Security, Potential, Goal Achievement, and Risky Choice Behavior

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Abstract

This paper develops a new model for risky choice behavior—Target Utility Theory (TUT)—that shed light on several puzzles of the decision making and financial literatures. In particular, TUT can explain the experimental evidence related to goal seeking behavior (Payne et al., 1980), preference for security/potential (Levy and Levy, 2002), and the effect of prior outcomes on risky choice behavior (Thaler and Johnson, 1990). Moreover, TUT can provide a framework to rationalize phenomena observed in the financial markets, such as the escalation of commitment (Staw, 1981), the disposition effect (Shefrin and Statman, 1985), and the increase in risk taking by investors that are obtaining below target returns (Coval and Shumway, 2005). Running a Logit model on several results of the decision making literature, I find that TUT significantly improves with respect to: Prospect Theory, Expected Utility Theory, SP/A theory, Regret Theory, and Disappointment Aversion.

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1 Introduction

Since the well known paradox of Allais (1953) the descriptive ability of Expected Utility Theory (Von Neumann and Morgenstern, 1947) has come under attack, and considerable effort has been devoted in finding a model that can properly describe how people face risk. The most prominent alternative to Expected Utility Theory (EUT) has been Prospect Theory (Kahneman and Tversky, 1979). However, mounting evidence casts doubt on the descriptive ability of Prospect Theory as well. For example, contrary to the predictions of Prospect Theory, several works (e.g., Levy and Levy, 2002) find evidence of preference for security/potential, i.e., downside risk aversion (risk aversion for losses) and preference for upside potential (risk seeking behavior for gains). Moreover, there is clear evidence that aspiration levels play an important role in the decision making process, posing a significant challenge to all existing models. In particular, it is still not clear why risky choice behavior is considerably affected by the presence of prior gains or losses (e.g., Thaler and Johnson, 1990), or by the overall probabilities of achieving relevant aspiration levels (e.g., Payne et al., 1980).

Understanding the effect of aspiration levels on risky choice behavior is of particular importance for the financial literature as well. In fact, an extensive literature documents that investors’ risk taking is affected by the need to achieve target returns. In particular, the empirical evidence is related to phenomena such as the escalation of commitment (the propensity to commit further resources in a failing course of action; e.g., Staw, 1981), the disposition effect (the higher propensity of holding stocks that are realizing paper losses; e.g., Shefrin and Statman, 1985), and investors’ portfolio choices (increase in risk taking by traders that are obtaining below target returns; e.g., Coval and Shumway, 2005).

The purpose of this paper, then, is to develop a new model for decision making under risk—Target Utility Theory—that can rationalize the phenomena that challenge existing theories and, in particular, clarify the effect of aspiration levels on risky choice behavior. Target Utility Theory (TUT) can improve with respect to existing models because it can capture three fundamental aspects that drive risky choice behavior: (i) goal seeking behavior (i.e., the desire of achieving relevant aspiration levels), (ii) preference for security/potential (i.e., downside risk aversion and preference for upside potential), (iii) and loss aversion (i.e., losses loom larger than gains).
The utility function of TUT is defined over gains and losses with respect to a reference point, or target payoff. Utility is equal to the sum of two components: a value function that captures the “pure” utility of each outcome, and an anticipatory feeling component that captures the expected regret of each lottery. The value function is concave for losses and convex for gains, implying preference for security/potential. The anticipatory feeling component, instead, creates a jump in utility at the reference point, implying goal seeking behavior. Finally, TUT is consistent with loss aversion because both components of the utility function are more pronounced for losses rather than gains.

The interaction between the value and the regret functions allows explaining why individuals alternate risk averse and risk seeking behavior. Since it emphasizes the importance of achieving the target payoff, the regret component induces risk seeking behavior for losses (in the negative domain, individuals are willing to take risk in order to avoid losses and decrease expected regret) and risk aversion for gains (in the positive domain, individuals want to avoid risk in order to obtain positive payoffs and decrease expected regret). The value component, instead, implies risk aversion for gains and risk seeking for losses. Depending on which one of the effects prevails, the model will predict either risk aversion or risk seeking behavior.

A particularly puzzling phenomena rationalized by TUT is the effect of prior outcomes on risky choice behavior. As shown by Thaler and Johnson (1990) individuals typically display (i) risk aversion after large losses, (ii) risk seeking behavior after relatively small losses (break-even effect), (iii) risk aversion after small gains, and (iv) risk seeking behavior after large gains (house money effect). In contrast with existing models, TUT can explain the observed behavior without requiring distortions in probabilities, or ad-hoc assumptions about the reference point. The concavity of TUT in the negative domain explains why individuals avoid risk after large losses that cannot be recovered. At the same time, the model is able to rationalize why individuals display risk seeking behavior when they are given the opportunity to break-even. In

1 Because of the jump at zero, TUT takes into account the overall probability of winning/losing.
2 Existing models can explain Thaler and Johnson (1990) results only by assuming that individuals sometimes integrate and sometimes segregate prior outcomes, therefore moving the reference point accordingly. In TUT, in contrast, the assumption is that individuals care about the payoff of the entire venture, i.e., they always integrate prior outcomes within the evaluation period.
this case, individuals prefer risky prospects that allow them to break-even and achieve their target, because these prospects entail lower expected regret. The opposite is true after prior gains. When facing small prior gains, individuals avoid risk in order to avoid regret. Finally, the convexity of the utility function in the positive domain is consistent with risk seeking behavior after large prior gains, as long as individuals are certain to obtain a final gain.

The TUT framework can also rationalize some phenomena related to investors’ behavior. The same logic used to clarify Thaler and Johnson (1990) results can be used to clarify phenomena such as the the escalation of commitment (Staw, 1981), the disposition effect (Shefrin and Statman, 1985), and the increase in risk taking by investors that are obtaining below target returns (Coval and Shumway, 2005). To shed light on the effect of aspiration levels on investors’ behavior, we can consider the portfolio choices of an investor facing prior outcomes and having TUT preferences defined over the gains/losses of the entire investment period. Consistent with the results of Coval and Shumway (2005), Brown et al. (1996), and Bowman (1980, 1982, 1984), the optimal response to prior outcomes is a V-shape risk profile: strong increase in risk taking when investors face prior losses, mild increase in risk taking when investors face prior gains, and minimum risk taking when investors are obtaining their target return.

Finally, Maximum Likelihood estimation results—obtained with a Logit model defined on a wide range of results of the empirical decision making literature—point out that TUT significantly improves with respect to: Expected Utility Theory, Original Prospect Theory (Kahneman and Tversky, 1979), Cumulative Prospect Theory (Tversky and Kahneman, 1992), SP/A theory (Lopes, 1987), Regret Theory (Loomes and Sudgen, 1982), Disappointment Aversion (Gul, 1991), and Expected Utility Theory with jumps (Diecidue and Van de Ven, 2008). Moreover, several model comparison tests on TUT confirm that goal seeking behavior, preference for security/potential, and loss aversion are jointly necessary to explain risky choice behavior. Finally, model comparison tests obtained with models that combine TUT and Prospect Theory point out that the factors implied by TUT are significant even after controlling for Prospect Theory, while the factors implied by Prospect Theory are driven out by TUT.

The reminder of the paper is organized as follows. Section 2 provides a brief review

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3 Regret is produced if the decision maker fails to obtain a final gain.
of the decision making literature. Section 3 describes TUT, showing how the model can explain the main phenomena that challenge existing theories. Section 4 outlines the empirical analysis, including the estimation methods and model comparison tests. Section 5 reports the estimation results. Section 6 focuses on the effect of aspiration levels on investment choices. Section 7 concludes.

2 Literature Review

2.1 Expected Utility Theory

Expected Utility Theory (EUT) has been the cornerstone of the individual decision making literature. The model was originally formulated by Bernoulli (1738) and first axiomatized by von Neuman and Morgenstern (1947). The basic principle of the theory is that decisions under risk are made as to maximize expected utility, the sum of the products of the utilities of outcomes and the probabilities that they will be obtained. For example, the expected utility of lottery $X = (x_1, p_1; \ldots; x_N, p_N)$ is:

$$E(U(W + X)) = \sum_{i=1}^{N} p_i U(W + x_i)$$

where $x_i$ is the outcome of lottery $X$ in state $i$, $p_i$ is the probability of state $i$, and $W$ is the initial wealth level of the decision maker.

Traditionally, it has been assumed that individual utility functions are concave functions of wealth (monotonically increasing in wealth, but increasing at a decreasing rate). The concavity of the utility function embodies the property of risk aversion, which is assumed to characterize “responsible” people that buy insurance and diversify investments. Risk aversion implies that any sure outcome $x$ is preferred to any gamble with an expected value of $x$. For example, risk aversion implies:

$$(50, 100\%) \succ (0, 50\%; 100, 50\%)$$

Risk averse individuals prefer $50 for sure, rather than a gamble that delivers $100 with 50% probability and $0 with 50% probability.
2.1.1 Phenomena Challenging Expected Utility Theory

Despite the widespread acceptance of EUT, there are reasons to question its descriptive ability. For example, Allais (1953) found preference patterns that violate fundamental implications of EUT. The phenomena studied by Allais are known as the “common consequence effect” and the “common ratio effect”.

Several economists also challenged the idea that individuals are always risk averse, as assumed in EUT. For example, Friedman and Savage (1948) claim that the fact that individuals buy both insurance and lottery tickets (and that most lotteries have more than one big prize) imply that the utility function must have two concave regions with a convex region in between. Markowitz (1952) pointed out several problems with the Friedman and Savage utility function. However, he showed that the problems are solved if we assume that decisions are based on change in wealth, and if the utility function has three inflection points. Markowitz utility function displays convexity for large losses, concavity for small losses, convexity for small gains, and concavity for large gains.

The so-called “fourfold pattern” is probably the best piece of empirical evidence showing that individuals alternate risk-averse and risk-seeking behavior. An example is provided by Kahneman and Tversky (1979):

\[
(500, 100\%) \succ (1000, 50\% ; 0, 50\%)
\]

\[
(5, 100\%) \prec (5000, 0.1\% ; 0, 99.9\%)
\]

\[
(-5, 100\%) \succ (-5000, 0.1\% ; 0, 99.9\%)
\]

\[
(-500, 100\%) \prec (-1000, 50\% ; 0, 50\%)
\]

First, individuals display risk aversion when they face lotteries with relatively high probabilities of obtaining positive outcomes. Second, individuals display risk-seeking behavior when they face lotteries with relatively low probabilities of obtaining positive outcomes (consistent with the purchase of lottery tickets). Regarding losses, we typically observe the opposite behavior: risk aversion for low probabilities of realizing large
losses (consistent with the purchase of insurance policies), and risk seeking behavior when individuals face high probabilities of realizing losses.

The empirical failure of EUT has stimulated researchers to formulate alternative theories that can account for the observed violations. These models have been generally labeled as “nonexpected utility” (for surveys, see Schoemaker, 1982; Stamerer, 2000; Schmidt, 2004).

2.2 Prospect Theory

Prospect Theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992) is the main challenging paradigm to EUT. The main features of Prospect Theory are:

1. Individuals make decisions based on change in wealth (like in Markowitz utility function), rather than on total wealth (as advocated by EUT).

2. Individuals maximize the expectation of a value function, $v(x)$, convex for losses and concave for gains (consistent with risk seeking for losses and risk aversion for gains). Moreover, $v(x)$ is steeper in the negative domain, in order to accommodate for loss aversion (the empirical observation that losses loom larger than gains).

3. Individuals subjectively distort probabilities. Instead of using the actual probability of each payoff, individuals make decisions based on a weighting probability function $w(p)$ that overstate low probabilities and understate high probabilities. In the original version of the model (usually defined as “Original Prospect Theory”, Kahneman and Tversky, 1979), $w(p)$ is defined over the probability of each outcome. In the later extension of the model (usually defined as “Cumulative Prospect Theory”, Tversky and Kahneman, 1992), instead, $w(p)$ is defined over cumulative probabilities.

4. The “framing” of alternative outcomes may strongly affect subjects’ choices.

Kahneman and Tversky (1979) showed that Prospect Theory can explain the phenomena that were challenging EUT (in particular, common consequence effect, common ratio effect, fourfold pattern, reflection effect).

\footnote{For more details, see Appendix A.1.}
2.2.1 Phenomena Challenging Prospect Theory

After its introduction, Prospect Theory gained popularity in the behavioral economics literature, and several works have found empirical evidence supporting the model. However, there are also several studies that find results that are at odds with the predictions of Prospect Theory (and also with other existing models of the decision theory literature). For example, several studies find evidence of preference for security/potential, i.e., risk aversion for losses and risk seeking for gains (e.g., Levy and Levy, 2002; Post and Levy, 2005; Levy, 2006), the opposite of what implied by the shape of Prospect Theory value function, and by its interaction with the weighting probability function. Moreover, several studies show that, within the Prospect Theory framework, it is difficult to rationalize the effect of aspiration levels on risky choice behavior. In particular, it is still not clear why risky choice behavior is considerably affected by the presence of prior gains or losses (Thaler and Johnson, 1990), or by the need to achieve specific target returns (Payne et al., 1980, 1981).

