Preliminary and Incomplete

Monetary Policy and the Uncovered Interest Rate Parity Puzzle[*]

David K. Backus[†], Federico Gavazzoni[‡], Chris Telmer[§] and Stanley E. Zin[¶]

December 2009

Abstract

High interest rate currencies tend to appreciate. This is the *uncovered interest rate parity (UIP) puzzle.* It is primarily a statement about *short-term* interest rates and how they are related to exchange rates. Short-term interest rates are strongly affected by monetary policy. The UIP puzzle, therefore, can be restated in terms of monetary policy. Do foreign and domestic monetary policies imply exchange rates that violate UIP? We represent monetary policy as foreign and domestic Taylor rules. Foreign and domestic pricing kernels determine the relationship between these Taylor rules and exchange rates. We examine different specifications for the Taylor rule and ask which can resolve the UIP puzzle. We find evidence in favor of a particular asymmetry. If the foreign Taylor rule responds to exchange rate variation but the domestic Taylor rule does not, then the model can resolve the puzzle.

---

[*] Thanks to Burton Hollifield for many helpful discussions.

[†] Stern School of Business, New York University; dbackus@stern.nyu.edu
[‡] Tepper School of Business, Carnegie Mellon University; federicogavazzoni@cmu.edu
[§] Tepper School of Business, Carnegie Mellon University; chris.telmer@cmu.edu
[¶] Stern School of Business, New York University; stan.zin@nyu.edu

1 Introduction

Uncovered interest rate parity (UIP) predicts that high interest rate currencies will depreciate relative to low interest rate currencies. Yet for many currency pairs and time periods we seem to see the opposite. The inability of asset-pricing models to reproduce this fact is what we refer to as the *UIP puzzle*.

The UIP evidence is primarily about *short-term* interest rates and currency depreciation rates. Monetary policy exerts substantial influence over short-term interest rates. Therefore, the UIP puzzle might be restated in terms of monetary policy: Why do countries with high interest rate *policies* have currencies that tend to appreciate relative to those with low interest rate *policies*? Or, more specifically, if foreign and domestic monetary policies follow simple Taylor rules, and if these Taylor rules affect exchange rates through equilibrium pricing kernels, then can monetary policies account for the UIP evidence?

There is a sizable literature that examines the link between UIP deviations and monetary policy.[1] But unlike much of this literature, we simplify the analysis by ignoring the economic role of money. Following the New Keynesian macroeconomics literature (*e.g.*, Clarida, Galí, and Gertler (1999)), the policy of the monetary authority acts directly on the short-term interest rate. In other words, monetary policy takes the form of a Taylor rule

$$i_t = \tau + \tau_1 \pi_t + z_t \quad , \tag{1}$$

where i_t is the nominal short-term interest rate, π_t is the inflation rate, z_t is a "policy shock," and τ and τ_1 are policy parameters. We also assume that the private sector can trade bonds. Therefore the nominal interest rate must also satisfy the standard (nominal) Euler equation,

$$i_t = -\log E_t \, n_{t+1} e^{-\pi_{t+1}} \quad , \tag{2}$$

where n_{t+1} is the real marginal rate of substitution. An equilibrium inflation rate process must satisfy both of these equations at each point in time, which requires inflation to solve the nonlinear stochastic difference equation:

$$\pi_t = -\frac{1}{\tau_1}\left(\tau + z_t + \log E_t \, n_{t+1} \, e^{-\pi_{t+1}}\right) \quad . \tag{3}$$

A solution to equation (3) is an endogenous inflation process, π_t, that is jointly determined by the response of monetary authority and the private sector to the

---

[1] See, for example, Alvarez, Atkeson, and Kehoe (2007), Backus, Gregory, and Telmer (1993), Bekaert (1994), Burnside, Eichenbaum, Kleshchelski, and Rebelo (2006), Canova and Marrinan (1993), Dutton (1993), Grilli and Roubini (1992), Lucas (1982), Macklem (1991), Marshall (1992), McCallum (1994) and Schlagenhauf and Wrase (1995).

