consumption-good-market-clearing condition links the price-dividend ratios \( f_t \) to the optimal

\(^{16}\)Some studies have assumed that loss-averse investors evaluate investment performance on an annual frequency (e.g., Benartzi and Thaler 1995; Barberis, Huang and Santos, 2001). Our results remain valid if we calibrate one period as one year in our economy. We have chosen the monthly frequency to be consistent with the literature using Epstein-Zin preferences (e.g., Bansal and Yaron, 2004).
consumption policy $\alpha_t$ as follows:

$$D_t = (1 - \alpha_t)(P_t + D_t) \Rightarrow f_t = \frac{1 - \alpha_t}{\alpha_t}. \quad (29)$$

Therefore, equations (13), (14) and (29) define a system for three unknowns: $f_t$, $\alpha_t$ and $R_{f,t}$.

Given the i.i.d. investment opportunities, we conjecture that

$$(f_t, \alpha_t, R_{f,t}) = (f, \alpha, R_f), \forall t. \quad (30)$$

The problem can be easily solved using any non-linear solver. Formally, we have the following proposition (the proof is in Appendix B):

**Proposition 1** In the representative-agent economy, (i) when $EIS \neq 1$ (i.e., when $\rho \neq 0$), the conditions that determine a constant price-dividend ratio $f$ and a constant risk-free rate $R_f$ are:

$$
\left(\frac{1 - \beta}{\beta}\right)^{1/\rho} = (1 - \beta)^{1/\rho} \left(\frac{1 + f}{f}\right)^{1/\rho} \mu(\theta_{t+1})
+ b (1 + f)^{1-1/\rho} E \left[ v \left( \left(\frac{1 + f}{f}\right)^{1/\rho} \theta_{t+1} - \left(\frac{1}{\beta}\right)^{1/\rho} \frac{E\left(\theta_{t+1}\right)^{1-1/\rho}}{E\left(\theta_{t+1}\right)} \right) \right] \quad (31)
$$

$$R_f = \left(\frac{1}{\beta}\right)^{1/\rho} \left(\frac{1 + f}{f}\right)^{1-1/\rho} \frac{E\left(\theta_{t+1}\right)^{1-1/\rho}}{E\left(\theta_{t+1}\right)}; \quad (32)$$

and (ii) when $EIS = 1$ (i.e., when $\rho = 0$), the price-dividend ratio is $f = \frac{\beta}{1 - \beta}$, and the constant risk-free rate $R_f$ is characterized by

$$
\left(\frac{\mu(\theta_{t+1}) E\left(\theta_{t+1}\right)^{1-1/\rho}}{E\left(\theta_{t+1}\right)}\right)^{1/\rho} = (1 - \beta)^{1/\rho} \beta^{1-1/\rho} R_f^{1-1/\rho}
$$

$$= \left(\frac{\mu(\theta_{t+1}) E\left(\theta_{t+1}\right)^{1-1/\rho}}{E\left(\theta_{t+1}\right)}\right)^{1/\rho} (1 - \beta)^{1-1/\rho} \beta^{1-1/\rho} \mu(\theta_{t+1}) R_f^{1-1/\rho} + b E \left[ v \left( \frac{\theta_{t+1}}{\beta} - R_f \right) \right]. \quad (33)
$$

Table 2 reports the annualized continuously-compounded equilibrium equity premiums
\( EPa = 12 [E (\log R_{t+1}) - \log R_f] \), risk-free rates \( (r_f^a = 12 \log (R_f)) \) and consumption policies \( (\alpha^a = 12\alpha) \) for a variety of combinations of preference-parameter values. For all combinations, we hold constant the time-patience parameter \( \beta \), the loss-aversion parameter \( \lambda \) and the relative-risk-aversion coefficient \( RA: \beta = 0.998, \lambda = 2.25 \) and \( RA = 1 \) (or \( \zeta = 0 \)). The choice of \( \beta \) is borrowed from Bansal and Yaron (2004) and it corresponds to an annual time-patience parameter of \( 0.976 = \beta^{12} \). The choice of \( \lambda \) is based on the estimation of Tversky and Kahneman (1992). When EIS is equal to one (i.e., \( \rho = 0 \)) and when there is no loss-aversion utility (i.e., \( b = 0 \)), setting \( RA = 1 \) (or \( \zeta = 0 \)) reduces the investor’s preference to an expected log utility, which is an important benchmark case in the market-selection literature (Breiman, 1961; Hakansson, 1971; De Long, Shleifer, Summers and Waldmann, 1991; Blume and Easley, 1992).

[INSERT TABLE 2 HERE]

Panels A, B and C correspond to different values of EIS: \( EIS = 1 \) (\( \rho = 0 \)), \( EIS = 0.8 \) (\( \rho = -0.25 \)) and \( EIS = 1.2 \) (\( \rho = 1/6 \)). To check the role of loss aversion, in each panel, parameter \( b \), which controls the relative importance of the loss-aversion utility in the investor’s preference, is set at three different values: \( b = 0, b = 0.0005 \) and \( b = 0.001 \). When \( b = 0 \), the investor’s preference does not exhibit loss aversion, and this economy has been well studied in the literature (e.g., Weil, 1989). When \( b > 0 \), the investor’s preference exhibits loss aversion; such an economy is the focus of behavioral finance; see Benartzi and Thaler (1995), Barberis, Huang and Santos (2001), and Barberis and Huang (2007, 2009). The choice of both positive values of \( b \) in the table, 0.0005 and 0.001, is justified by the investor’s attitudes to independent large and small monetary gambles: both parameterizations of the
investor’s preference satisfy Barberis and Huang’s conditions L and S (2007, p. 217 and p. 219). The last two columns of Table 2 report the premiums the representative agent would pay to avoid a large gamble and a small gamble, which are computed according to equation (34) in Barberis and Huang (2009, p. 1566).

Three notable patterns show up in Table 2. The first pattern regards the equity premium. In all three panels, when $b = 0$, that is, when loss aversion is absent in the investor’s preference, the equity premium is quite small (0.07%) relative to its historical value (6%), which is the well-known equity-premium puzzle. Once loss aversion is introduced, the equity premiums are raised significantly. For example, if $b = 0.001$, the model can generate an equity premium as high as 3.01%, which is more than 40 times the equity premium corresponding to an economy populated by only EZ-investors. The increased equity premiums still fall short of the empirical value, as in our model, the stock is a claim to the smooth aggregate consumption process, and, as a result of the constant equilibrium price-dividend ratios in equation (30), the stock returns are not volatile enough to cause the loss-averse investor to demand a large premium to hold the stock. This mismatch between the model-generated equity premium and the historical equity premium does not have any impact on our analysis.

The literature focuses on investors’ attitudes to independent monetary gambles, as it was, in part, the difficulty that researchers encountered in reconciling the equity premium with these attitudes that launched the equity-premium-puzzle literature in the first place. Barberis and Huang’s (2007) condition L is: “An individual with wealth of $75,000 should not pay a premium higher than $15,000 to avoid a 50:50 chance of losing $25,000 or gaining the same amount.” Their condition S is: “An individual with wealth of $75,000 should not pay a premium higher than $40 to avoid a 50:50 chance of losing $250 or gaining the same amount.”

Barberis, Huang and Santos (2001) have also studied the pricing impact of loss aversion in a representative-agent economy with dividends equal to consumption, and they report an equity premium of 1.26% as the relative-risk-aversion coefficient is equal to 1 (see the top part of their Table II), which is close to the equity premium generated in our model (1.22% when $b = 0.0005$).
What really matters is that the LA-investor exhibits first-order risk aversion and is more reluctant to hold the stock than the EZ-investor, which is exactly the reason why behavioral finance introduces loss aversion to explain the equity-premium puzzle.

The second pattern concerns the risk-free rate. In all three panels, the risk-free rate decreases with $b$. This occurs because as the investor is more concerned about fluctuations in the value of his financial wealth and as he is more loss averse, he is more inclined to allocate wealth to the safe asset to avoid the potential painful losses associated with the risky asset. This suggests that in a heterogeneous-agent economy populated by both the LA-investor and the EZ-investor, the bond is more attractive to the former than to the latter.

The third pattern is about the consumption policy. When EIS is equal to one, the investor’s monthly saving ratio is optimally chosen to be equal to the time-patience parameter, $\beta$. Therefore, in Panel A, the optimal consumption propensity $\alpha$ is independent of parameter $b$. However, when EIS is different from 1, $\alpha$ varies with $b$: $\alpha$ decreases with $b$ when EIS is less than 1 in Panel B, while $\alpha$ increases with $b$ when EIS is greater than 1 in Panel C. As is standard in the portfolio-choice problem for recursive preferences, two forces—the income effect and the substitution effect—are at play here. The asymmetric treatment of losses from gains in the loss-aversion utility tends to lower the value, measured in utility terms, of the investor’s future investment opportunities; that is, a higher $b$ tends to yield a lower $B^*_t$ in equation (9). This lowered $B^*_t$ has two effects on current consumption: it lowers consumption propensity through the income effect but raises consumption propensity through the substitution effect. When EIS is below 1, the income effect dominates, so that $\alpha$ decreases with $b$; when EIS is above 1, the substitution effect dominates, and the dependence of $\alpha$ on
b reverses as a result. The different responses of \( \alpha \) to \( b \) in different cases of EIS suggest that how loss aversion affects the LA-investor’s survival might depend on whether EIS is greater than or smaller than 1, as the literature suggests that saving behavior is a key determinant on survival. This issue will be examined in Section 4.3.