3 Target Utility Theory

3.1 The Model

The utility function of Target Utility Theory (TUT) is given by the sum of two components: a value function that captures the “pure” utility of each payoff, and an anticipatory feeling component that captures the expected regret/rejoice of each lottery. More specifically, the utility of a generic lottery $X$, when it’s compared with another lottery $Y$, is defined as follows:

$$U(X_Y) = \sum_{i=1}^{N} p_i F_{x_i}^- (v(x_i) - \sum_{j=1}^{J} p_j F_{y_j}^+ r(y_j))$$

$$+ \sum_{i=1}^{N} p_i F_{x_i}^+ (v(x_i) - \sum_{j=1}^{J} p_j F_{y_j}^- r(y_j))$$

(4)

Value—$v(x_i)$—and regret/rejoice—$r(y_j)$—functions are given by:

$$v(x_i) = \begin{cases} 
-\lambda (-x_i)^\psi & \text{if } x_i \leq 0 \\
\psi x_i & \text{if } x_i > 0 
\end{cases}$$

$$r(y_j) = \begin{cases} 
-\lambda (-y_j)^\gamma & \text{if } y_j \leq 0 \\
y_j^\gamma & \text{if } y_j > 0 
\end{cases}$$

(5)
The weighting functions $F^+$ and $F^-$ identify gains and losses, respectively, and satisfy $F^- = 1 - F^+$. We can assume that $F^+$ and $F^-$ are equal to the c.d.f. of a normal distribution.\(^5\) Therefore, the weighting functions are similar to indicator functions, but they are continuous around zero. Moreover, this definition of the weighting functions imply that outcomes equal to zero are classified as both gains and losses ($F^+_0 = F^-_0 = 1/2$).

As shown in Figure 1, the value function is concave for losses and convex for gains, implying preference for security/potential (risk aversion for losses and risk seeking behavior for gains). The regret/rejoice function, instead, creates a jump in utility at the reference point, implying goal seeking behavior (the desire of achieving relevant aspiration levels). Finally, the loss aversion parameter $\lambda$ takes into account the higher sensitivity to losses than gains of the same amount. It follows that the three parameters of the model—$\psi$, $\gamma$, $\lambda$—are all greater than one.\(^6\) Each parameter is associated with one of the factors captured by the model: $\psi$ controls for preference for security/potential, $\gamma$ controls for goal seeking behavior, and $\lambda$ controls for loss aversion.

Regret is defined as the negative feeling associated with the ex-post knowledge that a different past decision would have given a positive payoff, instead of the non-positive payoff obtained with the chosen prospect. Regret depends upon the positive outcomes of the alternative prospect, and it is experienced only when the decision maker fails to achieve the target return. Therefore, regret creates a negative shift in utility in the negative domain, equal to $-\sum_{j=1}^{J} p_j F^+_y r(y_j)$. Rejoice, instead, is associated with the avoidance of a loss, and it creates a positive shift in utility in the positive domain, equal to $-\sum_{j=1}^{J} p_j F^-_y r(y_j)$.

Because of the regret/rejoice component, the overall probabilities of realizing gains and losses play an important role in the decision making process. Therefore, since the model penalizes prospects that deliver a low probability of achieving the target return (i.e., a low probability of realizing gains), TUT implies goal seeking behavior.

\(^5\) The weighting function $F^+_{x_i}$ is equal to the c.d.f., defined on $x_i$, of a normal with mean 0 and standard deviation equal to 0.0001. $F^-_{x_i}$ is equal to the c.d.f., defined on $-x_i$, of a normal with mean 0 and standard deviation equal to 0.0001. Standard deviations are chosen to be arbitrary small, in order to make the weighting functions similar to indicator functions, but continuous around zero. Same considerations apply for $F^+_{y_j}$ and $F^-_{y_j}$.

\(^6\) The regret/rejoice function is assumed to be concave for losses and convex for gains, like the value function. The spirit of the model is that the impact of any outcome increases at an increasing rate (higher marginal impact of outcomes of higher magnitude).
The interaction of value and regret functions allows explaining why individuals alternate risk averse and risk seeking behavior, both in the domains of gains and losses. Regarding losses, the concavity of the value function implies risk aversion (downside risk aversion), while the regret function induces individuals to take risk in order to avoid losses (goal seeking behavior “from below”). Regarding gains, instead, the the convexity of the value function implies risk seeking behavior (preference for upside potential), while the rejoice function induces a individuals to avoid risk in order to order to realize gains (goal seeking behavior “from above”). Individuals will display either risk aversion or risk seeking behavior depending on which one of the two effect prevails: risk-aversion/risk-seeking implied by preference for security/potential, as opposed to risk-seeking/risk-aversion implied by goal seeking behavior.

3.2 Certainty Equivalent

The anticipatory feeling component of the utility function necessarily imply the comparison of any gamble with and alternative. This feature of the model can create problems that are common to all models involving regret/rejoice. For example, the necessity to compare any prospect with an alternative makes it difficult to define decisions over a set of more than two lotteries, since such choices may require a large number of pairwise comparisons, which in turn may lead to intransitive preferences. In addition, in some cases it is difficult (if not impossible) to find a valid alternative to the prospect under analysis (e.g., portfolio choice problems).

To solve these problems, we can rank lotteries according to their certainty equivalents. In other words, we can rank every lottery \( X \) according to the amount \( \text{ce}_X \), delivered for sure, such that \( X \sim \text{ce}_X \):

\[
U(X_{\text{ce}_X}) = U(\text{ce}_X) \quad (6)
\]

Utility is defined with the same logic used for pairwise comparison of two lotteries:

\[
U(X_{\text{ce}_X}) = \sum_{i=1}^{N} p_i F^{-}_{x_i} \left( v(x_i) - F^{+}_{ce_X} r(\text{ce}_X) \right) + \sum_{i=1}^{N} p_i F^{+}_{x_i} \left( v(x_i) - F^{-}_{ce_X} r(\text{ce}_X) \right) \quad (7)
\]
and

\[ U(ce_X) = F_{ce_X}^{-} \left( v(ce_X) - \sum_{i=1}^{N} p_i F_{x_i}^{+} r(x_i) \right) \]
\[ + F_{ce_X}^{+} \left( v(ce_X) - \sum_{i=1}^{N} p_i F_{x_i}^{-} r(x_i) \right) \]  \hspace{1cm} (8)

Everything else being equal, the model implies higher certainty equivalents\(^7\) for lotteries displaying: (i) higher probability of achieving the target payoff (goal seeking behavior), and (ii) lower dispersion of payoffs in the negative domain and higher dispersion of payoffs in the positive domain, as long as the probability of achieving the target payoff is not affected (preference for security/potential). These two predictions, which are unique to TUT, are the key drivers of the results of this paper.

### 3.3 Example: the Fourfold Pattern

In order to provide a few examples on how TUT works, let’s first consider the Fourfold pattern, shown in section 2.1. The certainty equivalents and implied choices predicted by TUT are as follows:\(^8\)

\[
\begin{array}{l}
(500, 100\%) \succ (1,000, 50\% ; 0, 50\%) \sim ce = 192.57 \\
(5, 100\%) \prec (5,000, 0.1\% ; 0, 99.9\%) \sim ce = 24.19 \\
(-5, 100\%) \succ (-5,000, 0.1\% ; 0, 99.9\%) \sim ce = -24.19 \\
(-500, 100\%) \prec (-1,000, 50\% ; 0, 50\%) \sim ce = -192.57
\end{array}
\]  \hspace{1cm} (9)

First of all, note that the certainty equivalents of lotteries with positive payoffs are the same as the certainty equivalents of lotteries with negative payoffs, but with opposite sign. This is because the value and regret/rejoice components in (5) have the same exponents in the positive and negative domains. A less parsimonious version of the

\(^7\)Since there is no closed formula solution to (8), the certainty equivalent must be found numerically.
\(^8\)These results are obtained by considering the parameter estimates shown in Table 2. Note that the certainty equivalents of the left hand side gambles are obviously equal to the payoffs—delivered for sure—of these gambles.
model could be parametrized with different $\psi$ and/or $\gamma$ for gains and losses, leading to asymmetric risk attitude in the positive and negative domains.

Regarding the first choice (relatively high probability of realizing gains), the certainty equivalent of the left hand side gamble is 500 (the prospect delivers 500 for sure), while the certainty equivalent of the right hand side gamble is only 192.57:

$$v(192.57) = 50\% v(1,000) + 50\% \frac{1}{2} (v(0) - r(192.57))$$  \hspace{1cm} (10)

Despite the convexity of the value function for gains (which would imply risk seeking behavior), the certainty equivalent is lower than the expected payoff of the gamble. This is because the gamble will generate regret with a 50% probability (the probability of obtaining nothing with that prospect). The probability of experiencing regret and the magnitude of potential regret are high enough to induce risk aversion.

If, instead, individuals face gambles with a relatively low probability of realizing large gains, the model predicts that the risk seeking behavior induced by the convexity of the value function in the positive domain (preference for upside potential) prevails over the risk averse behavior induced by the regret function (goal seeking “from above”). In fact, the certainty equivalent of the right hand side gamble of the second choice in (9) is greater than its expected payoff:

$$v(24.19) = 0.1\% v(5,000) + 99.9\% \frac{1}{2} (v(0) - r(24.19))$$  \hspace{1cm} (11)

The gamble generates regret with a 99.9% probability (the probability of obtaining nothing with that prospect), reducing the attractiveness of the lottery. However, the magnitude of potential regret is low, compared to the magnitude of the potential gain of this risky lottery. Since TUT assumes higher marginal impact for higher outcomes, in this case the magnitude of the potential gain is such that the risk seeking behavior induced by the convexity of the value function prevails over the risk averse behavior induced by the regret function.

The behavior observed for losses is the mirror image of what observed for gains: risk seeking behavior for prospects with high probabilities of losing money, and risk aversion for prospects with low probabilities of realizing large losses. According to TUT, the

\footnote{Note that the utility of the zero payoff is multiplied by $1/2$ because $F_0^- = F_0^+ = 1/2.$}
The certainty equivalent of the right hand side gamble in the fourth choice in (9) is:

\[ v(-192.57) = 50\% v(-1,000) + 50\% \frac{1}{2} (v(0) - r(-192.57)) \] (12)

The “risky” gamble is associated with rejoice (avoidance of a loss), which is experienced with 50% probability. In this case, the risk seeking behavior induced by the rejoice function (goal seeking “from below”) prevails over the risk averse behavior induced by the concavity of the value function in the negative domain (downside risk aversion). As a consequence, the certainty equivalent of the risky lottery is higher than its expected payoff (risk seeking behavior).

The opposite is true for choices with a relatively low probability of realizing a large loss (third choice in (9)). In this case, the risk aversion implied by the concavity of the value function prevails over the risk seeking behavior implied by the rejoice function. In fact, we have:

\[ v(-24.19) = 0.1\% v(-5000) + 99.9\% \frac{1}{2} (v(0) - r(-24.19)) \] (13)

Figure 2 shows how the certainty equivalent implied by TUT changes if we change the probability of obtaining non-zero payoffs in lotteries delivering either positive or negative payoffs. Consistent with the fourfold pattern, the model implies (i) risk aversion for lotteries with high probabilities of obtaining positive payoffs; (ii) risk seeking behavior for low probabilities of obtaining positive payoffs; (iii) risk aversion for low probabilities of obtaining negative payoffs; and (iv) risk seeking behavior for high probabilities of obtaining negative payoffs. The figure also shows that risk aversion for high probabilities of gains and risk seeking for low probabilities of losses are enhanced if we increase the magnitude of potential gains/losses (see section 3.6.2 on the “magnitude effect”).

### 3.4 Lotteries with Only Strictly Positive or Negative Payoffs

The target payoff of the regret/rejoice function is assumed to be equal to zero if the lottery under analysis deliver mixed outcomes. If, instead, the lottery delivers only
strictly positive or strictly negative payoffs, the reference point for the regret/rejoice function is the minimum payoff (in absolute terms) that can be achieved with all available prospects. In this case, regret (rejoice) is experienced if the decision maker fails to achieve (suffer) the minimum gain (loss). More generally, the reference point, $\tau$, of the regret/rejoice function is defined as:

$$\tau = \min_i (I_{x_i}^+ x_i + |I_{x_i}^- x_i|)$$

(14)

where $I_{x_i}^+$ is an indicator function equal to one if $x_i > 0$, and $I_{x_i}^- = 1 - I_{x_i}^+$. If all $x_i$ are strictly positive, then the reference point for regret is the minimum payoff that can be achieved. If, instead, all $x_i$ are strictly negative, then the reference point for rejoice is the minimum loss that must be suffered.