same underlying exogenous shocks. By substituting such a solution back into the Euler equation (2), we arrive at what Gallmeyer, Hollifield, Palomino, and Zin (2007) (GHPZ) refer to as a 'monetary policy consistent pricing kernel:' a (nominal) pricing kernel that depends on the Taylor-rule parameters τ and τ_1. For a two-country complete-markets model, the ratio of these pricing kernels determines the currency depreciation rate,

$$\frac{S_{t+1}}{S_t} = \frac{n^*_{t+1} e^{-\pi^*_{t+1}}}{n_{t+1} e^{-\pi_{t+1}}} \ , \tag{4}$$

where S_t denotes the nominal exchange rate (price of foreign currency in units of domestic), and asterisks denote foreign variables. Equations (1)–(4) (along with specifications for the shocks) fully characterize the joint distribution of interest rates and exchange rates and, therefore, any departures from UIP.

Given the Taylor rule in (1), we can ask whether the implied exchange rate process in (4) tends to appreciate when the implied interest rate in (2) is relatively low. If so, then the source of UIP deviations can be associated with this Taylor rule. Moreover, we can generalize the specification of the Taylor rule in equation (1) and analyze the consequences of alternative monetary policies for currency exchange rates. In addition, we can ask whether the Taylor rule parameters are identified by the UIP facts. Cochrane (2007) provides examples in which policy parameters and the dynamics of the shocks are not separately identified by the relationship between interest rates and inflation. Our framework has the potential for identifying monetary policy parameters from the properties of currency exchange rates.

In general, exchange rates movements can be driven by both real and nominal factors. We begin, in Section 3, by shutting down the real exchange rate channel so that the focus is exclusively on monetary policy (Section 4 relaxes this strong assumption). This means that $n_{t+1} = e^{-r}$ and $n^*_{t+1} = e^{-r^*}$, where r and r^* denote (constant) real interest rates. It implies that the real exchange rate is constant and that (continuously compounded) relative PPP holds exactly: $\log(S_{t+1}/S_t) = \pi_{t+1} - \pi^*_{t+1}$. It also implies that, with lognormality, the Euler equation (2) can be written as

$$i_t = r + E_t\, \pi_{t+1} - \frac{1}{2} Var_t(\pi_{t+1}). \tag{5}$$

Equation (5) indicates that all that really distinguishes our (initial) approach from the benchmark New-Keynesian setup is the conditional variance term. With homoskedastic inflation the nominal interest rate would satisfy the Fisher equation (up to a constant), the difference equation (3) would be linear, and the solution for inflation would be in the same class as, say, Clarida, Galí, and Gertler (1999). What would also be true, however, is that UIP would be satisfied (up to a constant) and Fama's (1984) well-known regression of the depreciation rate on the interest rate differential would yield a (population) slope coefficient of 1.0. Our paper would be finished before it even began. Stochastic volatility, therefore, is not a choice, it is a *requirement*. But since inflation is an endogenous process, we cannot simply assert a stochastic volatility process for inflation. Rather we must solve for the relationship between Taylor rule parameters, stochastic volatility and deviations from UIP.

An important precursor to our paper is McCallum (1994), which also derives implications for UIP as the solution to a linear rational expectations model characterized by a policy-type interest rate rule. We extend McCallum's work by endogenizing the currency risk premium which, in his paper, is exogenous.[2] This is an important step since it constrains the sense in which the UIP anomaly is driven by endogenous equilibrium *inflation risk*. That is, in our model, a shock is realized, the Taylor rule responds to that shock, and as a result so does inflation. Whether or not this shock commands a risk premium depends on the parameters of the model. We can then ask if the way in which monetary policy reacts to shocks is consistent with risk premiums that are capable of creating sizable deviations from UIP.