### 4.2 EIS=1: Portfolio Selection

In this subsection, we study the heterogeneous-agent economy and fix EIS at 1, so that both investors optimally choose to have a constant monthly consumption-wealth ratio: \( \alpha_{i,t}^* = 1 - \beta_i \), for \( i = EZ, LA \). To isolate the impact of loss aversion, we continue to assume that the preferences of both investors are otherwise identical except that the LA-investor derives loss-aversion utility, while the EZ-investor does not. So, except that \( b_{LA} > 0 \) and \( b_{EZ} = 0 \), all other parameters are the same across investors: \( \beta_{EZ} = \beta_{LA} \equiv \beta \), \( \rho_{EZ} = \rho_{LA} \equiv \rho \) and \( \zeta_{EZ} = \zeta_{LA} \equiv \zeta \). The assumption of a common time-patience parameter implies that both investors have the same endogenous saving rate, which is equal to \( \beta \). The focus of this subsection is therefore on how loss aversion changes the LA-investor’s portfolio decision, which in turn affects the LA-investor’s wealth accumulation and pricing impacts in a complete financial market.

#### 4.2.1 Wealth Dynamics and Survival

We follow the market-selection literature, such as Blume and Easley (2006), Yan (2008) and Kogan, Ross, Wang and Westerfield (2011), in defining the “extinction,” “survival” and “dominance” of the LA-investor in terms of his wealth shares as follows.
**Definition 2** The LA-investor is said to become extinct or vanish if
\[ \lim_{t \to \infty} \omega_t = 0, \text{almost surely (a.s.)}; \]
to survive if extinction does not occur; and to dominate the market if
\[ \lim_{t \to \infty} \omega_t = 1, \text{a.s..} \]

Our subsequent analysis suggests that the LA-investor will vanish via the channel of portfolio decisions if loss aversion causes him to be further from the log investor in terms of risk attitude than the EZ-investor. We also show that empirically-relevant parameter values typically lead to this result and that the wealth-selection process is extremely slow.

To illustrate how the LA-investor’s wealth shares (\( \omega_t \)) evolve over time, Table 3 reports their medians at times \( t = 60, 120, \) and 600 months (i.e., 5, 10 and 50 years) when the LA-investor has initial wealth shares of \( \omega_0 = 0.5 \) and both investors have a relative-risk-aversion coefficient of 1 (Panel A) or 3 (Panel B).\(^{19}\) We report the results for various values of \( \lambda \), which controls the degree of loss aversion. The technology parameters are fixed at the values in Table 1. The time-patience parameters are \( \beta_{EZ} = \beta_{LA} = 0.998 \) and parameter \( b_{LA} \) is set at 0.001. The medians of \( \omega_t \) are obtained from simulations. We first use the algorithm described in Appendix C to solve the equilibrium state-transition function \( \omega(\cdot, \cdot) \) and then use it to simulate \( N = 5,000 \) economies. For each economy, we make \( T = 600 \) independent draws of \( \theta_{t+1} \) from the distribution described in equation (1) to simulate a time series \( \{\theta_{t+1}\}_{t=1}^{T} \). We then use the solved function \( \omega(\cdot, \cdot) \) to calculate the next-period state \( \omega_{t+1} \). Finally the medians of \( \omega_t \) are estimated from the 5,000 simulated sample paths at time

\(^{19}\)The evolution patterns of \( \omega_t \) are robust to the choice of \( \omega_0 \).
We first note that the speed of wealth changes is excessively slow. In general, after 50 years (at \( t = 600 \)), on a typical sample path, \( \omega_t \) still maintains above 70% of its initial value. This suggests that the selection mechanism is not effective in terms of wealth, which is consistent with Yan (2008) who shows that it takes hundreds of years for an investor with incorrect beliefs to lose half of his wealth share in an economy populated with heterogeneous CRRA investors.

In terms of long-run survival, the results in Table 3 suggest that the insight in De Long, Shleifer, Summers and Waldman (1991) and Blume and Easley (1992) holds for recursive preferences in a general equilibrium setting: whether an investor survives depends on how close his/her preference is to log utility whose objective is to maximize growth rate of expected wealth. In Panel A of Table 3, the EZ-investor is the log investor, since \( RA_{EZ} = EIS_{EZ} = 1 \). We can see that for all values of \( \lambda \), the LA-investor’s wealth shares shrink as time passes. This suggests that in an economy populated with the log investor and the LA-investor, the LA-investor always loses out.

In Panel B of Table 3, we change the relative-risk-aversion coefficient of both investors from 1 to 3, so that the EZ-investor is no longer the log investor. In this case, we observe that the LA-investor sometimes survives, while at other times, he vanishes: when \( \lambda = 1.05 \), the LA-investor’s wealth shares increase over time, while when \( \lambda = 1.5, 2.25, \) or 3, his wealth shares decrease over time. When \( RA_{EZ} = 3 \), the EZ-investor is more risk averse than the log investor. If \( \lambda \) is close to 1, the loss aversion utility in the LA-investor’s preference is close to
the expected gains/losses (i.e., \( b_{LA} E_t [v(G_{LA,t+1})] \approx b_{LA} E_t (G_{LA,t+1}) \)). This makes the LA-investor’s preference as if it was generated from combining the EZ-investor’s preference and a risk-neutral investor’s preference, leading the LA-investor to hold portfolios corresponding to a greater risk tolerance. Therefore, the LA-investor can potentially be closer to the log investor in terms of risk attitude than the EZ-investor, which explains his long-run survival for the case of \( \lambda = 1.05 \). On the other hand, if \( \lambda \) is much greater than 1, as we believe is empirically likely, loss aversion penalizes losses much more than it rewards gains, making the LA-investor reluctant to invest in volatile stocks. This causes the LA-investor to mimic an investor who is more risk averse, and hence further from log utility, than the EZ-investor. Therefore, in this case, the LA-investor vanishes in the long run.

Table 4 further verifies the above intuition. It is difficult to directly compare the risk attitude of the LA-investor to that of the EZ-investor (and of the log investor). This is because, unlike the EZ-investor whose risk attitude is solely determined by parameter \( \zeta_{EZ} \), the LA-investor’s risk attitude is jointly determined, in a complicated way, by parameter \( \zeta_{LA} \), which captures traditional smooth risk aversion, and by parameters \( b_{LA} \) and \( \lambda \), which capture first-order risk aversion due to loss aversion. Nonetheless, we can compare their risk attitudes indirectly by examining the risk premiums generated by representative-agent economies—the equilibrium equity premium should be high if the representative agent strongly avoids risk. Table 4 adopts such an approach.

[INSERT TABLE 4 HERE]

Specifically, the first row of Table 4 reports the value of the equity premium, \( EP_{LA} \), in the representative-agent economy populated by the LA-investor for various values of \( \lambda \), while
other model parameters take the same values as those in Panel B of Table 3. The second row compares $E P_{LA}$ to $E P_{log}$ (=1.20 basis points), which is the equity premium emerging from the representative-log-investor economy. We find that $E P_{LA}$ is greater than $E P_{log}$ for all values of $\lambda$, suggesting that the LA-investor behaves as if he is more risk averse than the log investor. The third row compares $E P_{LA}$ to $E P_{EZ}$ (=3.60 basis points), the equity premiums emerging from the representative-agent economy populated with the EZ-investor. For $\lambda = 1.05$, we find that $E P_{LA} < E P_{EZ}$, suggesting the LA-investor behaves as if he is less risk averse than the EZ-investor. Given that the EZ-investor is more risk averse than the log investor ($R A_{EZ} = 3 > 1$), we now have the complete order among the three investors—i.e., for the case of $\lambda = 1.05$, the EZ-investor is more risk averse than the LA-investor, who is in turn more risk averse than the log investor. As a result, the LA-investor is closer to the log investor and survives in the long run. For other cases, a similar analysis shows that the ordering between the LA-investor and EZ-investor reverses, which implies that the LA-investor’s preference is further from the log utility and hence he eventually vanishes.

So far, our analysis of wealth shares suggests that the implication of loss aversion for wealth dynamics (and hence the long-run survival) can be largely understood by interpreting the LA-investor as a traditional investor who has a high risk-aversion coefficient. However, loss aversion and traditional smooth risk aversion are not observationally equivalent; they have dramatically different implications for price-impact dynamics, which we analyze next.
4.2.2 Price Impacts

State Prices The effectiveness of the market-selection mechanism should be judged by the price impact rather than by how the wealth shares of the LA-investor change over time, because what one really cares about is whether the behavior of asset prices can be approximated by models without the LA-investor. Since our economy is dynamically complete, there is a unique stochastic discount factor (SDF) or pricing kernel that determines the price of any security under no arbitrage. We can therefore measure price impacts based on this unique SDF.

Specifically, in each period \( t \), we define two one-period Arrow securities, “H-security” and “L-security.” The H-security pays one unit of consumption good in period \( t + 1 \) when \( \theta_{t+1} = \theta_H \) and zero units otherwise, while the L-security pays off when \( \theta_{t+1} = \theta_L \) and zero otherwise. Their prices are denoted by \( \phi_{H,t} \equiv \phi_H(\omega_t) \) and \( \phi_{L,t} \equiv \phi_L(\omega_t) \), respectively, which are usually called “state prices” in the literature. Normalizing the state prices by their respective probabilities gives rise to the SDF; that is, the SDF is \( \left( \frac{\phi_{H,t}}{\pi_H}, \frac{\phi_{L,t}}{\pi_L} \right) \). Given the computed risk-free-rate function \( R_{f,t}(\omega_t) \), the price-dividend-ratio function \( f(\omega_t) \) and the state-evolution function \( \omega(\omega_t, \theta_{t+1}) \), we can back out both state prices as follows:

\[
\begin{bmatrix}
\phi_{H,t} \\
\phi_{L,t}
\end{bmatrix} = \begin{bmatrix}
1 & 1 \\
(f_{t+1,H} + 1) \theta_H & (f_{t+1,L} + 1) \theta_L
\end{bmatrix}^{-1} \begin{bmatrix}
\frac{1}{R_{f,t}} \\
f_t
\end{bmatrix},
\]

where \( f_{t+1,H} \equiv f(\omega(\omega_t, \theta_H)) \) and \( f_{t+1,L} \equiv f(\omega(\omega_t, \theta_L)) \).