The reference point for the value function, instead, is always equal to zero, since the value function captures the pure utility associated with any outcome, and it ignores any anticipatory feeling associated with the failure of achieving the minimum possible gain (or avoiding the minimum loss).

Thaler and Johnson (1990) provide a few examples of choices on lotteries with only strictly positive/negative payoffs. For example, regarding gains, the authors found:

$$(21, 50\% ; 39, 50\%) \prec (30, 100\%)$$

(15)

Regarding the left hand side, since all payoffs are strictly positive, the reference point for the regret/rejoice function is 21 (the minimum payoff that can be achieved). In this case, the certainty equivalent predicted by TUT is 28.81:

$$v(28.81) = 50\% \left( v(21) - \frac{1}{2} r(28.81 - 21) \right) + 50\% v(39)$$

(16)

Risk aversion induced by goal seeking behavior prevails on preference for upside potential because the left hand side gamble has a 50% probability of not achieving the target return.\(^\text{10}\)

\(^{10}\)Note that considerations made in the case of the fourfold pattern apply to this case as well. In fact, TUT keep on predicting risk seeking behavior for lotteries with low probabilities of obtaining large positive payoffs, even if the minimum payoff is strictly positive.
Regarding losses, Thaler and Johnson (1990) found that:

\[ (-39, 50\% ; -21, 50\%) \succ (-30, 100\%) \] (17)

In this case, regarding the risky lottery on the left hand side, the reference point for regret/rejoice is the minimum loss that must be suffered: \(-21\). The certainty equivalent predicted by TUT is \(-28.81\):

\[ v(-28.81) = 50\% v(-39) + 50\% (v(-21) - 1/2 r(-28.81 + 21)) \] (18)

The left hand side gamble has a 50\% probability of achieving the target return, and risk seeking induced by goal seeking behavior prevails on downside risk aversion. As in the fourfold pattern example, the certainty equivalent for losses is the same as the certainty equivalent for gains, but with opposite sign. As pointed out earlier, this result can be avoided by imposing different parameters for the value and regret/rejoice functions in the positive and negative domains.

3.5 Phenomena Challenging Existing Models

3.5.1 Preference for Security/Potential: Experimental Evidence

Several studies find evidence of preference for security/potential, i.e., downside risk aversion (risk aversion for losses) and preference for upside potential (risk seeking behavior for gains). These results are in contrast with the S-shape value function of Prospect Theory, which implies exactly the opposite: risk seeking for losses and risk aversion for gains. Levy and Levy (2002) point out that most studies supporting Prospect Theory investigate preferences over gambles whose payoffs are confined either to the negative or to the positive domain. Conducting experiments on mixed prospects, Levy and Levy (2002) obtain results that are in contrast with the assertion that individuals are risk seeking for losses and risk averse for gains. For example:

\[ (-1,000, 25\% ; -800, 25\% ; 800, 25\% ; 2,000, 25\%) \succ \] (19)

\[ \succ (-1,600, 25\% ; -200, 25\% ; 1,200, 25\% ; 1,600, 25\%) \]
When facing mixed prospects, individuals prefer to decrease the dispersion of payoffs in the negative domain (consistent with downside risk aversion), and increase the dispersion of payoffs in the positive domain (consistent with preference for upside potential). Note that this behavior is observed only when we increase (decrease) the dispersion of strictly positive (negative) payoffs, without altering the overall probability of winning/losing.

Some authors argue that the curvature of Prospect Theory value function is much weaker than the curvature of the weighting probability function. As a consequence, in some cases Prospect Theory could imply downside risk aversion and preference for upside potential. But testing the interaction of Prospect Theory’s value function and weighting probability function, Levy (2006) found evidence contradicting that assertion. For example:

\[ (-875, 50\%; 2025, 50\%) \succ (-1000, 40\%; 1800, 60\%) \] (20)

This and other results of Levy (2006) provide evidence against the entire framework of Prospect Theory (S-shaped value function and Prospect Theory probability distortions are jointly rejected). Preferences in (20), instead, are consistent with the predictions of TUT. Despite the higher probability of losing, the left hand side lottery is preferred because of the higher magnitude of the potential gain and the lower magnitude of the potential loss. In other words, preferences in (20) confirm the intuition of TUT that the impact of payoffs increase at an increasing rate (higher marginal impact of outcomes with higher magnitude).

### 3.5.2 Preference for Security/Potential: Asset Pricing Literature

The idea that individuals are averse to downside risk and seek upside potential has also been investigated in the asset pricing literature. For example, Post and Levy (2005) use stochastic dominance criteria to show that the cross-section of stock returns can be rationalized by a utility function concave for losses and convex for gains, but not by a utility function convex for losses and concave for gains, or globally concave. Piccioni (2014) tests a CAPM based on asymmetric utility functions that can nest several models of the decision theory literature. The author finds that the cross-section of
stock returns is rationalized by a utility function displaying (i) a positive shift at zero, (ii) concavity for losses and mild convexity for gains, and (iii) a higher slope in the negative domain. Piccioni (2014) results are consistent with the main implications of TUT: the factors driving risky choice behavior (goal seeking behavior, preference for security/potential, and loss aversion) are also the factors driving the cross-section of stock returns. In other words, a utility function that can explain risky choice behavior can also explain how investors face risk, and, therefore, it can identify the sources of risk that drive assets’ returns.

Regarding preference for security/potential, the idea that investors distinguish between upside and downside volatility dates back to Roy (1952) “safety first” rule, where the risk of an investment is defined by its downside volatility, rather than its volatility. Supporting Roy’s intuition, a few years after developing the mean-variance portfolio theory, Markowitz (1959) pointed out that investors are interested in minimizing downside risk, rather than volatility, for two reasons: (i) only downside risk is relevant to an investor, and (ii) security returns may not be normally distributed. In particular, Markowitz argued that, due to investors’ concern with downside deviations in returns, semivariance (or lower partial second moment) is intuitively more appealing than variance as a measure of risk. After pointing out the benefits of the semivariance measure, Markowitz continued working with the variance measure because of its computational simplicity.

Experimental evidence supporting the idea that investors display downside risk aversion dates back to Mao (1970). Asked what they understood by the term “investment risk”, business executives provided the following statements (Mao, 1970, p.353):

“Risk is the prospect of not meeting the target rate of return. If you are one hundred percent sure of making the target return, then it is a zero risk proposition.”

“Risk is financial in nature. It is primarily concerned with downside deviations from the target rate of return. However, if there is a good chance of coming out better than forecast, that is negative risk (a sweetener) which is taken into account in determining the security of an investment.”
“I never worry about the project going above the target return. Risk is what might happen when the return is going to be less.”

Besides the extensive downside risk literature, there are also studies finding evidence of preference for upside potential. For example, Statman (2002) and Kumar (2009) find that investors are willing to pay a premium for lottery stocks, i.e., securities that provide high upside potential (high upside volatility). Moreover, Bali et al. (2011) find that investors are willing to pay a premium for securities that deliver extremely high returns with low probability. Lobe and Holz (2008) document the popularity of British Premium Bonds. Like other bonds, British Premium Bonds pay back the entire principal to bondholders at maturity. However, in lieu of periodic fixed coupons, coupons are determined by a lottery, and few randomly selected investors receive significantly higher payments. Therefore, British Premium Bonds provide both downside protection (principal fully paid at maturity) and upside potential (possibility of collecting very high coupons).

The ability of modeling preference for security/potential may also have important policy implications. For example, in his report on recent developments on the financial industry, Rajan (2005) pointed out that the incentive structure of investment managers today differs from the incentive structure of bank managers in the past also because today’s compensations typically imply less downside risk and more upside potential for generating investment returns. The incentive structure for bank managers in the past, instead, was characterized by extreme penalties on the downside and limited upside for salaries, making bankers extremely conservative. Today’s compensation schemes can create a variety of perverse behavior, making the financial market overall riskier.

3.5.3 Goal Seeking Behavior

Several studies investigate the effect of aspiration levels on decision making. For example, Camerer et al. (1997) find that New York cab drivers appear to be motivated to earn a daily target return. On rainy, busy days, their earnings per hour are high and they go home early. On hot summer days, when many New Yorkers prefer to take a walk, cabdrivers earn less per hour and work long hours to reach their target. Lopes (1987) find that farmers appear to have a minimum level of revenues that they want to achieve. Up to the subsistence level, they choose to cultivate “safe” crops with stable
returns. The remainder of their land is allocated to “risky” crops.

Further evidence of goal seeking behavior has been shown by Payne et al. (1980, 1981), who found that individuals are keen on achieving a target rate of return. For example, Payne et al. (1980) found that

\[(42, 20\%; 26, 40\%; 18, 40\%) \succ (86, 30\%; 26, 20\%; -10, 50\%)\]

\[(-10, 20\%; -26, 40\%; -34, 40\%) \prec (34, 30\%; -26, 20\%; -62, 50\%)\]

(21)

The first choice is consistent with risk aversion (the left hand side lottery has lower dispersion in payoffs). In the second choice, instead, individuals switch to risk seeking behavior. Note that the second set of gambles is simply obtained by subtracting 52 to all payoffs of the first set. An important point is that, regarding the first set of lotteries, individuals prefer the left hand side prospect, where all payoffs are strictly positive. After all payoff are reduced by 52, individuals prefer the right hand side prospect, where there’s a 30% probability of obtaining a positive payoff.

The pattern of preferences in (21) can be explained by TUT. The certainty equivalent of the left hand side gamble of the first choice is 25.33:

\[v(25.33) = 20\% v(42) + 40\% v(26) + 40\% (v(18) - 1/2 r(25.33 - 18))\]

(22)

In this case, all payoffs are strictly positive, and the regret function (regret is associated with the lowest payoff) plays a limited role in the decision making process. Regarding the right hand side gamble, instead, we have a 50% of a loss, and the regret/rejoice function plays a much more important role in the decision making process. In this case, the certainty equivalent is 15.54:

\[v(15.54) - 50\% r(-10) = 30\% v(86) + 20\% v(26) + 50\% (v(-10) - r(15.54))\]

(23)

The gamble generates regret with probability equal to 50%, which reduces its certainty equivalent. At the same time, the 50% probability of losing produces expected rejoice equal to \(-50\% r(-10)\) in the utility function of the certainty equivalent, which further contributes to reducing the certainty equivalent. Therefore, regarding the first choice in (21), TUT predicts risk aversion.
Regarding the second set of gambles in (21), the certainty equivalent of the left hand side lottery is −23.89:

\[ v(-23.89) = 20\% (v(-10) - 1/2 r(-23.89 + 10)) + 40\% v(-26) + 40\% v(-34) \] (24)

Since all payoffs are strictly negative, the rejoice function (rejoice is associated with the lowest loss of 10) plays a limited role in the decision making process. In the right hand side gamble, instead, there is a 30% probability of realizing a gain, which increases the importance of the rejoice component, increasing the desirability for the prospect. The certainty equivalent, in this case, is −8.33:

\[ v(-8.33) - 30\% r(34) = 30\% (v(34) - r(-8.33)) + 20\% v(-26) + 50\% v(-62) \] (25)

In this case, the gamble generates rejoice with probability equal to 30%, which increases its certainty equivalent. At the same time, the 30% probability of winning produces expected regret equal to −30\% r(34) in the utility function of the certainty equivalent, which further contributes to increasing the certainty equivalent. Therefore, regarding the second choice in (21), TUT predicts risk seeking behavior.

The empirical results related to the effect of aspiration levels on risky choice behavior provide mixed support to the two versions of Prospect Theory, and, in general, they can be rationalized only imposing the shape of the value function to be particularly pronounced, so that utility is particularly steep around the reference point and flat everywhere else. Such a parametrization wouldn’t allow the model to rationalize other phenomena of decision making under risk. As Payne (2005) points out, “... any descriptive theory choice among gambles with multiple outcomes [i.e., more than two payoffs] will need to include measures of the overall probability of a gain and the overall probability of a loss. Although the psychological meaningfulness of constructs reflecting the overall probabilities of winning and losing may seem obvious, such constructs are not part of the traditional expected utility model nor of the more recent nonlinear expectation models such as Prospect Theory or Cumulative Prospect Theory” (p.5).

In order to rationalize goal seeking behavior, Diecidue and Van de Ven (2008) develop a model that includes, into an expected utility representation, the overall probabilities of success and failure relative to the aspiration level. The authors show that
this turns up to be equivalent to expected utility with a discontinuous utility function: utility with an upward jump at the reference point (see Appendix A.2). In this respect, TUT is similar in spirit. However, the empirical analysis section of this work shows that the mere introduction of jumps in utility cannot improve our understanding of risky choice behavior. First of all, more structure is required in order to define the precise extent of the jump at zero. Moreover, it is necessary to interact goal seeking behavior with preference for security/potential.