2 Pricing Kernels and Currency Risk Premiums

We begin with a terse treatment of existing results in order to fix notation. The logarithm of the spot and one-period forward exchange rates, in units of U.S. dollars (USD) per unit of foreign currency (say, British pounds, GBP), are denoted s_t and f_t. USD and GBP one-period interest rates (continuously compounded) are denoted i_t and i_t^*. Covered interest parity implies that $f_t - s_t = i_t - i_t^*$. Fama's (1984) decomposition of the interest rate differential (forward premium) is

$$\begin{aligned} i_t - i_t^* = f_t - s_t &= \big(f_t - E_t s_{t+1}\big) + \big(E_t s_{t+1} - s_t\big) && (6) \\ &\equiv p_t + q_t && (7) \end{aligned}$$

This decomposition expresses the forward premium as the sum of q_t, the expected USD depreciation rate, and p_t, the expected payoff on a forward contract to receive USD and deliver GBP. We define the latter as the *foreign currency risk premium*. We define *uncovered interest parity* (UIP) as $p_t = 0$. The well-known rejections of UIP are manifest in negative estimates of the parameter b from the regression

$$s_{t+1} - s_t = c + b\big(i_t - i_t^*\big) + \text{residuals} \quad . \tag{8}$$

---

[2] Engel and West (2006) also study a model of how Taylor rules affect exchange rates. Their analysis, while focusing on a different set of questions, is related to McCallum's in that they interpret their 'policy shock' as an amalgamation of an actual policy shock and an exogenous risk premium. Our paper relates to theirs in that both derive an exchange rate process as the solution to a forward-looking difference equation. The main difference is that our deviations from UIP are endogenous.

The population regression coefficient can be written

$$b = \frac{Cov(q_t, p_t + q_t)}{Var(p_t + q_t)} \quad . \tag{9}$$

Fama (1984) noted that necessary conditions for $b < 0$ are

$$\begin{aligned} Cov(p_t, q_t) &< 0 \quad (10) \\ Var(p_t) &> Var(q_t) \quad (11) \end{aligned}$$

Our approach begins with the standard pricing-kernel equation,

$$b_t^{n+1} = E_t\, m_{t+1} b_{t+1}^n \quad , \tag{12}$$

where b_t^n is the U.S. dollar (USD) price of a nominal n-period zero-coupon bond at date t and m_t is the pricing kernel for USD-denominated assets. The one-period interest rate is $i_t \equiv -\log b_t^1$. An equation analogous to (12) defines the GBP-denominated pricing kernel, m_t^*, in terms of GBP-denominated bond prices, b_t^*.

Backus, Foresi, and Telmer (2001) translate Fama's (1984) decomposition into pricing kernel language. First, assume complete markets so that the currency depreciation rate is

$$s_{t+1} - s_t = \log\left(m_{t+1}^* / m_{t+1}\right) \tag{13}$$

Fama's (1984) decomposition becomes

$$\begin{aligned} i_t - i_t^* &= \log E_t\, m_{t+1}^* - \log E_t\, m_{t+1} && (14) \\ q_t &= E_t \log m_{t+1}^* - E_t \log m_{t+1} && (15) \\ p_t &= \left(\log E_t\, m_{t+1}^* - E_t \log m_{t+1}^*\right) - \left(\log E_t\, m_{t+1} - E_t \log m_{t+1}\right) && (16) \\ &= Var_t(\log m_{t+1}^*)/2 - Var_t(\log m_{t+1})/2 \quad , && (17) \end{aligned}$$

where equation (17) is only valid for the case of conditional lognormality. Basically, Fama's (1984) conditions state that the means and the variances must move in opposite directions and that the variation in the variances must exceed that of the means.

Our objective is to write down a model in which $b < 0$. Inspection of equations (10) and (17) indicate that a necessary condition is that p_t vary over time and that, for the lognormal case, the log kernels *must* exhibit stochastic volatility.

3 Nominal Exchange Rates and Taylor Rules

Domestic (U.S.) monetary policy is described by a Taylor rule of the form

$$i_t = \tau + \tau_1 \pi_t + z_t \ \ , \tag{18}$$

where π_t is the (continuously-compounded) inflation rate and z_t is a 'policy shock'. There are many alternative specifications for Taylor rules. A good discussion related to asset pricing is Ang, Dong, and Piazzesi (2007). We begin with this relatively simple specification for reasons of tractability and clarity.