[INSERT FIGURE 1 HERE]

In Panel (a) of Figure 1, we plot the two state prices \( \phi_{H,t} \) and \( \phi_{L,t} \) as functions of \( \omega_t \) for the parameter configuration in Panel A of Table 3 with \( \lambda = 2.25 \). For each \( \omega_t \), the price \( \phi_{L,t} \)
of the L-security is higher than the price $\phi_{H,t}$ of the H-security. This is because both the EZ- and the LA-investors have a higher marginal utility when they are relatively poor facing a low dividend realization. In addition, the price $\phi_{L,t}$ increases with $\omega_t$, while the price $\phi_{H,t}$ decreases with $\omega_t$: relative to the EZ-investor, the LA-investor is harmed more by the low dividend realization, and thus he is more willing to pay a high price for the L-security that pays off when $\theta_{t+1} = \theta_L$ and a low price for the H-security paying off when $\theta_{t+1} = \theta_H$.

We use the ratio of the price $\phi_{H,t}$ in the heterogeneous-agent economy to that in the representative-agent economy with only the EZ-investor, $\phi_{H,EZ}$, to capture the LA-investor’s price impact on the state price of the H-security. The denominator $\phi_{H,EZ}$ in this ratio of $\frac{\phi_{H,t}}{\phi_{H,EZ}}$ controls for other factors, such as the dividend process, that affect asset prices in both economies. Similarly, we use the ratio $\frac{\phi_{L,t}}{\phi_{L,EZ}}$ to capture the LA-investor’s price impact on the state price of the L-security. According to Panel (a) of Figure 1, we know that $\frac{\phi_{H,t}}{\phi_{H,EZ}} < 1$ and $\frac{\phi_{L,t}}{\phi_{L,EZ}} > 1$; that is, relative to the representative-agent economy with only the EZ-investor, the H-security is “underpriced” and the L-security is “overpriced” in the heterogeneous-agent economy. The two ratios $\frac{\phi_{H,t}}{\phi_{H,EZ}}$ and $\frac{\phi_{L,t}}{\phi_{L,EZ}}$ jointly describe the LA-investor’s price impact on the SDF.

Table 5 reports the medians of $\frac{\phi_{H,t}}{\phi_{H,EZ}}$ and $\frac{\phi_{L,t}}{\phi_{L,EZ}}$ in a variety of economies. The primitive parameters take the same values as those in Table 3. Unlike Table 3, Table 5 only reports results for the case of $\lambda = 2.25$ and not for the cases of $\lambda = 1.05, 1.5$ and 3: the case of $\lambda = 1.05$ is not empirically plausible, while the results in cases of $\lambda = 1.5$ and 3 are similar to those in the case of $\lambda = 2.25$.\textsuperscript{20} In addition, unlike Table 3 where the LA-investor starts

\textsuperscript{20}In Table 3, we are interested in these three cases of $\lambda = 1.05, 1.5$ and 3, because we want to understand
with half of the aggregate wealth, Table 5 assumes that the LA-investor’s initial wealth shares can take four possible values, \( \omega_0 = 0.1, 0.5, 0.9 \) or 1, because, as will become clear shortly, we will use these different \( \omega_0 \)’s to illustrate different aspects of the effectiveness of the market-selection mechanism. In particular, when \( \omega_0 = 1 \), the economy becomes a representative-agent economy with only the LA-investor, which generates the maximal price impact of loss aversion.

[INSERT TABLE 5 HERE]

We observe that in all cases with \( \omega_0 < 1 \), as time passes, the LA-investor gradually loses his impact on the state prices (and hence on the SDF), because the medians of \( \frac{\phi_{H,t}}{\phi_{H,EZ}} \) and \( \frac{\phi_{L,t}}{\phi_{L,EZ}} \) gradually approach 1. So, the LA-investor is unlikely to have a large price impact in the long run. This result is not surprising given that, according to Table 3, the LA-investor’s wealth share shrinks over time for the parameterizations considered in Table 5.

The surprising result is that the market-selection mechanism is quite effective in terms of the selection speed of price impacts. This occurs for two reasons. First, introducing the EZ-investor immediately reduces the price impact of loss aversion at time 0. For example, in Panel A of Table 5, the L-security is overpriced by as much as 20% in the representative-agent economy with only the LA-investor relative to the representative-agent economy with only the EZ-investor (i.e., \( \frac{\phi_{L,t=0}}{\phi_{L,EZ}} = 1.20 \) when \( \omega_0 = 1 \)). However, in the heterogeneous-agent economy with both investors, it is overpriced only by 1% when \( \omega_0 = 0.5 \) (i.e., \( \frac{\phi_{L,t=0}}{\phi_{L,EZ}} = 1.01 \) when \( \omega_0 = 0.5 \)). Second, even when \( \omega_0 \) is large and the price impact of loss aversion is thus initially large, it is likely that a small drop in wealth shares leads to a large drop in price whether the insights on survival in Blume and Easley (1992) can be extended to recursive preferences.
impact. For instance, in Panel A of Table 5, at time 0 when $\omega_0 = 0.9$, the L-security is overpriced by nearly 10%, but after five years at $t = 60$, 35% of this overpricing vanishes (i.e., when $\omega_0 = 0.9$, $\frac{L_{t=0}}{L_{t=EZ}} = 1.10$ and $\frac{L_{t=60}}{L_{t=EZ}} = 1.06$).

The reason underlying the above strong selection force in terms of price impact is as follows. In the heterogeneous-agent economy, the LA-investor may choose not to participate in the stock market and hold a fully-insured position in equilibrium. As a result, the state prices are determined by the indifference curve of the smooth Epstein-Zin preferences and they are not directly affected by loss aversion. To illustrate this, in Panel (b) of Figure 1, we plot the Edgeworth box that depicts the equilibrium determination when $\omega_t = 0, 0.5$ and 1, and in Panel (c), we plot the LA-investor’s optimal stock investment $s_{LA,t}$ as a function of $\omega_t$. The other parameter configuration is still fixed at the one of Panel (a) of Figure 1. Comparing Panels (a) with (c), we find that the two state prices $\phi_H$ and $\phi_L$ differ the most from their counterparts produced by the Epstein-Zin preferences when the LA-investor participates in the stock market; that is, $\phi_{H,t}$ and $\phi_{L,t}$ differ the most from $\phi_{H,EZ}$ and $\phi_{L,EZ}$ for those states of $\omega_t$ with $s_{LA,t} > 0$.

This observation is also reflected in the Edgeworth box analysis in Panel (b): when $\omega_t = 0.5$, the LA-investor chooses a fully-insured allocation on the 45\degree line, and his indifference curve is kinked at the equilibrium allocation. Thus, the two state prices are determined by the marginal rate of substitution which corresponds to the slope of the indifference curve of the smooth Epstein-Zin preference; that is, the (dashed) budget line is tangent to the indifference curve of the EZ-investor. This is very similar to the way that the prices are determined in the representative-agent economy with only the EZ-investor, and as a result,
the slopes of the two (dashed) budget lines in cases of \( \omega_t = 0.5 \) and of \( \omega_t = 0 \) are almost parallel to each other. In contrast, when \( \omega_t = 1 \), the LA-investor is the only investor in the economy and has to hold both Arrow-securities in equilibrium, and thus the two state prices are determined by his indifference curve. As a consequence, the budget line for the case of \( \omega_t = 1 \) is much flatter than it is with \( \omega_t = 0 \).

**Equity Premiums** To make the above SDF analysis more intuitive, we next examine the impact of loss aversion on the equity premium. This analysis of the equity-premium impact is also useful as the most robust implication of loss aversion in standard representative-agent models is to raise the equity premium. Specifically, we use the ratio of the conditional equity premium \( EP_t \) in the heterogeneous-agent economy to that in the representative-agent economy with only the EZ-investor, \( EP_{EZ} \), to capture the equity-premium impact of loss aversion. We also report the medians of this ratio of \( \frac{EP_t}{EP_{EZ}} \) in Table 5.

We find that the implication for the equity premium is the same as that for the two state prices. Specifically, Table 5 shows that the medians of \( \frac{EP_t}{EP_{EZ}} \) gradually decline toward 1. So in the long run, the impact of the LA-investor on the equity premium will disappear. Perhaps more important is that in the short run, the market-selection speed is still fast in terms of equity premiums. Again, this is reflected in two ways. First, introducing the EZ-investor immediately reduces the equity-premium impact of loss aversion at time 0. For example, in Panel A of Table 5, in representative-agent economies with the LA-investor, loss aversion raises equity premiums by as much as 19 times the value produced by Epstein-Zin preferences. However, in the heterogeneous-agent economy, when \( \omega_0 = 0.5 \), \( \frac{EP_{0.5}}{EP_{EZ}} \) drops
sharply to about 2 at time 0. Second, even when \( \omega_0 \) is high and the equity-premium impact of loss aversion is initially large, it is likely that the (small, short-run) decline in wealth share for the loss averse investor leads to a large decline in his impact on the equity-premium. For instance, in Panel A of Table 5, at time 0 when \( \omega_0 = 0.9 \), \( \frac{EP_{t=0}}{EP_{EZ}} \) is nearly 10. However, after five years (\( t = 60 \)), \( \frac{EP_{t=60}}{EP_{EZ}} \) falls to about 7 and after 50 years (\( t = 600 \)), \( \frac{EP_{t=600}}{EP_{EZ}} \) further drops to a value near 3.