### 3.5.4 Effect of Prior Outcomes on Risky Choice Behavior

The experimental evidence related to the effect of prior outcomes on risky choice behavior has represented a puzzle up to these days. Preferences elicited in the presence of prior outcomes not only are difficult to explain by themselves, but they are also difficult to reconcile with the behavior observed in the absence of prior gains and losses. These phenomena cannot be explained by existing models, independently of their parametrization.

Thaler and Johnson (1990) study how prior outcomes affect risky choice behavior, finding the following pattern of preferences:

\[
\begin{align*}
\text{prior: } &-30 & ( -9, 50\%; 9, 50\%) &\prec (0, 100\%) \\
\text{prior: } &-9 & ( -9, 50\%; 9, 50\%) &\succ (0, 100\%) \\
\text{prior: } &+9 & ( -9, 50\%; 9, 50\%) &\prec (0, 100\%) \\
\text{prior: } &+30 & ( -9, 50\%; 9, 50\%) &\succ (0, 100\%)
\end{align*}
\]

Individuals reject the risky prospect when they’re facing a prior loss of $30; they accept the gamble when they’re facing a loss of $9; they reject the gamble when they’re winning $9; and they accept the gamble when they’re winning $30. Assuming that utility is defined over the final payoffs of the entire endeavor, preferences in (26) are equivalent to:

\[
\begin{align*}
( -39, 50\%; -21, 50\%) &\prec ( -30, 100\%) \\
( -18, 50\%; 0, 50\%) &\succ ( -9, 100\%) \\
( 0, 50\%; 18, 50\%) &\prec ( 9, 100\%) \\
( 21, 50\%; 39, 50\%) &\succ ( 30, 100\%)
\end{align*}
\]

\[
(26)
\]

\[
(27)
\]

\[11\] In other words, individuals always integrate prior outcomes obtained within the evaluation period.
Under the integration assumption, the first choice in (27) represents evidence of downside risk aversion (aversion toward risk if all payoffs are in the negative domain).\textsuperscript{12} The second and third choices, instead, are evidence of goal seeking behavior. In the second choice, people are willing to take risk, in order to have a chance to break-even. In the third choice, instead, individuals display risk aversion in order to make sure to realize a gain. Finally, the fourth choice is evidence of preference for upside potential (risk seeking behavior for gains).\textsuperscript{13}

Thaler and Johnson (1990) argue that their results can be rationalized by Prospect Theory if specific assumptions are made regarding the integration/segregation of prior outcomes. Their assumptions imply that individuals sometimes segregate and sometimes integrate prior outcomes, following a quasi-hedonic editing rule that partially follows the hedonic editing rule (according to which people edit gambles in a way that would make them appear as good as possible).

In contrast, TUT can explain the choice pattern in (27) by simply assuming that individuals integrate prior outcomes. Regarding the first choice (prior loss of $30), the certainty equivalent of the risky gamble is $\text{-30.76}$:

$$v(-30.76) = 50\% \cdot v(-39) + 50\% \cdot v(-21)$$

(28)

Since there is no regret/rejoice effect (all final payoffs are strictly negative), concavity for losses implies risk aversion. Note that in all choices in (26) the reference point of both value and regret/rejoice functions is equal to zero, since the different stages of the game involve both non-positive and non-negative payoffs for all gambles.

Regarding the second choice (prior loss of $9), the certainty equivalent of the risky gamble is $\text{-7.83}$:

$$v(-7.83) = 50\% \cdot v(-18) + 50\% \cdot \frac{1}{2} \left(v(0) - r(-7.83)\right)$$

(29)

The effect of anticipatory feelings (expected rejoice associated with the risky gamble: 50\% probability of breaking even) induces individuals to take risk. In fact, in this case,

\textsuperscript{12}This is an example of no-break-even effect: individuals display risk aversion when prior losses cannot be recovered.

\textsuperscript{13}The fourth choice is an example of the so called “house money effect”, i.e., the willingness to take risk after gains, as long as there is no danger of ending up with nothing.
by taking risk individuals can break-even and avoid losses.

Regarding the third choice (prior gain of $9), the certainty equivalent of the risky gamble is 7.83:

\[ v(7.83) = 50\% \frac{1}{2} (v(0) - r(7.83)) + 50\% v(18) \] (30)

In this case, the effect of anticipatory feelings (expected regret associated with the risky gamble: 50% probability of not realizing any gain) induces risk aversion. Individuals want to avoid risk in order to make sure of obtaining a gain.

Finally, regarding the fourth choice (prior gain of $30), the certainty equivalent of the risky gamble is 30.76:

\[ v(30.76) = 50\% v(21) + 50\% v(39) \] (31)

Since there is no regret/rejoice effect (all final payoffs are strictly positive), convexity for gains implies risk seeking behavior.

To the best of my knowledge, TUT is the first model that is able to explain the effect of prior outcomes on risky choice behavior, assuming a consistent framework regarding the integration/segregation of prior outcomes, and without requiring distortions in probabilities.

### 3.6 Empirical Evidence: Other Results

#### 3.6.1 Gain-Loss Separability

Wu and Markle (2008) investigate a premise of Prospect Theory: the valuation of gains and losses is separate (gain-loss separability). The authors conduct experimental studies that demonstrate systematic violations of the double-matching axiom, an axiom that is necessary for gains-loss separability. An example of violation of gain-loss separability found by Wu and Markle (2008) is:

\[
\begin{align*}
( -3,000, 50\% ; 4,200, 50\% ) & \succ ( -4,500, 25\% ; 3,000, 75\% ) \\
( -3,000, 50\% ; 0, 50\% ) & \prec ( -4,500, 25\% ; 0, 75\% ) \\
( 0, 50\% ; 4,200, 50\% ) & \prec ( 0, 25\% ; 3,000, 75\% )
\end{align*}
\] (32)
Wu and Markle (2008) present a more detailed empirical investigation demonstrating that the violation of gain-loss separability illustrated above is not unique. As in (32), the authors document a reversal between preferences for mixed gambles (i.e., gambles involving both gains and losses) and the associated gain and loss gambles: mixed gamble $X$ is preferred to mixed gamble $Y$, but the gain and loss portions of $Y$ is preferred to the gain and loss portions of $X$. Wu and Markle (2008) argue that the observed choice patterns are consistent with a process in which individuals are less sensitive to probability differences when choosing among mixed gambles than when choosing among either gain or loss gambles.

Wu and Markle (2008) find that their results can be rationalized by Prospect Theory only if the parameters used to fit choices over mixed gambles significantly differ from the parameters used to fit choices over single-domain gambles. Moreover, the new parametrization would compromise the ability of explaining phenomena studied in previous works, such as the fourfold pattern, the reflection effect, the common ratio and common consequence effects, etc.

In contrast, TUT can explain the observed behavior without a specific parametrization. This is because the model is consistent with the intuition that the decision process is driven by both the magnitude of the potential outcomes and by the overall probabilities of realizing gains/losses.

Regarding the second and third choice in (32), in the TUT framework the overall probability of winning/losing plays an important role in moving preferences toward the right hand side gambles. Regarding the first choice, instead, the magnitudes of the potential gains/losses mitigate the effect of the overall probabilities of winning/losing. The left hand side gambles have lower probabilities of realizing gains (and higher probabilities of losing), but also lower potential losses and higher potential gains. Since in TUT the impact of potential outcomes increases with their magnitude (concavity for losses and convexity for gains), in this case the model implies preference for the left hand side lottery.

### 3.6.2 Effect of Multiplying and Translating Outcomes and Probabilities

Several studies consider the effect of multiplying or translating prospects’ outcomes or probabilities, finding evidence consistent with the idea that both goal seeking behavior
and preference for security/potential drive risky choice behavior. For example, Holt and Laury (2002) document that, in the positive domain, we witness an increase in risk aversion when all payoffs of all prospects are multiplied by the same factor (magnitude effect). In the TUT framework, this result is consistent with the effect of goal seeking behavior on decision making: in the positive domain individuals are more motivated to realize gains (and avoid risk) if the potential payoffs are larger in magnitude. It’s important to point out that Holt and Laury (2002) increase the dispersion of payoffs by multiplying all outcomes of lotteries that deliver two strictly positive payoffs. Their results (increase in risk aversion by increasing the dispersion of payoffs) are different from Levy and Levy (2002) results (decrease in risk aversion by increasing the dispersion of payoffs in the positive domain) because in Holt and Laury (2002) the increase in magnitude affects the target payoff as well (with strictly positive payoffs, the target is equal to the minimum payoff). If, instead, the increase in dispersion were to be applied only to two or more payoffs that are strictly above the target payoff, TUT would predict an decrease in risk aversion, like it is observed in Levy and Levy (2002).

Lopes and Odean (1999), instead, compare the effect of either multiplying (scaling) or adding (shifting) a common factor to all payoffs of lotteries delivering either gains or losses. The authors find that the prospects that before the shift had a high overall probability of a zero outcome became relatively much more attractive to the subjects after the shift than the prospects with a small probability on a zero outcome. Prospects that before the shift had a high probability of a zero outcome later had a high overall probability of a gain. The prospects with a small probability on a zero outcome did not become much more attractive because they already had a high overall probability of a gain. The scaling did not influence choices much, which is understandable because the overall probability of a gain is unaffected by this operation. Lopes and Odean (1999) results provide more evidence supporting the idea that the overall probabilities of realizing positive payoffs play an important role in the decision making process.

Finally, Fennema and Wakker (1997) consider how preferences are affected by shifting the probability mass of extreme versus intermediate payoffs, without affecting the overall probability of gaining/losing. Consistent with preference for security/potential, the authors find that individuals prefer to increase the probability of higher positive payoffs (rather than intermediate gains), and to reduce the probability of extreme losses.
Fennema and Wakker (1997) argue that their results are consistent with a weighting probability function that favors extreme-outcome moves over middle-outcome moves, providing more support for Cumulative Prospect Theory, rather than Original Prospect Theory. Fennema and Wakker (1997) are also consistent with a reverse S-shaped value function.

Taken together, the results of Holt and Laury (2002), Lopes and Odean (1999), and Fennema and Wakker (1997) provide further evidence supporting the predictions resulting from the interaction of value and regret/rejoice components in TUT. The support toward Prospect Theory, instead, is mixed (some results support the original formulation of Prospect Theory, others support the later improvement based on cumulative probabilities), and parameters required to rationalize these phenomena typically deviate significantly from those proposed in previous works, leading to violations of previous results.

3.6.3 Repeated Games

Since the publication of Samuelson’s theorem (1963), considerable effort has been devoted to the comparison of risky choice behavior in single played and repeated games. In his paper, “Risk and Uncertainty: A Fallacy of Large Numbers,” Samuelson describes a lunchtime conversation in which he offered an attractive bet to one of his colleagues: “flip a fair coin; heads you win $200, tails you lose $100.” The colleague declined Samuelson’s offer but announced his willingness to take a series of 100 such bets. This behavior—turning down one bet but accepting many—struck Samuelson as inconsistent and induced him to prove a theorem. The theorem states that if an individual is unwilling to take a single play of a bet at any wealth level that could obtain over a series of such bets (here current wealth minus $10,000 to plus $20,000) then she should not accept multiple plays of the same bet, assuming that utility is globally concave. The “fallacy of large numbers” is the erroneous belief that the variance of outcomes decreases as the number of trials increases.

Since the work of Samuelson, several researchers have focused their attention on risky choice behavior in repeated games, and the empirical evidence is in line with
Samuelson’s colleague preferences.\textsuperscript{14} In the empirical analysis I consider some of these works: Benartzi and Thaler (1999), Keren and Wagenaar (1987), Keren (1991), Shefrin (2000), Langer and Weber (2001). The main result is that, in contrast with EUT, TUT can explain the switch in preferences between single played and repeated games. The key role here is played by goal seeking behavior. In fact, by repeating favorable bets, we witness an increase in the overall probability of realizing gains. The increase in the overall probability of winning makes the repeated games more attractive in the TUT setting (but is not taken into account by other models). Repeated favorable bets are also attractive because the increase in variance—with respect to single played bets—is more on the upside, rather than the downside.

3.6.4 Insurance Behavior

An extensive literature investigates risk taking in the domain of losses. These works have relevant implications for the study of insurance behavior. For example, Schmeckman and Kunreuther (1981) find that:

\begin{align*}
( -10,000, 0.1\% ; 0, 99.9\%) & \prec ( -10, 100\%) \\
( -50, 20\% ; 0, 80\%) & \succ ( -10, 100\%) \\
( -10,000, 0.1\% ; 0, 99.9\%) & \prec ( -10, 100\%) \\
( -10,000, 99.9\% ; 0, 0.1\%) & \succ ( -9,990, 100\%) \\
( -100, 1\% ; 0, 99\%) & \prec ( -1, 100\%) \\
( -1,000,000, 1\% ; 0, 99\%) & \succ ( -10,000, 100\%)
\end{align*}

The first set of choices points out that individuals display risk aversion when facing lotteries with low probabilities of large losses, and risk seeking behavior when facing intermediate probabilities of moderate losses. The second set of choices, instead, emphasizes risk seeking behavior toward large probabilities of large losses. Finally, the third set of choices focuses on the effect of the magnitude of the potential losses. In this case, we observe higher willingness to take risk if we increase the magnitude of

\textsuperscript{14} Note that Samuelson’s statement is correct (and Samuelson’s colleague preferences are inconsistent) if we assume a globally concave utility function.
potential losses (without changing the underlying probabilities of losing money).

The interaction of goal seeking behavior and preference for security/potential can clarify the observed behavior. The first four choices in (33) are consistent with the fourfold pattern. The last two choices, instead, emphasize the importance of avoiding large losses.

The ability to explain Schoemaker and Kunreuther (1981) and other results related to risky choice behavior in the domain of losses is not unique to TUT, but the empirical evidence in the domain of losses provides an important set of results to calibrate the model and verify its ability at explaining decision making under risk.

3.7 Results Challenging Target Utility Theory

Several preferences elicited in the studies of the empirical analysis are not consistent with the predictions of TUT. These preferences, in general, do not violate the fundamental assumptions of the model, but are difficult to rationalize with the parameters used to explain the generality of the phenomena under analysis. Interestingly, the highest failure rate of TUT is related to studies that investigate the effect on risky choice behavior of the overall probability of winning (see table 5 and section 5.1 for more details). These phenomena, in general, are related to very strong goal seeking behavior, even too strong for TUT. For example, one of the choices that cannot be explained by the model is related to Payne (2006):

\[(85, 30\%; 65, 5\%; 0, 25\%; -55, 15\%; -90, 25\%) \succ (85, 30\%; 65, 5\%; -25, 25\%; -55, 15\%; -65, 25\%)\]

In this and some other examples (especially in Levy and Levy, 2009, and Baucells and Heukamp, 2006) individuals express very strong goal seeking behavior. In other words, individuals seems to pay very close attention to the overall probabilities of winning money. Even though Target Utility Theory implies goal seeking behavior, the model cannot explain these observations because it is calibrated to provide a comprehensive view of how people face risk. Therefore, the model achieves a balance between goal seeking behavior and preference for security/potential, which cannot explain the strong
preference to increase the overall probability of winning that is sometimes observed.

Another set of results contradicting the predictions of TUT deals with the exact opposite behavior. In fact, there are choices that are consistent with low goal seeking behavior and strong preference for security and potential. For example, Thaler and Johnson (1990) found that:

\[
(-4.5, 67\% ; 10.5, 33\%) \succ (0.5, 100\%)
\] (35)

In this case, individuals prefer the risky lottery on the left hand side even though it delivers a 67% probability of a loss (note that the expected payoff of the risky lottery is equal to 0.5). It is possible to explain this specific preference with TUT only if we attach a particularly strong parameter to preference for security/potential (i.e., \(\psi\) high enough), and reduce the effect of goal seeking behavior and loss aversion. However, such a parametrization would not allow to explain the other phenomena under analysis.

Finally, the other set of results that create some problem for TUT is represented by choices in which individuals express strong risk aversion toward favorable one stage bets. One of such choices is in Keren and Wagenaar (1987):

\[
(15, 100\%) \succ (25, 80\%; 0, 20\%)
\] (36)

In this case, individuals express strong risk aversion (the expected payoff of the risky lottery is 20). In the logic of TUT, individuals are showing strong goal seeking behavior and low preference for upside potential. As in the previous cases discussed in this section, the parametrization of TUT that is consistent with this specific choice would compromise the ability to explain other phenomena.

4   Empirical Analysis

4.1   Maximum Likelihood Estimation

To test the performance of TUT and several models of the decision making literature, I run a Maximum Likelihood estimation on a Logit model defined on a wide range
of results of the empirical decision making literature. Following Lopes and Odean (1999), I find the optimal parameters $\hat{\theta}$ that maximize the Log-Likelihood function $LL(\theta)$ of each model:

$$\max_{\theta} LL(\theta) = \sum_i \log(p_i)$$ (37)

The probability $p_i$ of each observation is obtained with the following Logit model:

$$p_i = \frac{1}{1 + e^{-\kappa_1 (CE_{Wi} - CE_{Li})^\gamma_2}}$$ (38)

where $CE_{Wi}$ is the certainty equivalent of the winning prospect in choice $i$, and $CE_{Li}$ is the certainty equivalent of the losing prospect in choice $i$.

Models are valuated according to their ability to explain as many phenomena as possible (i.e., the ability of maximizing the number of choices according to which $CE_{Wi} > CE_{Li}$). In fact, a model able to explain all phenomena under analysis would display $LL \approx 0$, while the worst possible model would have $LL \approx -\infty$.

4.2 Model Comparison Tests

Model comparison tests can be used to verify the significance of the parameters of all models under analysis. Regarding TUT, the significance of each parameter corresponds to the importance of each factor captured by the model: $\psi$ controls for preference for security/potential, $\gamma$ controls for goal seeking behavior, and $\lambda$ controls for loss aversion. In addition, we can test:

- The importance of either value or regret/rejoice components of TUT;
- The shape of value and regret/rejoice functions, which can be constrained by imposing $\alpha, \gamma$ to be above or below one; and
- Whether TUT components are significant after controlling for Prospect Theory, and vice versa.

15 The models under analysis are discussed in section 5.1 and appendix A. All models are estimated on the sample defined in section 4.3.
Model comparison tests are conducted using the following Likelihood Ratio test:

\[ LR_m = -2 (LL(\hat{\theta}_R) - LL(\hat{\theta})) \sim \chi^2_m \] (39)

where \( LL(\hat{\theta}_R) \) is the log-likelihood of the restricted model. The test is distributed according to a chi-square distribution with \( m \) degrees of freedom (\( m \) is equal to the number of restricted parameters). A low value of the \( LR_m \) test implies that the restrictions on the parameters cannot be rejected because the log-likelihood function does not decrease much by imposing the restrictions.

### 4.3 Data

Each model is tested on 326 choices elicited from several experiments conducted in the last six decades. The studies under analysis can be classified in six groups:16


- **Group 5**: gain-loss separability (27 choices). Wu and Markle (2008)


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16 All data on payoffs and probabilities are available upon request.
5 Results

5.1 Main results

Table 1 shows the main results of the empirical analysis. The table shows the maximum log-likelihood and the failure rates of the following models: Target Utility Theory, Original Prospect Theory (Kahneman and Tversky, 1979), Cumulative Prospect Theory (Tversky and Kahneman, 1992), Security Potential/Aspiration (Lopes, 1987), Regret Theory (Bell, 1982, 1983; Loomes and Sudgen, 1982), Disappointment Aversion (Gul, 1991), Expected Utility Theory with Jumps (Diecidue and Van de Ven, 2008), and Expected Utility Theory (power utility: constant relative risk aversion). All models under analysis significantly improve with respect to a non-informative model that states default indifference between all prospects, achieving a log-likelihood of $-225.96$.

It is clear that TUT represents an improvement with respect to the existing models. In particular, TUT can improve with respect to Prospect Theory: improvement of the log-likelihood function from $-130.75$ to $-118.62$, and reduction of the failure rate from 25.15% to 16.87%. Consistent with some existing evidence, the original version of Prospect Theory—Original Prospect Theory—fairs better than the later improvement of the model—Cumulative Prospect Theory.

Looking at table 5, it appears that Cumulative Prospect Theory encounters more difficulty than Original Prospect Theory at explaining phenomena related to the importance of the overall probability of winning (group 3), and preferences over repeated games (group 6). Original Prospect Theory, instead, does a surprisingly good job at explaining the phenomena related to goal seeking behavior (group 3), even better than TUT. The point is that the parameters I find to be optimal over the entire sample significantly differ from the original parameters proposed in the early literature (see

17 Appendix A provides more details about all models studied in the empirical analysis. Each model is tested on the dataset described in section 4.3.

18 With a non-informative model we have that $CE_W = CE_L$, for every choice $i$. Since the sample used in the maximum likelihood estimation is made of 326 observations, the non-informative model achieves a log-likelihood of $LL = 326 \cdot log(1/2) = -225.96$. The p-values of all models in table 1 confirm the statistically significant improvement with respect to the non-informative model. All p-values are obtained from a likelihood ratio test comparing the models of table 1 with the non-informative model. The number of degrees of freedom of each test is equal to the number of parameters of each model, since the non-informative model has zero parameters.
In particular, consistent with recent works (e.g., Wu and Markle, 2008),
the optimal curvature of the value function is particularly pronounced. Therefore, the
value function is similar to the sum of two indicator functions, resulting very steep
around zero and flat everywhere else. This feature of the model allows explaining the
importance of the overall probabilities of winning documented in the studies in group 3,
but it compromises the ability of explaining other phenomena, first of all the original
results rationalized in Kahneman and Tversky (1979).

Table 5 also shows the failure rates of the two versions of Prospect Theory with
the original parameters proposed by Kahneman and Tversky (1979), and Tversky and
Kahneman (1999). Not surprisingly, the original parametrization does a much better
job with the “classic” phenomena of group 1, but it displays a significant deterio-
rating on other groups (in particular, on results related to goals seeking behavior in
group 3, and gain-loss separability in group 5).

Regarding TUT, the model does a surprisingly good job at explaining the classic
phenomena of group 1. Equally surprising may appear the high failure rate in group 3
(studies on goal seeking behavior). The point, emphasized in section 3.7, is that in
many choices individuals express particularly strong preference for increasing the over-
all probabilities of realizing gains, or avoiding losses. In principle, TUT can rationalize
such behavior, but the parametrization required to do so would considerably diminish
the relative importance of preference for security/potential. Since the goal is to provide
a comprehensive view of how people face risk, the model achieves a balance between
goal seeking behavior and preference for security/potential, which cannot explain the
strong preference to increase the overall probability of winning that is sometimes ob-
served. Similar considerations can be made for the results of groups 4 and 6 that
contradicts the predictions of TUT.

Table 1 also shows the results of three models sharing some intuition with TUT:
Expected Utility Theory with jumps (Diecidue and Van de Ven, 2008), Regret The-
ory (Bell, 1982, 1983; Loomes and Sudgen, 1982), and SP/A theory (Lopes 1987).
Regarding Expected Utility Theory with jumps, Table 1 clearly points out that the
mere introduction of jumps in Expected Utility Theory cannot significantly improve

\footnote{Note that this group of choices contains some results not considered in Kahneman and Tversky (1979), and Tversky and Kahneman (1999). Here’s why the failure rate is greater than zero.}
the performance of the model: more structure in the definition of the jump is required to elucidate decision making under risk. In TUT this structure is defined by the regret/rejoice component, which differs from the standard definition of regret found in Regret Theory. In fact, regret in TUT is defined in such a way that it emphasizes the overall probabilities of obtaining the target return. This property allows to improve with respect to the models defined in Bell (1982, 1983) and Loomes and Sudgen (1982). Regarding SP/A theory, the model was developed to capture the concepts of preference for security/potential and goal seeking behavior, mainly by applying distortions in probabilities (i.e., by using weighting probability functions on the cumulative probabilities, modifying Cumulative Prospect Theory weighting functions). However, the estimation results point out that these concepts are captured more effectively by Target Utility Theory components (value function for preference for security/potential, and regret/rejoice components for goal seeking behavior), rather than distortions on probabilities.

5.2 Tests on Target Utility Theory: Shape and Components

Table 11 shows the log-likelihood obtained by imposing several restrictions on TUT parameters. These tests are conducted to test the importance of the factors implied by TUT: goals seeking behavior, preference for security/potential, and loss aversion.

The first set of tests investigates the shape of TUT value function, therefore testing the importance of preference for security/potential. Table 11 shows that there is a significant deterioration in performance if we impose alternative shapes on TUT value function: highly significant change in log-likelihood if we impose concavity for both losses and gains, or convexity for losses and concavity for gains, or linearity for both losses and gains. Similar results are obtained if we impose alternative shapes to the regret/rejoice function, confirming the general intuition of the model: the marginal impact of each outcome grows with its magnitude.

Table 11 also reports the test on the importance of the regret/rejoice function, therefore testing goals seeking behavior. The table shows that the regret/rejoice component is statistically significant: the value function of TUT, by itself, is not able to explain risky choice behavior. Moreover, table 11 shows the results obtained removing the value function from TUT, i.e., the results obtained with a model featuring only
regret/rejoice. Such a model (a pure jump model with a flat utility function with a shift at zero) fairs significantly worse than the complete version of TUT.

The last test of table 11 investigates the importance of the loss aversion parameter $\lambda$. By imposing $\lambda = 1$ (i.e., no higher impact of losses rather than gains of the same amount), the model doesn’t experience a significant deterioration in performance. It appears that the concept of loss aversion can be captured by the other components of TUT. The results of this test, by themselves, do not reject the hypothesis that individuals are averse at realizing losses. The test simply states that aversion toward the realization of losses can be captured by the regret/rejoice component of TUT, and by the implied goal seeking behavior. Therefore, a more parsimonious version of TUT with only two parameters could be used. However, without the loss aversion parameter $\lambda$, TUT could not rationalize one interesting phenomenon: the aversion toward symmetric bets.