Again, the strong equity-premium-selection force is driven by the optimal withdrawal of the LA-investor from the risky asset market. We plot in Panel (d) of Figure 1 the conditional equity premium \( EP_t \) as a function of \( \omega_t \) for the same parameter configuration as in Panel (a). There is a kink in the equity-premium function in Panel (d), and the location of the kink is determined by the level of the wealth share at which the LA-investor starts to buy the stock in Panel (c). The equity-premium function is rather steep around the kink, and therefore, as \( \omega_t \) declines from 1 toward 0, \( \frac{EP_t}{EP_{EZ}} \) declines dramatically. This high sensitivity of \( EP_t \) to \( \omega_t \) explains the two strong forces driving the effectiveness of market selection in terms of the equity premium.

This analysis makes it clear that in our economy, wealth shares and price impact do not move in the same order—the selection process is slow in terms of wealth shares, but is fast in terms of price impact. This differentiates our model from other selection models using traditional CRRA preferences which show that the selection speed is the same no matter whether it is measured in terms of wealth or price impacts (e.g., Yan 2008; Fedyk, Heyerdahl-Larsen and Walden, 2012). The difference between wealth dynamics and price impact dynamics highlighted by our analysis also complements Kogan, Ross, Wang and
Westerfield (2006, 2011) and Cvitanić and Malamud (2011) who underscore that survival and price impact are two distinct concepts in the long run.

To summarize, our analysis in Section 4.2 shows that in the unit-EIS case, for empirically relevant values (i.e., when \( \lambda = 1.5, 2.25 \) and 3 in Table 3), loss aversion, through portfolio selection, causes the LA-investor to vanish in the long run, and so does his price impact. The market-selection process is excessively slow in terms of wealth shares, but in terms of price impact, the selection mechanism is much more effective. In the following subsection, we show that all of these results carry over to cases of a non-unit EIS.

### 4.3 EIS\( \neq 1 \): Saving Behavior

The analysis in Section 4.1 suggests that when EIS is not equal to 1, loss aversion can change the investor’s saving behavior, which might affect the investor’s wealth accumulation and survival prospects. In this subsection, we investigate this possibility in the heterogeneous-agent economy. Again, we assume that the preferences of both investors are identical except that \( b_{LA} > 0 \) and \( b_{EZ} = 0 \).

When \( EIS_{EZ} = EIS_{LA} > 1 \), the intuition in the representative-agent economies implies that the LA-investor consumes more than the EZ-investor, which harms his survival prospects. We know from the previous subsection that in the absence of different saving behaviors, the LA-investor already loses to the EZ-investor. So this extra force coming from saving should cause the LA-investor to vanish faster. This is indeed the case, as verified by Panel A of Table 6, which reports the medians of \( \omega_t \) and \( \frac{EP_{LA}}{EP_{EZ}} \) in years 5 (\( t = 60 \)),
10 \( (t = 120) \), and 50 \( (t = 600) \), for the case of \( EIS_{EZ} = EIS_{LA} = 1.2 \). The technology parameters are fixed at the values in Table 1, while the other preference parameters are \( \beta_{EZ} = \beta_{LA} = 0.998 \), \( RA_{EZ} = RA_{LA} = 1 \), \( \lambda = 2.25 \) and \( b_{LA} = 0.001 \).\(^{21}\) Indeed, both \( \omega_t \) and \( \frac{EP_t}{EP_{EZ}} \) decline over time, suggesting that the LA-investor is losing his wealth share and equity-premium impact in the long run. Also, the selection speed is slow in terms of wealth, but it is fast in terms of the equity-premium impact, and hence the market selection mechanism is effective.

When \( EIS_{EZ} = EIS_{LA} < 1 \), the analysis in Section 4.1 suggests that the LA-investor saves more than the EZ-investor, which favors his survival. As a result, two counteracting forces are at play here: portfolio decisions work against the LA-investor’s wealth accumulation, while consumption decisions benefit it. It thus becomes nontrivial to explore whether the saving force is strong enough to reverse the result in the previous subsection. Panel B of Table 6 presents the medians of \( \omega_t \) and \( \frac{EP_t}{EP_{EZ}} \) for the case of \( EIS_{EZ} = EIS_{LA} = 0.8 \). The other parameter values are identical to those used in Panel A. We still find that all our results go through in this case, implying that in calibrated economies, the difference in saving behaviors induced by the common small EIS shared by our investors is not large enough to cause the LA-investor to survive and have a large price impact.

To better understand the effect of savings, Figure 2 plots the consumption policies for

\(^{21}\)The results are insensitive to the choice of these parameter values, as long as the LA-investor remains more cautious in buying the stock.
both investors and the equilibrium risk-free-rate function under the parameter configuration of Panel B of Table 6. Panel (a) of Figure 2 shows that, although the difference in the endogenous monthly saving rates of the two investors can be large, with a maximum of 23 basis points (which corresponds to a difference of 2.8% in annual saving rates) achieved at $\omega_t = 1$, this difference declines sharply, as the LA-investor loses wealth over time. The intuition is as follows: as $\omega_t$ decreases, the EZ-investor controls more wealth, and because she saves less relative to the LA-investor, the risk-free rate increases, which in turn raises the LA-investor’s future income because he tends to invest in the risk-free rate (as suggested by Section 4.1) and, as a result of anticipating this rise in the permanent income, he consumes more. Then, once his wealth-eroding investment positions start to reduce his wealth share, he also saves less, making his situation even worse in terms of wealth accumulation. This explains why the difference in the endogenous saving behavior cannot overcome the disadvantage coming from the portfolio decisions of the LA-investor.

In sum, for the non-unitary EIS case, if the LA-investor differs from the EZ-investor only in that he derives loss aversion utility, then for empirically-relevant parameter values, he will lose his wealth share over time, and his price impact diminishes along the way. The selection speed remains slow in terms of wealth, but fast in terms of price impact.

5 Multi-Dimensional Heterogeneity in Preferences

So far, we have assumed that the LA-investor and the EZ-investor differ only in one dimension: the LA-investor derives loss aversion utility, while the EZ-investor does not. However,
it is highly likely that they also differ in other dimensions. This section examines the effect of additional differences in the investors’ preferences.

Before presenting this analysis, we briefly discuss what kind of heterogeneity might be plausible. In principle, in addition to loss aversion utility, investors can be different in the following three dimensions: traditional risk aversion (parameter \( \zeta \)), EIS (parameter \( \rho \)), and time preference (parameter \( \beta \)). Risk aversion might not be a good candidate, as the very reason why the literature introduces loss aversion is to increase the LA-investor’s effective risk aversion, which serves to generate a high equity premium. So, to make the analysis empirically relevant, any perturbation of parameter \( \zeta \) should not reverse the order of the investors’ risk attitudes and would not change the results. Researchers have not reached a consensus regarding the “reasonable” value for the EIS or the time discount rate.\(^{22}\) We therefore choose to investigate the effect of a differing EIS or a differing time-patience parameter, and we find that these parameters indeed matter dramatically for long-run survival through their affect on saving decisions.

\section*{5.1 Different EIS Parameters}

To stack the deck against the EZ-investor, we need to assume that the LA-investor has a larger EIS than the EZ-investor, so that he would save more than the EZ-investor in

\(^{22}\)Some studies estimate the EIS to be well above 1 (e.g., Hansen and Singleton, 1982; Attanasio and Weber, 1989; Guvenen, 2001; Vissing-Jorgensen, 2002), while others estimate it to be well below 1 (e.g., Hall, 1988; Epstein and Zin, 1991; Campbell, 1999). See Guvenen (2006) for a comprehensive review of the empirical evidence on the heterogeneity in the EIS across the population. Similarly, the calibrations of the time-patience parameter \( \beta \) are widely dispersed, with its annualized counterpart \( \beta^{12} \) ranging from 0.89 (Campbell and Cochrane, 1999) to 1.1 (Brennan and Xia, 2001).
a growing economy. Specifically, we set $EIS_{LA} = 0.8$, and decrease $EIS_{EZ}$ to determine when the LA-investor will survive for an economy with the technology parameters fixed at the values in Table 1 and the other preference parameters fixed at $\beta_{EZ} = \beta_{LA} = 0.998$, $RA_{EZ} = RA_{LA} = 1$, $\lambda = 2.25$ and $b_{LA} = 0.001$. It turns out that when $EIS_{EZ} = 0.5$, the LA-investor starts to dominate the economy. The result is driven by the different saving behaviors induced by the different EIS. Panel (a1) of Figure 3 depicts the whole simulated probability-density functions (p.d.f.s) of the LA-investor’s wealth shares assuming that $\omega_0 = 0.5$, where the p.d.f.s are estimated non-parametrically from the simulation data. We can see that the p.d.f.s shift to the right as time passes, suggesting that the LA-investor tends to dominate the market in the long run. Panel (a2) shows that the difference in saving rates does not drop much when $\omega_t$ declines from intermediate levels of $\omega_t$, because when $\omega_t$ decreases, the EZ-investor consumes more as a result of a strong income effect of the increased risk-free rate. Therefore, when the wealth share of the LA-investor declines due to his portfolio decisions, his advantage in terms of saving behavior helps him.