Taken together, the results of table 11 point out that, in order to improve our understanding of risky choice behavior, we need a model that is able to jointly capture the three concepts of goal seeking behavior, preference for security/potential, and loss aversion. The three factors are jointly important to provide a comprehensive explanation of decision making under risk.

5.3 Tests on Target Utility Theory and Prospect Theory

Table 12 shows the results obtained with the model that best combines TUT and Prospect Theory. The model displays the value and regret/rejoice functions of TUT, with the addition of the probability weighting function of Original Prospect Theory. The introduction of distortions of probabilities as advocated by Original Prospect Theory (over-weighting low probabilities and under-weighting high probabilities) cannot significantly improve the performance of the model. Moreover, the probability weighting function of Original Prospect Theory may lead to counterintuitive predictions and to the violation of First Order Stochastic Dominance, as pointed out by Tversky and Kahneman (1992). These problems led to the development of Cumulative Prospect Theory, where distortions to probabilities are applied to cumulative probabilities. However, I find that using cumulative (or cumulative distorted) probabilities lead to a deterioration in performance.
Table 13 shows the results obtained by running the tests of table 11 on the model combining TUT and Prospect Theory. In this regard, similar considerations can be made regarding the significance of TUT components, and the validity of the assumptions made on their characteristics. In particular, when combining TUT and Prospect Theory components, the S-shaped value function and probability distortions are rejected. Overall, the results of tables 13 and 12 point out that the factors implied by TUT (goal seeking behavior and preference for security/potential) are significant even after controlling for Prospect Theory, while the factors implied by Prospect Theory (distortions in probability and S-shaped value function) are driven out by TUT.

6 Prior Outcomes and Investment Choices

6.1 Disposition effect and Escalation of Commitment

An extensive literature studies the effect of aspiration levels on investors’ attitude toward risk. In particular, the literature has focused its attention on investors’ reluctance to realize losses. For example, Genesove and Mayer (2001) and Case and Shiller (1988) find that homeowners are reluctant to sell houses for less than the purchase price. Kaustia (2004) shows that investors are averse to sell shares of an IPO that are trading below the offer price. The zombie-loan literature, instead, describes the tendency of Japanese banks to keep financing insolvent borrowers rather than writing off the loans.

Perhaps the most prominent trading anomaly in financial economics is the disposition effect (Shefrin and Statman, 1985), defined as the tendency to sell stocks that are winners (have gains relative to the purchase price) more often than losers. To illustrate the difficulty of loss realization, Shefrin and Statman cite the manual for stock brokers of Gross (1982):

Many clients, however, will not sell anything at a loss. They don’t want to give up the hope of making money on a particular investment, or perhaps they want to get even before they get out. The “getevenitis” disease has

---

probably wrought more destruction on investment portfolios than anything else. Rather than recovering to an original entry price, many investors plunge sickeningly to even deeper losses.

The disposition effect follows the literature on the so called “escalation of commitment” (Staw, 1981), which refers to the tendency of decision makers to persist with a failing course of action. This behavior arises when people have to decide whether to cease a questionable project (in which considerable resources have already been invested), or to commit more effort and resources into that venture.

The principal psychological explanation of the escalation of commitment and disposition effect is based on Prospect Theory. In the Prospect Theory framework, the failure to segregate prior losses induces the decision maker to frame future decisions as choices between losses. It follows that, being the Prospect Theory value function convex in the domain of losses, the integration of prior losses will induce individuals to be risk seeking, and to commit further resources to the failing venture.

However, the explanation of the escalation of commitment based on Prospect Theory contradicts the no-break-even effect, i.e. the reluctance to take risk after losses if there’s no possibility to break even (Thaler and Johnson, 1990). According to Prospect Theory, if individuals integrate prior losses, they should exhibit risk seeking behavior even if they can’t change their “losing status”, given the convexity of the value function in the domain of losses. Instead, individuals display risk seeking behavior only if they can recover prior losses (break-even effect). To explain both phenomena (no-break-even and break-even effects) we need a utility function that, like TUT, implies both aversion toward downside risk and goal seeking behavior.

### 6.2 Prior Outcomes and Portfolio Choices

Further support for the hypothesis that previous gains/losses can affect investors’ attitude toward risk has been found by Coval and Shumway (2005). The authors study the investment behavior of professional investors of the Chicago Board of Trade, by splitting the the trading day into two periods (morning and afternoon), and testing whether traders with profitable mornings decrease their afternoon risk-taking. Their main finding is that traders that lost money during a morning session took significantly
above average risk in the afternoon of the same day. At the same time, investors that were gaining money took below average risk in the afternoon; some increase in the willingness to take risk was observed for traders that obtained the highest profits in the morning. These results are consistent with the break-even and the house money effects documented by Thaler and Johnson (1990), and, more generally, with the observed reluctance of realizing losses.

The empirical evidence on the effect of prior outcomes on portfolio choices dates back to Bowman (1980, 1982, 1984), who documented the so called “risk-return paradox”: the negative risk-return association (higher risk and lower returns) that is observed for companies experiencing losses. In a related study conducted on 3,330 firms in 85 industries, Fiegenbaum (1990) documents the following findings:

1. A negative association between risk and return for organizations that are obtaining returns below their target return (higher risk taking the lower the below-target return);

2. A positive association between risk and return for organizations that are obtaining returns above their target return (higher risk taking the higher the above-target return);

3. The relationship between risk and return is steeper for organizations being below their target, rather than above their target.

Similar results are shown by Brown et al. (1996), who find that fund managers that are under-performing their benchmark in mid-year take higher risk during the last part of the year.

6.3 Target Utility Theory: Investing with Prior Outcomes

The same logic used to rationalize Thaler and Johnson (1990) results (in particular, the break-even effect) can be used to explain the phenomena of the financial literature described in section 6.1 (disposition effect and escalation of commitment).21

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21 Explaining the escalation of commitment in the TUT framework requires the assumption that utility is defined on the gains/losses of the entire endeavor (integration of prior outcomes). Regarding the disposition effect, we need the additional assumption of narrow framing: the investment is viewed in isolation with respect to the rest of the portfolio of the investor.
To further investigate whether TUT can explain the effect of prior outcomes on portfolio choices (section 6.2), we can consider the investment problem of an investor facing prior outcomes and having TUT preferences defined over the overall gains/losses of the entire investment period (in other words, the investor integrates prior outcomes in her decisions). The main assumptions of the investment problem are:

- There are three periods: \( t = 0, t = 1, \) and \( t = 2 \). The investor has an initial wealth equal to \( W_0 \). Utility is defined over the change in wealth, in real terms, between time \( t = 0 \) and time \( t = 2 \): \( W_2 - W_0 (1 + \pi) \). With \( \pi \) equal to the inflation rate between time \( t = 0 \) and time \( t = 2 \).

- At time \( t = 1 \) the investor is facing a prior gain/loss equal to \( W_0 x \), where \( x \) is the return on her wealth between time \( t = 0 \) and time \( t = 1 \). The prior return \( x \) is treated as an exogenous variable.

- At time \( t = 1 \), the investor has to decide how much to invest in a risky asset (delivering a return of \( r \), normally distributed with mean \( \mu \) and variance \( \sigma^2 \)), and a risk-free security (delivering a return of \( r_F \), for sure). There are no constraints on the percentage of her wealth, \( w \), invested in the risky asset. If \( w > 1 \), the investor borrow money at the rate \( r_B \).

Given the assumptions above, utility is defined over:

\[
W_0 (1 + x) (w (1 + r) + (1 - w) (1 + r_F)) - W_0 (1 + \pi)
\]  
(40)

Figure 3 shows the optimal portfolio weight of the risky asset at time \( t = 1 \), as a function of the prior gain/loss on the investor’s wealth.\(^{23}\) The optimal response to prior outcomes is a V-shaped risk profile. In fact, TUT predicts a strong increase in risk taking (higher weight invested in the risky asset) if the investor is facing prior losses, with a very strong increase in risk taking when the prior is such that investing

\(^{22}\) With borrowing—i.e., \( w > 1 \)—we have: \( W_0 (1 + x) (w (1 + r) + (1 - w) (1 + r_B)) - W_0 (1 + \pi) \).

\(^{23}\) Returns of risky and risk-free assets are calibrated with the US data. Assuming an annual horizon (one year between \( t = 0 \) and \( t = 2 \)), and assuming that time \( t = 1 \) corresponds to halfway through the year, we have semiannual (real) data: \( \mu = 4.05\% \), \( \sigma = 11.03\% \), \( \pi = 1.31\% \), and \( r_F = 0.45\% \). The borrowing rate is assumed to increase with the amount borrowed: \( r_B = r_F \cdot w \). Investor’s initial wealth is $1,000,000. The parameters of TUT are those shown in Table 2.
in the risk free asset doesn’t allow to break even (in real terms). At the same time, TUT predicts an increase in risk taking if the investor is facing prior gains, but this increase is much smaller than that associated with prior losses. Risk taking is minimum when investors are achieving their target.

The predictions of TUT are consistent with the main results of the studies cited above in section 6.2. It appears that TUT can explain the effect of prior outcomes on investment choices, which has represented a puzzle to these days.

7 Conclusions

This paper introduces a new model for risky choice behavior—Target Utility Theory (TUT)—that can rationalize several puzzles of the empirical decision making literature. Besides explaining the phenomena already rationalized by existing models (phenomena such as the fourfold pattern, the reflection effect, and Allais paradoxes), TUT provides a comprehensive explanation to the evidence related to preference for security/potential (e.g., Levy and Levy, 2002; Post and Levy, 2005; Levy, 2006), the effect of aspiration levels on risky choice behavior (Payne et al., 1980, 1981), decision making in the presence of prior gains and losses (Thaler and Johnson, 1990), and gain-loss separability (Wu and Markle, 2008). Moreover, TUT can shed light on how investment behavior is affected by prior gains and losses, providing a framework to rationalize the escalation of commitment (Staw, 1981), the disposition effect (Shefrin and Statman, 1985), and the observed increase in risk taking by investors that are facing prior losses (e.g. Coval and Shumway, 2005; Brown et al., 1996; Bowman, 1980).

The utility function of TUT is defined over gains and losses with respect to a reference point, or target payoff. In both positive and negative domains, the utility function is given by the sum of two components. First, a value function that captures the “pure” utility associated with each outcome. Second, an anticipatory feeling component that captures the expected regret/rejoice associated with each lottery. The value function is concave for losses and convex for gains, implying preference for security/potential. The anticipatory feeling component creates an upward shift in utility at the reference point, implying goal seeking behavior. Finally, TUT is consistent with loss aversion because the utility function is more pronounced for losses rather than gains.
Testing the model on a wide range of results of the empirical decision making literature, I find that all factors implied by TUT (goal seeking behavior, preference for security/potential, and loss aversion) are necessary to explain risky choice behavior, and to improve with respect to existing models. In particular, several model comparison tests point out that TUT factors (goal seeking behavior and preference for security/potential) are significant even after controlling for Prospect Theory, while the factors implied by Prospect Theory (distortions in probability and S-shaped value function) are driven out by TUT.

This paper can be extended in several directions. For example, in a follow up study, Piccioni (2014) shows that a Capital Asset Pricing Model based on a utility function with the same characteristics of TUT (concave for losses and convex for gains; steeper in the negative domain; and with an upward shift at zero) can explain the cross-section of stock returns and identify the sources of risk that drive size, value, and momentum anomalies. Moreover, the TUT framework can be extended to examine other phenomena of the decision making literature, like intertemporal choices (in particular, hyperbolic discounting: higher impatience in the short run), or preferences over delayed risky lotteries (in particular, the “anxiety” literature: higher risk aversion in the short run).

### A Models of the Decision Making Literature

#### A.1 Prospect Theory

According to Prospect Theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992), any gamble \( X = (x_1, p_1; x_2, p_2; ...) \) is valued according to:

\[
V(X) = \sum_{i=1}^{N} w(p_i) v(x_i)
\]  

(41)

where \( v(x_i) \) is the value function equal to:

\[
v(x_i) = \begin{cases} 
-\lambda (-x_i)^{\alpha^-} & \text{if } x_i \leq 0 \\
\alpha^+ x_i & \text{if } x_i > 0 
\end{cases}
\]  

(42)
The value function is convex for losses ($\alpha_- < 1$) and concave for gains ($\alpha_+ < 1$). The probability function overvalues low probabilities, undervalues high probabilities, and it satisfies: $w(0) = 0$ and $w(1) = 1$.