[INSERT FIGURE 3 HERE]

5.2 Different Time Patience Parameter $\beta$

We conduct a similar exercise as we did in examining the effect of different EIS parameters. Specifically, we set $\beta_{LA} = 0.998$ and decrease $\beta_{EZ}$ to examine when the LA-investor dominates the market in the long run. The survival result is very sensitive to the time discount rate: a slight difference in $\beta$, as small as 0.002, can overturn the effect of the
LA-investor’s portfolio decisions on his survival prospects. To illustrate this sensitivity, we set $EIS_{EZ} = EIS_{LA} = 1.2$, which means that the deck is set against the LA-investor, as he would consume more than the EZ-investor if they had a common $\beta$. Other preference parameters are fixed at $RA_{EZ} = RA_{LA} = 1$, $\lambda = 2.25$ and $b_{LA} = 0.001$, and the technology parameters are fixed at the values in Table 1. Panel (b1) of Figure 3 shows that, as time passes, the p.d.f.s of $\omega_t$ shift to the right, suggesting that the LA-investor is accumulating wealth at a faster rate than the EZ-investor. Panel (b2) displays the large difference in the endogenous monthly saving ratios induced by the time-patience parameter. The minimum of this difference is 0.13%, and the maximum is 0.35%. These large magnitudes account for the LA-investor’s eventual prosperity.

The analysis in this section suggests that whether investors optimize over savings decisions is critical for long-run survival. This echoes the statements made by Yan (2008) and Cvitanić and Malamud (2011). Both papers present selection models with CRRA preferences. Our analysis sharpens their results because the recursive preferences in our model can separate risk aversion from EIS, which is the parameter determining saving decisions. Whether the LA-investor survives depends crucially on his saving behavior and it is an empirical issue. The recent empirical study by Tanaka, Camerer and Nguyen (2010) found that loss-averse investors also save less, which therefore suggests that they are likely to lose out in the long run.
6 Related Research and Discussions

This paper contributes to two strands of literature. The first is the market-selection literature, which studies what types of investors survive and have a price impact in a dynamic economy. So far, this literature has primarily focused on selection over beliefs and not over preferences. Although the idea of market selection dates back to the early 1950s (Alchian, 1950; Friedman, 1953), rigorous analysis of this idea has only recently been done. De Long, Shleifer, Summers and Waldman (1991) are the first who cast doubts on the idea of market selection. They rely on partial-equilibrium analysis and show that investors with incorrect beliefs can survive. Blume and Easley (1992) show that incorrect beliefs can be an advantage for survival in models with endogenous asset prices, but exogenous savings decisions. They also show that if savings rates are exogenous, and all investors are expected-utility maximizers with correct beliefs, then complete-asset-markets economies select for investors whose preferences are logarithmic. Sandroni (2000) and Blume and Easley (2006) endogenize both savings and portfolio decisions and show that if all investors are expected-utility maximizers, then selection in bounded, complete-asset-markets economies is determined only by beliefs and discount factors. In particular, attitudes toward risk do not matter for selection in these economies. Yan (2008), Blume and Easley (2009) and Kogan, Ross, Wang and Westerfield (2011) demonstrate that in economies with unbounded endowments, investors with incorrect beliefs may survive. Kogan, Ross, Wang and Westerfield (2006, 2011) and Cvitanić and Malamud (2011) point out that survival and price impact can be different concepts in the

\(^{23}\)One exception is Condie (2008), which analyzes the market-selection problem for an economy populated with ambiguity-averse investors and expected-utility investors.
long run. A survey of market-selection and asset-pricing results is provided in Blume and Easley (2010).

Investors in all of the above models have time-separable utility functions. Borovička (2012) has recently studied the belief-selection problem in an economy with Epstein-Zin preferences and found that agents with distorted beliefs are not driven out of the market for an empirically-relevant range of parameters. Other studies on market selection consider issues related to incomplete markets (Coury and Sciubba, 2005; Sandroni, 2005; Blume and Easley, 2006; Gallmeyer and Hollifield, 2008; Cao, 2011), imperfect competition (Palomino, 1996; Kyle and Wang, 1997), feedback from security prices to cash flows (Hirshleifer, Subrahmanyan and Titman, 2006), limited attention (Hirshleifer, Lim and Teoh, 2011), comparison of trading rules (Blume and Easley, 1992; Amir, Evstigneev, Hens and Schenk-Hoppé, 2005; Böhm and Wenzelburger, 2005), small vs. large economies (Massari, 2013), and asymmetric information and learning (Mailath and Sandroni, 2003; Sciubba, 2005; Cogley and Sargent, 2009). Instead of studying belief selection, this paper analyzes preference selection in frictionless and complete-market economies, and it is the first study on the market-selection problem between loss aversion and Epstein-Zin preferences.

The second strand of related literature considers the role of loss aversion in determining trading behavior, asset prices and trading volumes. Loss aversion, that investors are more sensitive to reductions in the value of their financial wealth than to gains, is a key feature of prospect theory which was introduced by Kahneman and Tversky (1979). Berke-

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24 There is also a large literature examining the specific pricing implications of heterogeneous risk attitudes and beliefs, including Basak and Cuoco (1998), Basak (2005), Bhamra and Uppal (2009), Basak and Yan (2010), and Xiong and Yan (2010), among others.
Barberis and Xiong (2009) study the optimal portfolio choice problem under loss aversion.\textsuperscript{25} Benartzi and Thaler (1995) were the first to use loss aversion to explain the equity-premium puzzle. Barberis, Huang and Santos (2001) extend Benartzi and Thaler’s setting to a dynamic model and find that combining loss aversion and the “house-money effect” helps to explain the behavior of the aggregate stock market. Barberis and Huang (2001) find that loss aversion is also useful in understanding the value effect in the cross-section of stock returns. Grünea and Semmler (2008) study a production economy and find that a model incorporating loss aversion can match data much better than pure consumption-based, asset-pricing models. McQueen and Vorkink (2004) show that loss aversion helps to explain the asymmetric GARCH properties of stock returns. Barberis and Huang (2007, 2009) propose a preference specification that incorporates both loss aversion and narrow framing and study its applications in portfolio choice and asset pricing.

All of the above-mentioned asset-pricing models are conducted in a representative agent framework. Gomes (2005), Gabaix (2007) and Berkelaar and Kouwenberg (2009) explore the interaction between loss-averse investors and expected-utility maximizers. In particular, Chapman and Polkovnichenko (2009) also point out that the pricing implications of first-order-risk-aversion preferences are sensitive to heterogeneity in a one-period, two-agent economy, because of endogenous withdrawal by nonexpected-utility agents from holding the risky asset. However, all these studies are done in finite-horizon models (usually two-period

\textsuperscript{25}There are also some empirical studies testing the trading implications of loss aversion, and the existing evidence is mixed (e.g., Genesove and Mayer, 2001; Ben-David and Hirshleifer, 2012; Meng, 2012).
models) and are therefore unable to answer the questions of whether loss-averse investors survive and affect prices in the long run and of whether price and wealth evolve according to the same dynamics. So, our fully dynamic model complements those studies.

We want to caution that our market-selection results should not be interpreted as implying that prospect theory cannot affect prices in the presence of arbitrageurs. Our analysis only identifies conditions under which loss aversion, which is only one element of prospect theory, matters in determining asset prices, and highlights the importance of the interactions between loss-averse behavioral investors and non-loss-averse rational investors for price dynamics. Whether loss aversion affects prices depends on whether these conditions identified by our model are satisfied, and that is an empirical question. In addition, our analysis only incorporates the loss-aversion feature of prospect theory and ignores its two other features, namely, diminishing sensitivity and probability weighting, which have also been separately studied in the asset-pricing literature (e.g., Barberis and Huang, 2008; Li and Yang, 2013). So, even if loss aversion does not affect prices in some scenarios, the other two features of prospect theory may still do so.

Meanwhile, our analysis indeed provides new insights into the implications of loss aversion in financial markets. For example, Figure 1 implies that it is difficult for loss aversion to simultaneously explain the equity premium puzzle and the stock-market-nonparticipation puzzle, as in order for loss aversion to generate a high equity premium, it is necessary for the LA-investor to hold the risky asset. More important is that Table 5 suggests that the price impact of loss-averse investors typically vanishes at a higher order than their wealth shares. Thus, even if they initially control a large fraction of the wealth in the economy, and
so have short-run effects on asset pricing, their pricing impact diminishes quickly, an insight that cannot be obtained from static models or representative-agent dynamic models.

7 Conclusion

We construct a model to study the wealth and price implications of loss aversion in the presence of “arbitrageurs” with Epstein-Zin preferences. Our analysis shows that the market-selection mechanism is effective. If loss-averse investors and arbitrageurs only differ in their attitudes toward loss aversion, then loss-averse investors vanish and have no effect on long-run asset prices for an empirically-relevant range of parameters. This is because loss aversion causes investors to act as if they are more risk averse than arbitrageurs. In the short run, although the selection process is excessively slow in terms of wealth shares, it is very effective in terms of price impact, because the first-order-risk-aversion feature of loss aversion causes loss-averse investors to endogenously reduce their participation in the stock market. We also find that the market selects for those investors with high saving motives which can be generated by high EIS or time-patience parameters.

Our analysis provides insights into the conditions under which loss aversion can affect asset prices in a dynamic financial market. Our results suggest that introducing heterogeneity is important for determining price dynamics. Empirical studies are needed to examine whether and to what extent real investors who exhibit loss aversion are different from those who do not, which in turn, with the help of our framework, is useful in further quantifying the pricing effect of preference heterogeneity. In addition, at a higher level, we believe that
our findings should hold for other types of first-order-risk-aversion preferences, such as ambiguity aversion and disappointment aversion. This is because the intuitions for our long-run and short-run results—that loss aversion raises effective risk aversion and that loss aversion implies first-order risk aversion—are shared by these other types preferences as well. We leave such a general analysis for future research.

Appendix

A. First-Order Conditions for the Case of EIS=1

This appendix derives the conditions that define the investor’s optimal decisions when the EIS takes the value of 1 ($\rho_i = 0$). In this case, the aggregator function has the Cobb-Douglas form:

$$H_i (C, X) = C^{1-\beta_i} X^{\beta_i}.$$ 

The Bellman equation becomes

$$A_{i,t} W_{i,t} = \max_{C_{i,t},s_{i,t}} C_{i,t}^{1-\beta_i} \left[ \mu_i \left[ J_i (W_{i,t+1}, I_{t+1}) | I_t \right] + b_i E_t \left[ v \left( G_{i,t+1} \right) \right] \right]^{\beta_i}$$

$$= W_{i,t} \max_{\alpha_{i,t}, s_{i,t}} \alpha_{i,t}^{1-\beta_i} (1 - \alpha_{i,t})^{\beta_i} \left[ \mu_i \left[ A_{i,t+1} M_{i,t+1} | I_t \right] + b_i E_t \left[ v \left( s_{i,t} (R_{t+1} - R_{f,t}) \right) \right] \right]^{\beta_i}$$

$$= W_{i,t} \max_{\alpha_{i,t}} \alpha_{i,t}^{1-\beta_i} (1 - \alpha_{i,t})^{\beta_i} \max_{s_{i,t}} \left[ \mu_i \left[ A_{i,t+1} M_{i,t+1} | I_t \right] + b_i E_t \left[ v \left( s_{i,t} (R_{t+1} - R_{f,t}) \right) \right] \right]^{\beta_i}.$$ 

Therefore, the optimal consumption policy can be explicitly solved:

$$\max_{\alpha_{i,t}} \alpha_{i,t}^{1-\beta_i} (1 - \alpha_{i,t})^{\beta_i} \Rightarrow \alpha_{i,t}^* = 1 - \beta_i.$$ 

As a result,

$$A_{i,t} = (1 - \beta_i)^{1-\beta_i} \beta_i^{\beta_i} \max_{s_{i,t}} \left[ \mu_i \left[ A_{i,t+1} M_{i,t+1} | I_t \right] + b_i E_t \left[ v \left( s_{i,t} (R_{t+1} - R_{f,t}) \right) \right] \right]^{\beta_i}.$$ 

(35)
The partial-equilibrium problem is therefore summarized by the above equation, which involves solving the optimal-investment-decision function, \(s_i(\cdot)\), and the value function, \(A_i(\cdot)\). So, relative to the case of a non-unit EIS, one can avoid numerically solving the investor’s consumption policy, as it is given by equation (34), but he needs to numerically solve the investor’s indirect value function using equation (35).

The first-order conditions to the portfolio choice problem are

\[
FOC_{i,+} = \left[ E_t (A_{i,t+1} M_{i,t+1}^\zeta) \right]^{1/\zeta_i - 1} E_t [A_{i,t+1} M_{i,t+1}^{\zeta_i - 1} (R_{t+1} - R_{f,t})] + b_i E_t [v (R_{t+1} - R_{f,t})] = 0, \text{ for } s_{i,t}^* > 0,
\]

\[
FOC_{i,-} = \left[ E_t (A_{i,t+1} M_{i,t+1}^\zeta) \right]^{1/\zeta_i - 1} E_t [A_{i,t+1} M_{i,t+1}^{\zeta_i - 1} (R_{t+1} - R_{f,t})] - b_i E_t [v (R_{f,t} - R_{t+1})] = 0, \text{ for } s_{i,t}^* < 0,
\]

where \(FOC_{i,+} \leq 0 \text{ and } FOC_{i,-} \geq 0, \text{ for } s_{i,t}^* = 0.

In particular, for the EZ-investor, \(b_{EZ} = 0\), and the above first-order conditions boil down to

\[
E_t [A_{EZ,t+1} M_{EZ,t+1}^{\zeta_{EZ} - 1} (R_{t+1} - R_{f,t})] = 0.
\]

**B. Proof of Proposition 1**

(i) Suppose \(\rho \neq 0\). By equation (29), we can express \(\alpha\) in terms of \(f\): \(\alpha = \frac{1}{1+f}\). Combining this expression with \(M_{i,t+1} = R_{t+1} = \frac{f+1}{f} \theta_{t+1}\) and \(s_{i,t} = 1\), we can rewrite equations (13) and (14) as follows:

\[
\left( \frac{1 - \beta}{\beta} \right)^{1/\rho} \left( \frac{1}{f} \right)^{1-1/\rho} = (1 - \beta)^{1/\rho} \frac{(1 + f)^{1/\rho}}{f} \mu (\theta_{t+1}) + b E [v (R_{t+1} - R_{f})], \quad (36)
\]
\[ (1 - \beta)^{1/\rho} (1 + f)^{1/\rho - 1} \left[ E \left( \theta_{t+1}^\kappa \right) \right]^{1/\kappa - 1} E \left[ \theta_{t+1}^{\kappa - 1} \left( \frac{f + 1}{f} \theta_{t+1} - R_f \right) \right] + b E \left[ v \left( R_{t+1} - R_f \right) \right] = 0. \]  

(37)

We use (36) to get the expression of \( b E \left[ v \left( R_{t+1} - R_f \right) \right] \), which is then plugged into equation (37) to obtain (32). Finally, plugging back (32) into (36) yields (31).

(ii) Suppose \( \rho = 0 \). Since \( \alpha = 1 - \beta \), by equation (29), we have

\[ f = \frac{1 - \alpha}{\alpha} = \frac{\beta}{1 - \beta}. \]

Using the above equation together with \( M_{i,t+1} = R_{t+1} = \frac{f + 1}{f} \theta_{t+1} \) and \( s_{i,t} = 1 \), we can rewrite the first-order condition in Appendix A and the value function in (35) as follows:

\[ A \left[ E \left( \theta_{t+1}^\kappa \right) \right]^{1/\kappa - 1} E \left( \theta_{t+1}^{\kappa - 1} \right) R_f = A \frac{1}{\beta} \mu \left( \theta_{t+1} \right) + b E \left[ v \left( \frac{\theta_{t+1}}{\beta} - R_f \right) \right], \tag{38} \]

\[ A \frac{1}{\beta} \mu \left( \theta_{t+1} \right) + b E \left[ v \left( \frac{\theta_{t+1}}{\beta} - R_{f,t} \right) \right] = \frac{A^{1/\beta}}{(1 - \beta)^{1-\beta/\beta}}. \tag{39} \]

These two equations combine to imply:

\[ A \left[ E \left( \theta_{t+1}^\kappa \right) \right]^{1/\kappa - 1} E \left( \theta_{t+1}^{\kappa - 1} \right) R_f = \frac{A^{1/\beta}}{(1 - \beta)^{1-\beta/\beta}} \Rightarrow \]

\[ A = \left( A \left[ E \left( \theta_{t+1}^\kappa \right) \right]^{1/\kappa - 1} E \left( \theta_{t+1}^{\kappa - 1} \right) \right)^{\frac{1}{1-\beta/\beta}} R_f^{\frac{1}{1-\beta/\beta}} (1 - \beta)^{\frac{\beta}{1-\beta}}, \]

which is then inserted back to (38), yielding equation (33).

C. Numerical Algorithm

This appendix sketches the procedure used to numerically solve the model. We focus on the non-unit EIS case \( \rho_t \neq 0 \), and the solution procedure for the unit EIS case is slightly different. The algorithm is developed based on Kubler and Schmedders (2003) and is summarized as follows.

**Step 0**: Define a finite grid on \([0,1]\). Choose two continuous functions, \( \alpha_{EZ}^0(\cdot) \) and \( \alpha_{LA}^0(\cdot) \),
as initials for the investors’ consumption-policy functions. These initials define the initial for the price-dividend-ratio function, $f^0(\cdot)$, through equation (22). Then on each grid point $\omega_t$, go through steps 1-4.

**Step 1:** Given functions $\alpha^0_{EZ}(\cdot)$ and $\alpha^0_{LA}(\cdot)$, suppose that the LA-investor allocates nothing on the stock; that is, $s^1_{LA}(\omega_t) = 0$. Then use both investors’ value functions, equation (13), the EZ-investor’s first-order condition, equation (17), and the state-transition functions, equation (27), to solve five unknowns: $\alpha^*_{EZ,t}$, $\alpha^*_{LA,t}$, $R_f,t$, $\omega_{t+1,H}$, $\omega_{t+1,L}$, where $\omega_{t+1,H}$ and $\omega_{t+1,L}$ are the next-period wealth shares when $\theta_{t+1} = \theta_H$ and $\theta_L$, respectively.

**Step 2:** Plug the solved $\alpha^*_{LA,t}$, $R_f,t$, $\omega_{t+1,H}$ and $\omega_{t+1,L}$ into equations (14) and (15) to get $FOC_{LA,+}$ and $FOC_{LA,-}$. If $FOC_{LA,+} \leq 0$ and $FOC_{LA,-} \geq 0$, then set $\alpha^{n+1}_{EZ}(\omega_t) = \alpha^*_{EZ,t}$ and $\alpha^{n+1}_{LA}(\omega_t) = \alpha^*_{LA,t}$. If $FOC_{LA,+} > 0$, then go to Step 3; otherwise, go to Step 4.