The original version of the model, usually defined as Original Prospect Theory (Kahneman and Tversky, 1979), differs from the later improvement, usually defined as Cumulative Prospect Theory (Tversky and Kahneman, 1992), because of the difference in the probability weighting function. Regarding Original Prospect Theory, the probability weighting function equal to:

$$w(p_i) = \frac{p_i^w}{(p_i^w + (1 - p_i)^w)^{1/w}} \quad (43)$$

Regarding Cumulative Prospect Theory, the probability weighting function is equal to:

$$w(p) = \begin{cases} 
\frac{P_i^{w-}}{(P_i^{w-} + (1 - P_i)^{w-})^{1/w-}} & \text{for losses} \\
\frac{D_i^{w+}}{(D_i^{w+} + (1 - D_i)^{w+})^{1/w+}} & \text{for gains} 
\end{cases} \quad (44)$$

In Cumulative Prospect Theory the probability weighting function is applied to cumulative probabilities $P_i$ for losses, and decumulative probabilities $D_i$ for gains. The function $w(p)$ continues to overweight low probabilities and underweight high probabilities, but distortions are different for gains and losses.

**A.2 Expected Utility Theory with Jumps**

Diecidue and Van de Ven (2008) develop a model that includes into an expected utility representation the overall probabilities of success and failure relative to the aspiration level. The authors show that this turns up to be equivalent to expected utility with discontinuous utility function (utility with a positive jump at the reference point).

In Diecidue and Van de Ven model, the expected utility of any gamble $X$ is:

$$E(U(W + X)) = \sum_{i=1}^{N} p_i \tilde{U}(W + x_i) \quad (45)$$
where $x_i$ is the outcome of lottery $X$ in state $i$, $p_i$ is the probability of state $i$, and $W$ is the initial wealth level. If we assume constant relative risk aversion, we have:

$$
\hat{U}(W + x_i) = \begin{cases} 
\frac{(W+x_i)^{1-\alpha}}{1-\alpha} & \text{if } x_i \leq 0 \\
\frac{(W+x_i)^{1-\alpha}}{1-\alpha} + \lambda & \text{if } x_i > 0
\end{cases}
$$

(46)

Because of the positive jump in utility equal to $\lambda$, when we compute expected utility we consider the overall probability of achieving the target return:

$$
E(U(W + X)) = \sum_{i=1}^{N} p_i \hat{U}(W + x_i) = \lambda \sum_{x_i > 0} p_i + \sum_{i=1}^{N} p_i \frac{(W + x_i)^{1-\alpha}}{1-\alpha}
$$

(47)

A.3 SP/A Theory

SP/A Theory (Lopes, 1987) is a dual criterion model in which the process of choosing between lotteries entails integrating two logically and psychologically separate criteria. Gambles are valuated according to a function $f$ that depends on two criteria: security-potential criterion ($SP$), and aspiration criterion ($A$). The general version of $f$ is:

$$
SP/A(X) = f(SP, A)
$$

(48)

In the empirical analysis I estimate the function:

$$
f(SP, A) = \lambda_{SP} SP^{\alpha_{SP}} + \lambda_A A^{\alpha_A}
$$

(49)

In order to approximate for a wide range of functional forms, I do not impose any constraints on the parameters of (49).

Regarding the $SP$ criterion, the functional form proposed by Lopes is:

$$
SP(X) = \sum_{i=1}^{N} h(D_i) (x_i - x_{i-1})
$$

(50)
with $x_i > x_{i-1}$, and the decumulative weighting function $h(D_i)$ has the form

$$h(D_i) = w D_i^{1+q} + (1 - w) (1 - (1 - D_i)^{q+1})$$

(51)

for both gains and losses. As Lopes and Odean (1999) point out, “the equation is derived from the idea that subjects assess lotteries from the bottom up (a security-minded analysis), or the top down (a potential-minded analysis), or both (a cautiously hopeful analysis).”

The aspiration criterion is defined over the probability of obtaining payoffs above the reference point:

$$A(X) = p(x > 0)$$

(52)

## A.4 Regret Theory

Regret Theory (Bell, 1982, 1983; Loomes and Sudgen, 1982) involves the comparison of all outcomes delivered by any pair of lotteries $X$ and $Y$. For all outcomes $x_i$ and $y_j$ utility is given by:

$$U(x_i, y_j) = Q(u(W + x_i) - u(W + y_j))$$

(53)

Bell (1982) referred to $u$ as a value function measuring strength of preference, or incremental value. Loomes and Sugden (1982) referred to $u$ as a choiceless utility function, which reflects the utility the decision maker would derive from an outcome $x_i$ if he experienced it without having chosen it. Function $u$ is assumed to be globally concave, and can be defined as:

$$u(W + x_i) = \frac{(W + x_i)^{1-\alpha}}{1 - \alpha}$$

(54)

with $1 - \alpha < 1$. The function $Q$, instead, captures regret. The function is strictly increasing and has the symmetry property: $-Q(z) = Q(-z)$. Regret aversion, which generates the distinctive predictions of regret theory, implies that $Q$ is convex. We can define $Q(z)$ as:

$$Q(z) = \text{sign}(z) \left(|z|^\gamma\right)$$

(55)

with $\gamma > 1$. 

44
### A.5 Disappointment Aversion

Gul (1991) develops a theory of Disappointment Aversion in which payoffs below the certainty equivalent of any lottery generate disappointment, while payoffs above the certainty equivalent are elating. The certainty equivalent of prospect \( X \) is implicitly defined by:

\[
u(ce_X) = \sum_{i=1}^{N} p_i \left( u(W + x_i) + A(u(W + x_i) - u(W + ce_X)) I_{x_i < ce_X} \right)
\]  

(56)

where \( A \) is the disappointment aversion coefficient. If \( 0 < A < 1 \), individuals are disappointment averse, and care more about downside rather than upside risk. The utility function \( u \) is globally concave, and it can be defined as:

\[
u(W + x_i) = \frac{(W + x_i)^{1-\alpha}}{1-\alpha}
\]

(57)

with \( 1 - \alpha < 1 \).

### References


Table 1: Maximum Likelihood Estimation, All Models under Analysis

<table>
<thead>
<tr>
<th>Model</th>
<th>Log-Likelihood</th>
<th># Par.</th>
<th>Failure Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target Utility Theory</td>
<td>−118.62</td>
<td>3</td>
<td>16.87 %</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Original Prospect Theory</td>
<td>−130.75</td>
<td>4</td>
<td>25.15 %</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cumulative Prospect Theory</td>
<td>−173.25</td>
<td>5</td>
<td>30.67 %</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SP/A</td>
<td>−186.47</td>
<td>6</td>
<td>44.48 %</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expected Utility Theory</td>
<td>−212.21</td>
<td>2</td>
<td>39.88 %</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EUT with Jumps</td>
<td>−200.35</td>
<td>3</td>
<td>38.04 %</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regret Theory</td>
<td>−216.23</td>
<td>3</td>
<td>48.47 %</td>
</tr>
<tr>
<td></td>
<td>(0.0016)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Disappointment Aversion</td>
<td>−210.42</td>
<td>3</td>
<td>36.50 %</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The table shows the results of the Maximum Likelihood estimation of the Logit model of equation (37). The Failure Rate is the percentage of choices not explained by each model. In parenthesis we have the p-values of the likelihood ratio tests that compare each model under analysis with a non-informative model that states default indifference between all lotteries (see sections 4.2 and 5.1). The number of degrees of freedom of each test is equal to the number of parameters of each model plus the two extra parameters of the Logit model in equation (37). Each model is tested on the sample defined in section 4.3. Models under analysis are discussed in section 5.1 and appendix A.
Table 2: Maximum Likelihood Estimation, Target Utility Theory

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log-Likelihood</td>
<td>$-118.62$ (0.0000)</td>
<td></td>
</tr>
<tr>
<td>Failure Rate</td>
<td>$16.87%$</td>
<td></td>
</tr>
<tr>
<td># Parameters</td>
<td>$3$</td>
<td></td>
</tr>
<tr>
<td>$\psi$: value function</td>
<td>$1.5990$ (0.0000)</td>
<td></td>
</tr>
<tr>
<td>$\gamma$: regret/rejoice function</td>
<td>$2.1610$ (0.0000)</td>
<td></td>
</tr>
<tr>
<td>$\lambda$: loss aversion</td>
<td>$1.0645$ (0.9999)</td>
<td></td>
</tr>
<tr>
<td>$\kappa_1$: $LL$ parameter (linear)</td>
<td>$1.3367$ (0.1453)</td>
<td></td>
</tr>
<tr>
<td>$\kappa_2$: $LL$ parameter (exponent)</td>
<td>$0.2325$ (0.0000)</td>
<td></td>
</tr>
</tbody>
</table>

The table shows the estimation results of Target Utility Theory. The model is estimated with Maximum Likelihood estimation (Logit model of equation (37)) on the sample discussed in section 4.3. In parenthesis we have the p-values of the likelihood ratio tests that check the significance of each parameter; all parameters are tested against the null hypothesis that they are equal to one (which would imply no goal seeking behavior, no preference for security/potential, no loss aversion, no effect of the $LL$ parameters).  

54
<table>
<thead>
<tr>
<th></th>
<th>OPT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log-Likelihood</td>
<td>$-130.75$</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Failure Rate</td>
<td>25.15%</td>
</tr>
<tr>
<td># Parameters</td>
<td>4</td>
</tr>
<tr>
<td>$\alpha_-$: value function, losses</td>
<td>0.0483</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
</tr>
<tr>
<td>$\alpha_+$: value function, gains</td>
<td>0.2000</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
</tr>
<tr>
<td>$\lambda$: loss aversion</td>
<td>3.4336</td>
</tr>
<tr>
<td></td>
<td>(0.1967)</td>
</tr>
<tr>
<td>$w$: probability weighting</td>
<td>0.6454</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
</tr>
<tr>
<td>$\kappa_1$: $LL$ parameter (linear)</td>
<td>0.0018</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
</tr>
<tr>
<td>$\kappa_2$: $LL$ parameter (exponent)</td>
<td>2.4964</td>
</tr>
<tr>
<td></td>
<td>(0.0868)</td>
</tr>
</tbody>
</table>

The table shows the estimation results of Original Prospect Theory (Kahneman and Tversky, 1979). The model is estimated with Maximum Likelihood estimation (Logit model of equation (37)), on the set of results discussed in section 4.3. In parenthesis we have the p-values of the likelihood ratio tests that check the significance of each parameter. The null hypothesis is that each OPT parameter is equal to one (so that we have risk neutrality, no loss aversion, no probabilities distortions, and no effect of the $LL$ parameters).
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log-Likelihood</td>
<td></td>
<td>−173.25</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Failure Rate</td>
<td></td>
<td>30.67%</td>
<td></td>
</tr>
<tr>
<td># Parameters</td>
<td></td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>$\alpha_-$: value function, losses</td>
<td></td>
<td>0.0265</td>
<td>(0.0002)</td>
</tr>
<tr>
<td>$\alpha_+$: value function, gains</td>
<td></td>
<td>0.0245</td>
<td>(0.0026)</td>
</tr>
<tr>
<td>$\lambda$: loss aversion</td>
<td></td>
<td>1.0000</td>
<td>(0.9999)</td>
</tr>
<tr>
<td>$w_-$: probability weighting, losses</td>
<td></td>
<td>1.0000</td>
<td>(0.9999)</td>
</tr>
<tr>
<td>$w_+$: probability weighting, gains</td>
<td></td>
<td>1.0000</td>
<td>(0.9999)</td>
</tr>
<tr>
<td>$\kappa_1$: LL parameter (linear)</td>
<td></td>
<td>0.3422</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>$\kappa_2$: LL parameter (exponent)</td>
<td></td>
<td>0.4489</td>
<td>(0.0001)</td>
</tr>
</tbody>
</table>
The table shows the estimation results of Cumulative Prospect Theory (Tversky and Kahneman, 1992). The model is estimated with Maximum Likelihood estimation (Logit model of equation (37)), on the set of results discussed in section 4.3. In parenthesis we have the p-values of the likelihood ratio tests that check the significance of each parameter. The null hypothesis is that each CPT parameter is equal to one (so that we have risk neutrality, no loss aversion, no probabilities distortions, and no effect of the $LL$ parameters).
The table shows the failure rate of Target Utility Theory, Original Prospect Theory, and Cumulative Prospect Theory, by breaking the sample in several groups of choices. Regarding Original Prospect Theory and Cumulative Prospect Theory, in parenthesis we have the failure rates obtained with the parameters originally proposed by Kahneman and Tversky (1979)—$\alpha_- = 0.88$, $\alpha_+ = 0.88$, $\lambda = 2.25$, $w = 0.69$—and Tversky and Kahneman (1992)—$\alpha_- = 0.88$, $\alpha_+ = 0.88$, $\lambda = 2.25$, $w_- = 0.69$, $w_+ = 0.61$ (see appendix A.1).

<table>
<thead>
<tr>
<th>Group Description</th>
<th>TUT</th>
<th>Original PT</th>
<th>Cumulative PT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1: classic phenomena</td>
<td>0.00%</td>
<td>44.44%</td>
<td>44.44%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(25.93%)</td>
<td>(22.22%)</td>
</tr>
<tr>
<td>Group 2: effect of changing the dispersion of payoffs</td>
<td>6.52%</td>
<td>30.43%</td>
<td>26.09%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(28.26%)</td>
<td>(10.87%)</td>
</tr>
<tr>
<td>Group 3: effect of changing the overall probabilities</td>
<td>24.84%</td>
<td>19.11%</td>
<td>29.30%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(40.76%)</td>
<td>(43.31%)</td>
</tr>
<tr>
<td>Group 4: effect of prior outcomes</td>
<td>17.39%</td>
<td>39.13%</td>
<td>39.13%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(26.09%)</td>
<td>(39.13%)</td>
</tr>
<tr>
<td>Group 5: gain-loss separability</td>
<td>0.00%</td>
<td>14.81%</td>
<td>18.52%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(48.15%)</td>
<td>(66.67%)</td>
</tr>
<tr>
<td>Group 6: repeated games</td>
<td>19.57%</td>
<td>28.26%</td>
<td>34.78%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(23.91%)</td>
<td>(28.26%)</td>
</tr>
<tr>
<td>Total</td>
<td>16.87%</td>
<td>25.15%</td>
<td>30.06%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(34.97%)</td>
<td>(36.50%)</td>
</tr>
</tbody>
</table>
Choices are classified in the following groups:

- **Group 1: Allais paradoxes, common consequence effect, common ratio effect, fourfold pattern, reflection effect, insurance behavior (27 choices).** Kahneman and Tversky (1979), Redelmeier and Tversky (1992), Shefrin (2005), Schoemaker and Kunreuther (1981), Hersey and Schoemaker (1980).


- **Group 4: effect of prior outcomes (23 choices).** Thaler and Johnson (1990).

- **Group 5: gain-loss separability (27 choices).** Wu and Markle (2008)

Table 6: Maximum Likelihood Estimation, SP/A Theory

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log-Likelihood</td>
<td>-186.47</td>
<td>0.0000</td>
</tr>
<tr>
<td>Failure Rate</td>
<td>44.48%</td>
<td></td>
</tr>
<tr>
<td># Parameters</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>$w$: weight in weighting function</td>
<td>1.0000</td>
<td>0.9999</td>
</tr>
<tr>
<td>$q$: exponent in weighting function</td>
<td>8761.9600</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\lambda_{SP}$: linear term in SP</td>
<td>3.7889</td>
<td>0.0002</td>
</tr>
<tr>
<td>$\alpha_{SP}$: exponent in SP</td>
<td>0.5636</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\lambda_A$: linear term in A</td>
<td>534.6100</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\alpha_A$: exponent in A</td>
<td>0.7353</td>
<td>0.0335</td>
</tr>
<tr>
<td>$\kappa_1$: $LL$ parameter (linear)</td>
<td>0.0841</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\kappa_2$: $LL$ parameter (exponent)</td>
<td>0.4900</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
The table shows the estimation results of SP/A Theory (Lopes, 1987). The model is estimated with Maximum Likelihood estimation (Logit model of equation (37)), on the set of results discussed in section 4.3. In parenthesis we have the p-values of the likelihood ratio tests that check the significance of each parameter. The null hypothesis is that each parameter is equal to one.
Table 7: Maximum Likelihood Estimation, Expected Utility Theory

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log-Likelihood</td>
<td>-212.21</td>
<td>0.0000</td>
</tr>
<tr>
<td>Failure Rate</td>
<td>39.88%</td>
<td></td>
</tr>
<tr>
<td># Parameters</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>$\alpha$: relative risk aversion</td>
<td>0.9973</td>
<td>0.0115</td>
</tr>
<tr>
<td>$W$: initial wealth</td>
<td>12,266.66</td>
<td></td>
</tr>
<tr>
<td>$\kappa_1$: LL parameter (linear)</td>
<td>0.2614</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\kappa_2$: LL parameter (exponent)</td>
<td>0.2268</td>
<td>0.0169</td>
</tr>
</tbody>
</table>

The table shows the estimation results of Expected Utility Theory (with constant relative risk aversion). The model, discussed in section 2.1, is estimated with Maximum Likelihood estimation (Logit model of equation (37)), on the set of results discussed in section 4.3. In parenthesis we have the p-values of the likelihood ratio tests that check the significance of each parameter. The risk aversion parameter is tested against the null hypothesis of risk neutrality ($\alpha=1$).
The table shows the estimation results of Expected Utility Theory with Jumps (Diecidue and Van de Ven, 2008). The model is estimated with Maximum Likelihood estimation (Logit model of equation (37)), on the set of results discussed in section 4.3. In parenthesis we have the p-values of the likelihood ratio tests that check the significance of each parameter. The risk aversion parameter is tested against the null hypothesis of risk neutrality ($\alpha=1$); while $\lambda = 0$ is the null hypothesis for the jump.
Table 9: Maximum Likelihood Estimation, Disappointment Aversion

<table>
<thead>
<tr>
<th></th>
<th>Disappointment Aversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log-Likelihood</td>
<td>−210.42</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Failure Rate</td>
<td>36.50 %</td>
</tr>
<tr>
<td># Parameters</td>
<td>3</td>
</tr>
<tr>
<td>( \alpha ): relative risk aversion</td>
<td>0.8726</td>
</tr>
<tr>
<td></td>
<td>(0.0002)</td>
</tr>
<tr>
<td>( W ): initial wealth</td>
<td>68,498.31</td>
</tr>
<tr>
<td>( A ): disappointment aversion</td>
<td>−0.78</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
</tr>
<tr>
<td>( \kappa_1 ): ( LL ) parameter (linear)</td>
<td>0.2400</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
</tr>
<tr>
<td>( \kappa_2 ): ( LL ) parameter (exponent)</td>
<td>0.2400</td>
</tr>
<tr>
<td></td>
<td>(0.0005)</td>
</tr>
</tbody>
</table>

The table shows the estimation results of Disappointment Aversion (Gul, 1991). The model is estimated with Maximum Likelihood estimation (Logit model of equation (37)), on the set of results discussed in section 4.3. In parenthesis we have the p-values of the likelihood ratio tests that check the significance of each parameter. The risk aversion and disappointment parameters are tested against the null hypothesis of risk neutrality and no disappointment aversion.
### Table 10: Maximum Likelihood Estimation, Regret Theory

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log-Likelihood</td>
<td>−216.23</td>
<td>(0.0000)</td>
<td></td>
</tr>
<tr>
<td>Failure Rate</td>
<td>48.47%</td>
<td></td>
<td></td>
</tr>
<tr>
<td># Parameters</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>α: relative risk aversion</td>
<td>0.7826</td>
<td>(0.0002)</td>
<td></td>
</tr>
<tr>
<td>W: initial wealth</td>
<td>143,500.00</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>γ: regret aversion</td>
<td>4.60</td>
<td>(0.0014)</td>
<td></td>
</tr>
<tr>
<td>κ₁: LL parameter (linear)</td>
<td>0.8150</td>
<td>(0.9999)</td>
<td></td>
</tr>
<tr>
<td>κ₂: LL parameter (exponent)</td>
<td>0.2174</td>
<td>(0.0000)</td>
<td></td>
</tr>
</tbody>
</table>

The table shows the estimation results of Regret Theory (Bell, 1982, 1983; Loomes and Sudgen, 1982). The model is estimated with Maximum Likelihood estimation (Logit model of equation (37)), on the set of results discussed in section 4.3. In parenthesis we have the p-values of the likelihood ratio tests that check the significance of each parameter. The risk aversion and regret parameters are tested against the null hypothesis of risk neutrality and no regret aversion.
The table shows the results of model comparison tests on Target Utility Theory. The table shows the Maximum Likelihood that is obtained by imposing the restrictions. The p-values are obtained from the Likelihood ratio test discussed in section 4.2.
Table 12: Maximum Likelihood Estimation, Target Utility Theory and Original Prospect Theory Combined

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log-Likelihood</td>
<td>$-117.32$</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Failure Rate</td>
<td>$17.18%$</td>
<td></td>
</tr>
<tr>
<td># Parameters</td>
<td>$4$</td>
<td></td>
</tr>
<tr>
<td>$\psi$: value function</td>
<td>$1.5590$</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>$\gamma$: regret/rejoice function</td>
<td>$2.1490$</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>$\lambda$: loss aversion</td>
<td>$1.0375$</td>
<td>(0.9999)</td>
</tr>
<tr>
<td>$w$: probability weighting</td>
<td>$0.9703$</td>
<td>(0.1063)</td>
</tr>
<tr>
<td>$\kappa_1$: $LL$ parameter (linear)</td>
<td>$1.2635$</td>
<td>(0.2123)</td>
</tr>
<tr>
<td>$\kappa_2$: $LL$ parameter (exponent)</td>
<td>$0.2575$</td>
<td>(0.0000)</td>
</tr>
</tbody>
</table>

The table shows the estimation results of the model that best combines Target Utility Theory and Original Prospect Theory. The model is estimated with Maximum Likelihood estimation (Logit model of equation (37)) on the sample discussed in section 4.3. In parenthesis we have the p-values of the likelihood ratio tests that check the significance of each parameter; all parameters are tested against the null hypothesis that they are equal to one (which would imply no goal seeking behavior, no preference for security/potential, no loss aversion, no distortion in probability, no effect of the $LL$ parameters).
### Table 13: Comparison Tests, Target Utility Theory Original Prospect Theory

<table>
<thead>
<tr>
<th>Model</th>
<th>Log-Likelihood</th>
<th># Restrictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target Utility Theory</td>
<td>−118.62</td>
<td>-</td>
</tr>
<tr>
<td>Original Prospect Theory</td>
<td>−130.75</td>
<td>-</td>
</tr>
<tr>
<td>Models combined</td>
<td>−117.32</td>
<td>-</td>
</tr>
</tbody>
</table>

Tests on the shape of the value function:

<table>
<thead>
<tr>
<th></th>
<th>Log-Likelihood</th>
<th># Restrictions</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convexity for losses &amp; Concavity for gains</td>
<td>−130.49</td>
<td>2</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>No Loss Aversion (λ = 0):</td>
<td>−117.33</td>
<td>1</td>
<td>(0.9999)</td>
</tr>
<tr>
<td>No Regret/Rejoice</td>
<td>−136.72</td>
<td>1</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>No Weighting Probability function</td>
<td>−118.62</td>
<td>1</td>
<td>(0.1063)</td>
</tr>
</tbody>
</table>

The table shows the results of model comparison tests using the model that best combines Target Utility Theory and Original Prospect Theory. The table shows the Maximum Likelihood that is obtained by imposing the restrictions. The p-values are obtained from the Likelihood ratio test, discussed in section 4.2, that compares the Log-Likelihood of the model that combines Target Utility Theory and Original Prospect Theory, with and without imposing the restrictions.
Figure 1: Target Utility Theory

The graph shows the utility function of Target Utility Theory. As discussed in section 3.1, the utility function is given by the sum of two components. First, a value function that is concave for losses and convex for gains. Second, an anticipatory feeling component that captures expected regret and rejoice, creating the negative shift in utility in the negative domain and the positive shift in the positive domain. The shape of the value function is consistent with preference for security/potential (i.e., downside risk aversion and preference for upside potential). Moreover, because of the shift at zero created by the anticipatory feeling component, TUT implies goal seeking behavior (i.e., the importance of achieving relevant aspiration levels, or target returns). Finally, TUT is consistent with loss aversion (i.e., the empirical observation that losses loom larger than gains), because the utility function is more pronounced for losses rather than gains.
Figure 2: Target Utility Theory, Certainty Equivalent

\[(a) \ ce_x \sim (10, p; 0, 1-p) \]
\[(b) \ ce_x \sim (100, p; 0, 1-p) \]

The graph shows how the certainty equivalent of two lotteries—\((10, p; 0, 1-p)\) and \((100, p; 0, 1-p)\)—evolves by changing the underlying probabilities of the outcomes. The results are obtained with the parameters of Target Utility Theory estimated in table 2. The blue curves represent the certainty equivalents, while the red lines the expected values of the lotteries. A certainty equivalent lower (greater) than the expected value implies risk aversion (risk seeking). Similar curves are obtained for losses.
Figure 3: Investing with Prior Outcomes

The graph shows the solution to the investment problem discussed in section 6.3. The graph shows the optimal portfolio weight of the risky asset, as a function of the prior gain/loss on the investor’s wealth. Returns of risky and risk-free assets are calibrated with the US semiannual (real) data: $\mu = 4.05\%$, $\sigma = 11.03\%$, $\pi = 1.31\%$, and $r_F = 0.45\%$. The borrowing rate is assumed to increase with the amount borrowed: $r_B = r_F \cdot w$. Investor’s initial wealth is $1,000,000.$