**Step 3:** Use both investors’ value functions, equation (13), the EZ-investor’s first-order equation, (17), the LA-investor’s first-order condition for a positive investment, equation (14), and the state-transition function, equation (27), to solve six unknowns: $\alpha^*_{EZ,t}$, $\alpha^*_{LA,t}$, $R_f,t$, $\omega_{t+1,H}$, $\omega_{t+1,L}$, $s^*_{LA,t}$. Set $\alpha^{n+1}_{EZ}(\omega_t) = \alpha^*_{EZ,t}$ and $\alpha^{n+1}_{LA}(\omega_t) = \alpha^*_{LA,t}$.

**Step 4:** Use both investors’ value functions, equation (13), the EZ-investor’s first-order equation, (17), the LA-investor’s first-order condition for a negative investment, equation (15), and the state-transition function, equation (27), to solve six unknowns: $\alpha^*_{EZ,t}$, $\alpha^*_{LA,t}$, $R_f,t$, $\omega_{t+1,H}$, $\omega_{t+1,L}$, $s^*_{LA,t}$. Set $\alpha^{n+1}_{EZ}(\omega_t) = \alpha^*_{EZ,t}$ and $\alpha^{n+1}_{LA}(\omega_t) = \alpha^*_{LA,t}$.

**Step 5:** Check whether the following stop criterion is satisfied:

$$\max_{\omega_t} \left\| \begin{pmatrix} \alpha^{n+1}_{EZ}(\cdot) \\ \alpha^{n+1}_{LA}(\cdot) \\ f^{n+1}(\cdot) \end{pmatrix} - \begin{pmatrix} \alpha^n_{EZ}(\cdot) \\ \alpha^n_{LA}(\cdot) \\ f^n(\cdot) \end{pmatrix} \right\| < \tau,$$

where $\tau$ is an error tolerance. If yes, then the algorithm terminates, and the next step is to
set the consumption and investment policy functions and the risk-free-rate function as those solved in the last round. Otherwise, increase \( n \) by 1 and go to Step 1.

References


Table 1: Technology-Parameter Values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\pi_H$</th>
<th>$\pi_L$</th>
<th>$\theta_H$</th>
<th>$\theta_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>0.5</td>
<td>0.5</td>
<td>1.0126</td>
<td>0.9906</td>
</tr>
</tbody>
</table>

This table reports the technology-parameter values used in the computation of equilibria. The calibration takes one period to be one month. The consumption-growth-rate parameters $\theta_H$ and $\theta_L$ are calibrated to match the historical mean (1.84%) and volatility (3.79%) of the annual log consumption growth rate.
Table 2: Asset Prices and Consumption Policies in Representative-Agent Economies

<table>
<thead>
<tr>
<th>Panel A: $EIS = 1$</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$b = 0$</td>
<td>0.07</td>
<td>4.17</td>
<td>2.40</td>
<td>4289</td>
<td>0.42</td>
</tr>
<tr>
<td>$b = 0.0005$</td>
<td>1.22</td>
<td>3.03</td>
<td>2.40</td>
<td>5076</td>
<td>21</td>
</tr>
<tr>
<td>$b = 0.001$</td>
<td>2.65</td>
<td>1.59</td>
<td>2.40</td>
<td>5894</td>
<td>42</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: $EIS = 0.8$</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$b = 0$</td>
<td>0.07</td>
<td>4.63</td>
<td>2.86</td>
<td>4289</td>
<td>0.42</td>
</tr>
<tr>
<td>$b = 0.0005$</td>
<td>1.13</td>
<td>3.31</td>
<td>2.60</td>
<td>5021</td>
<td>19</td>
</tr>
<tr>
<td>$b = 0.001$</td>
<td>3.01</td>
<td>0.95</td>
<td>2.13</td>
<td>6079</td>
<td>47</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C: $EIS = 1.2$</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$b = 0$</td>
<td>0.07</td>
<td>3.86</td>
<td>2.09</td>
<td>4289</td>
<td>0.42</td>
</tr>
<tr>
<td>$b = 0.0005$</td>
<td>1.27</td>
<td>2.86</td>
<td>2.29</td>
<td>5111</td>
<td>22</td>
</tr>
<tr>
<td>$b = 0.001$</td>
<td>2.53</td>
<td>1.82</td>
<td>2.50</td>
<td>5832</td>
<td>41</td>
</tr>
</tbody>
</table>

This table reports the continuously-compounded annualized equilibrium equity premiums ($EP^a = 12 [E (\log R_{t+1}) - \log R_f]$), risk-free rates ($r_f^a = 12 \log (R_f)$) and consumption propensities ($\alpha^a = 12\alpha$), assuming that investors are identical in preferences. Panels A, B and C correspond to different values of EIS: $EIS = 1$ ($\rho = 0$), $EIS = 0.8$ ($\rho = -0.25$) and $EIS = 1.2$ ($\rho = 1/6$). Parameter $b$ controls the relative importance of loss aversion utility in the investor’s preferences. For all combinations, the following three preference parameters are fixed at constant: $\beta = 0.998$, $\lambda = 2.25$ and $RA = 1$ (or $\zeta = 0$). The technology parameters are fixed at the values in Table 1. $q_L$ ($q_S$) is the premium a representative agent with wealth of $75,000 would pay to avoid a 50:50 bet to gain or lose $25,000 ($250).
Table 3: Survival of the LA-Investor When $EIS = 1$

Panel A: $RA_{EZ} = RA_{LA} = 1 \ (\zeta_{EZ} = \zeta_{LA} = 0)$

<table>
<thead>
<tr>
<th></th>
<th>$\lambda = 1.05$</th>
<th>$\lambda = 1.5$</th>
<th>$\lambda = 2.25$</th>
<th>$\lambda = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 60$</td>
<td>0.4897</td>
<td>0.4897</td>
<td>0.4897</td>
<td>0.4897</td>
</tr>
<tr>
<td>$t = 120$</td>
<td>0.4853</td>
<td>0.4853</td>
<td>0.4853</td>
<td>0.4853</td>
</tr>
<tr>
<td>$t = 600$</td>
<td>0.4547</td>
<td>0.4534</td>
<td>0.4534</td>
<td>0.4534</td>
</tr>
</tbody>
</table>

Panel B: $RA_{EZ} = RA_{LA} = 3 \ (\zeta_{EZ} = \zeta_{LA} = -2)$

<table>
<thead>
<tr>
<th></th>
<th>$\lambda = 1.05$</th>
<th>$\lambda = 1.5$</th>
<th>$\lambda = 2.25$</th>
<th>$\lambda = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 60$</td>
<td>0.5026</td>
<td>0.4810</td>
<td>0.4782</td>
<td>0.4782</td>
</tr>
<tr>
<td>$t = 120$</td>
<td>0.5039</td>
<td>0.4667</td>
<td>0.4633</td>
<td>0.4633</td>
</tr>
<tr>
<td>$t = 600$</td>
<td>0.5110</td>
<td>0.3702</td>
<td>0.3641</td>
<td>0.3641</td>
</tr>
</tbody>
</table>

This table reports the medians of the LA-investor’s wealth shares ($\omega_t$) at times $t = 60$, 120 and 600 months when the LA-investor has initial wealth shares of $\omega_0 = 0.5$ and both investors have a relative-risk-aversion coefficient of 1 (Panel A) or 3 (Panel B). Both investors have a unit EIS: $EIS_{EZ} = EIS_{LA} = 1$. They have the same time-patience parameter: $\beta_{EZ} = \beta_{LA} = 0.998$. In the LA-investor’s preferences, parameter $b_{LA}$ is set as 0.001, and the loss-aversion coefficient $\lambda$ can take four possible values: $\lambda = 1.05, 1.5, 2.25$ and 3. The technology parameters are fixed at the values in Table 1. The medians are estimated from 5000 simulated sample paths at time $t$. 

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Table 4: Risk Attitude and Survival

<table>
<thead>
<tr>
<th></th>
<th>$\lambda = 1.05$</th>
<th>$\lambda = 1.5$</th>
<th>$\lambda = 2.25$</th>
<th>$\lambda = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$EP_{LA}$ (bps)</td>
<td>3.42</td>
<td>21.96</td>
<td>23.82</td>
<td>43.97</td>
</tr>
<tr>
<td>$EP_{LA} &gt; EP_{log}$ (=1.20bps)</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>$EP_{LA} &gt; EP_{EZ}$ (=3.60bps)</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Is LA further from log?</td>
<td>(log&lt;LA&lt;EZ)</td>
<td>Y</td>
<td>(log&lt;EZ&lt;LA)</td>
<td>Y</td>
</tr>
<tr>
<td></td>
<td>(log&lt;LA&lt;EZ)</td>
<td>(log&lt;EZ&lt;LA)</td>
<td>(log&lt;EZ&lt;LA)</td>
<td>(log&lt;EZ&lt;LA)</td>
</tr>
<tr>
<td>Does LA vanish?</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
</tbody>
</table>

This table illustrates the relationship between the risk attitude and the survival prospects of the LA-investor. The variables $EP_{LA}$, $EP_{log}$ and $EP_{EZ}$ are monthly equity premiums (in basis points) in the representative-agent economies populated with the LA-investor, the log investor and the EZ-investor, respectively. The technology parameters are fixed at the values in Table 1. The other preference parameters are set at the following values: $b_{LA} = 0.001$, $EIS_{EZ} = EIS_{LA} = 1$, $\beta_{EZ} = \beta_{LA} = 0.998$ and $RA_{EZ} = RA_{LA} = 3$. “Y” and “N” represent “Yes” and “No” respectively. The expression of “log<LA<EZ” means that in terms of risk attitude, the LA-investor behaves as if he is more risk averse than the log investor, and the EZ-investor is more risk averse than the LA-investor. A similar explanation applies to the expression of “log<EZ<LA.”
Table 5: Price Impact of the LA-Investor When $EIS = 1$

Panel A: $RA_{EZ} = RA_{LA} = 1$ ($\zeta_{EZ} = \zeta_{LA} = 0$)

<table>
<thead>
<tr>
<th></th>
<th>Medians of $\phi_{H,t}/\phi_{H,EZ}$</th>
<th>Medians of $\phi_{L,t}/\phi_{L,EZ}$</th>
<th>Medians of $EP_t/EP_{EZ}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_0 = 0.1$</td>
<td>$0.9988$</td>
<td>$0.9891$</td>
<td>$0.9026$</td>
</tr>
<tr>
<td>$\omega_0 = 0.5$</td>
<td>$0.9988$</td>
<td>$0.9895$</td>
<td>$0.9368$</td>
</tr>
<tr>
<td>$\omega_0 = 0.9$</td>
<td>$0.9988$</td>
<td>$0.9897$</td>
<td>$0.9507$</td>
</tr>
<tr>
<td>$\omega_0 = 1$</td>
<td>$0.9988$</td>
<td>$0.9909$</td>
<td>$0.9784$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$t = 0$</th>
<th>$t = 60$</th>
<th>$t = 120$</th>
<th>$t = 600$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_0 = 0.1$</td>
<td>$0.9963$</td>
<td>$0.9965$</td>
<td>$0.9965$</td>
<td>$0.9970$</td>
</tr>
<tr>
<td>$\omega_0 = 0.5$</td>
<td>$0.9717$</td>
<td>$0.9737$</td>
<td>$0.9750$</td>
<td>$0.9825$</td>
</tr>
<tr>
<td>$\omega_0 = 0.9$</td>
<td>$0.8709$</td>
<td>$0.8956$</td>
<td>$0.9225$</td>
<td>$0.9685$</td>
</tr>
<tr>
<td>$\omega_0 = 1$</td>
<td>$0.8113$</td>
<td>$0.8113$</td>
<td>$0.8113$</td>
<td>$0.8113$</td>
</tr>
</tbody>
</table>

Panel B: $RA_{EZ} = RA_{LA} = 3$ ($\zeta_{EZ} = \zeta_{LA} = -2$)

<table>
<thead>
<tr>
<th></th>
<th>Medians of $\phi_{H,t}/\phi_{H,EZ}$</th>
<th>Medians of $\phi_{L,t}/\phi_{L,EZ}$</th>
<th>Medians of $EP_t/EP_{EZ}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_0 = 0.1$</td>
<td>$0.9963$</td>
<td>$0.9717$</td>
<td>$0.8709$</td>
</tr>
<tr>
<td>$\omega_0 = 0.5$</td>
<td>$0.9965$</td>
<td>$0.9737$</td>
<td>$0.8956$</td>
</tr>
<tr>
<td>$\omega_0 = 0.9$</td>
<td>$0.9965$</td>
<td>$0.9750$</td>
<td>$0.9225$</td>
</tr>
<tr>
<td>$\omega_0 = 1$</td>
<td>$0.9970$</td>
<td>$0.9825$</td>
<td>$0.9685$</td>
</tr>
</tbody>
</table>

This table reports the medians of the LA-investor’s price impacts at $t = 0$, 60, 120 and 600 months when $\omega_0 = 0.1$, 0.5, 0.9 and 1, and both investors have a relative-risk-aversion coefficient of 1 (Panel A) or 3 (Panel B). The price impacts are measured by $\phi_{H,t}/\phi_{H,EZ}$, $\phi_{L,t}/\phi_{L,EZ}$ and $EP_t/EP_{EZ}$, where $\phi_{H,t}$, $\phi_{L,t}$ and $EP_t$ are the state prices and the conditional monthly equity premium in the heterogeneous-agent economy and where $\phi_{H,EZ}$, $\phi_{L,EZ}$ and $EP_{EZ}$ are their counterparts in the representative-agent economy with EZ-investor. The preference parameters are: $EIS_{EZ} = EIS_{LA} = 1$, $\beta_{EZ} = \beta_{LA} = 0.998$, $b_{LA}=0.001$ and $\lambda=2.25$. The technology parameters are fixed at the values in Table 1.
Table 6: Survival and Price Impact of the LA-Investor When $EIS \neq 1$

<table>
<thead>
<tr>
<th></th>
<th>Panel A: EIS=1.2</th>
<th>Panel B: EIS=0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_0 = 0.1$</td>
<td>0.0984</td>
<td>0.0986</td>
</tr>
<tr>
<td>$\omega_0 = 0.5$</td>
<td>0.4889</td>
<td>0.4905</td>
</tr>
<tr>
<td>$\omega_0 = 0.9$</td>
<td>0.8358</td>
<td>0.8662</td>
</tr>
<tr>
<td>$\omega_0 = 1$</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Medians of $\omega_t$

<table>
<thead>
<tr>
<th>$t$</th>
<th>$\omega_0 = 0.1$</th>
<th>$\omega_0 = 0.5$</th>
<th>$\omega_0 = 0.9$</th>
<th>$\omega_0 = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 60$</td>
<td>0.0979</td>
<td>0.4838</td>
<td>0.7954</td>
<td>1</td>
</tr>
<tr>
<td>$t = 120$</td>
<td>0.0979</td>
<td>0.4838</td>
<td>0.7954</td>
<td>1</td>
</tr>
<tr>
<td>$t = 600$</td>
<td>0.0960</td>
<td>0.4463</td>
<td>0.6316</td>
<td>1</td>
</tr>
</tbody>
</table>

Medians of the equity-premium impact $EP_t/EP_{EZ}$

<table>
<thead>
<tr>
<th>$t$</th>
<th>$EP_t/EP_{EZ}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 0$</td>
<td>1.1126, 2.0328, 16.6277, 18.10</td>
</tr>
<tr>
<td>$t = 60$</td>
<td>1.1107, 1.9867, 8.0234, 18.10</td>
</tr>
<tr>
<td>$t = 120$</td>
<td>1.1100, 1.9661, 5.8613, 18.10</td>
</tr>
<tr>
<td>$t = 600$</td>
<td>1.1076, 1.8284, 2.8241, 18.10</td>
</tr>
</tbody>
</table>

This table reports the medians of the LA-investor’s wealth shares $\omega_t$ and the equity-premium impacts $EP_t/EP_{EZ}$ at times $t = 60, 120$ and $600$ months when both investors have a common non-unit EIS parameter. In Panel A, $EIS_{EZ} = EIS_{LA} = 1.2$, and in Panel B, $EIS_{EZ} = EIS_{LA} = 0.8$. Both investors have the same time-patience parameter and relative-risk-aversion parameters: $\beta_{EZ} = \beta_{LA} = 0.998$ and $RA_{EZ} = RA_{LA} = 1$ ($\zeta_{EZ} = \zeta_{LA} = 0$). In the LA-investor’s preferences, parameter $b_{LA}$ is set as 0.001, and the loss-aversion coefficient $\lambda$ is set as 2.25. The technology parameters are fixed at the values in Table 1. The medians are estimated from 5000 simulated sample paths at time $t$. 
Figure 1: Asset Prices and Stock-Market Participation

Panels (a), (c) and (d) respectively graph the two state prices ($\phi_H$ and $\phi_L$), the LA-investor’s stock investment policy ($s_{LA,t}$), and the conditional equity premium ($EP_t=Et(R_{t+1}-R_{f,t})$) as functions of the LA-investor’s wealth share $\omega_t$. Panel (b) uses the Edgeworth box to depict the equilibrium determination when the LA-investor’s wealth share $\omega_t$ is 0, 0.5 and 1. “IC” refers to “indifference curve” in this panel. The preference parameters are $EIS_{EZ}=EIS_{LA}=1$, $RA_{EZ}=RA_{LA}=1$, $\beta_{EZ}=\beta_{LA}=0.998$, $\lambda=2.25$ and $b_{LA}=0.001$. The technology parameters are fixed at the values in Table 1.
This figure depicts investors’ consumption policies in Panel (a) and the equilibrium risk-free-rate function in Panel (b) when $EIS_{EZ} = EIS_{LA} = 0.8$. The other preference parameters are $RA_{EZ} = RA_{LA} = 1$, $\beta_{EZ} = \beta_{LA} = 0.998$, $\lambda = 2.25$ and $b_{LA} = 0.001$. The technology parameters are fixed at the values in Table 1.
Figure 3: Survival and Consumption Policies when Investors Have Different EIS or Time-Patience Parameters

This figure depicts the probability-density functions (p.d.f.s) of the LA-investor’s wealth shares ($\omega_t$) at $t=60, 120, 600$ and $2400$ months in Panels (a1) and (b1), as well as the consumption policies of both investors in Panels (a2) and (b2) when both investors either have different EIS parameters or time-patience parameters. Specifically, in Panels (a1) and (a2), investors have different EIS parameters ($EIS_{EZ}=0.5$ and $EIS_{LA}=0.8$), while they share a common time-patience parameter ($\beta_{EZ}=\beta_{LA}=0.998$). In Panels (b1)-(b2), they have different time-patience parameters ($\beta_{EZ}=0.996$ and $\beta_{LA}=0.998$), while they share a common EIS parameter ($EIS_{EZ}=EIS_{LA}=1.2$). The other preference parameters are $RA_{EZ}=RA_{LA}=1$, $\lambda=2.25$ and $b_{LA}=0.001$. The technology parameters are fixed at the values in Table 1. The p.d.f.s are estimated non-parametrically from 5000 simulated data. At time 0, each investor has half of the aggregate wealth; that is, $\omega_0=0.5$. 