The process for z_t is a AR(1) with stochastic volatility:

$$\begin{aligned} z_t &= \varphi_z z_{t-1} + v_{t-1}^{1/2} \varepsilon_t & (19) \\ v_t &= \theta_v(1-\varphi_v) + \varphi_v v_{t-1} + \sigma_v w_t & (20) \end{aligned}$$

where ε_t and w_t are *i.i.d.* standard normal. Recall that stochastic volatility is not an option. It is a requirement. The only issue is where it comes from. Since our goal is to emphasize monetary policy we specify stochastic volatility as arising from the nominal policy rule.

Turning to asset pricing, nominal pricing kernel, $m_{t+1} = n_{t+1} \exp(-\pi_{t+1})$, is comprised of a real piece n_{t+1} and nominal piece, $\exp(-\pi_{t+1})$. The (nominal) short interest rate rate is $i_t = -\log E_t \, m_{t+1}$. As discussed in the introduction, we turn off the real component of the kernel, implying that

$$\begin{aligned} i_t &= -\log E_t \, e^{-\pi_{t+1}} & (21) \\ &= E_t \, \pi_{t+1} - \frac{1}{2} Var_t(\pi_{t+1}) \ \ , & (22) \end{aligned}$$

where, for simplicity, we've set the real interest rate to zero. The Taylor rule (18) and the Euler equation (22) imply that inflation must satisfy the following difference equation:

$$\pi_t = -\frac{1}{\tau_1}\left(\tau + z_t + E_t \, \pi_{t+1} - \frac{1}{2} Var_t(\pi_{t+1})\right) \ \ . \tag{23}$$

Guess that the solution has the form,

$$\pi_t = a + a_1 z_t + a_2 v_t \ \ . \tag{24}$$

Instead of solving equation (23) forward, just substitute equation (24) into the Euler equation (22), compute the moments, and then solve for the a_i coefficients by matching up the result with the Taylor rule (18). This gives,

$$\begin{aligned} a &= \frac{C-\tau}{\tau_1} & (25) \\ a_1 &= \frac{1}{\varphi_z - \tau_1} & (26) \\ a_2 &= \frac{1}{2(\varphi_z-\tau_1)^2(\varphi_v-\tau_1)} & (27) \end{aligned}$$

where

$$C \equiv a + a_2\theta_v(1-\varphi_v) - (a_2\sigma_v)^2/2 \tag{28}$$

Inflation and the short rate are:

$$\begin{aligned}
\pi_t &= \frac{C-\tau}{\tau_1} + \frac{1}{\varphi_z-\tau_1} z_t + \frac{1}{2(\varphi_z-\tau_1)^2(\varphi_v-\tau_1)} v_t && (29)\\
i_t &= C + \frac{\varphi_z}{\varphi_z-\tau_1}\, z_t + \frac{\tau_1}{2(\varphi_z-\tau_1)^2(\varphi_v-\tau_1)}\, v_t && (30)\\
&= C + \varphi_z a_1 z_t + \tau_1 a_2 v_t && (31)
\end{aligned}$$

The pricing kernel can now be written

$$\begin{aligned}
-\log m_{t+1} &= C + (\sigma_v a_2)^2/2 + a_1\varphi_z z_t + a_2\varphi_v v_t + a_1 v_t^{1/2}\varepsilon_{t+1} + \sigma_v a_2 w_{t+1}\\
&= D + \frac{1}{\varphi_z-\tau_1}\varphi_z z_t + \frac{\varphi_v}{2(\varphi_z-\tau_1)^2(\varphi_v-\tau_1)} v_t && (32)\\
&\quad + \frac{1}{\varphi_z-\tau_1} v_t^{1/2}\varepsilon_{t+1} + \frac{\sigma_v}{2(\varphi_z-\tau_1)^2(\varphi_v-\tau_1)} w_{t+1}
\end{aligned}$$

where

$$D \equiv C + (\sigma_v a_2)^2/2 \tag{33}$$

Now consider a foreign country, say the U.K.. Denote all foreign variables with an asterisk. The foreign Taylor rule is

$$i_t^* = \tau^* + \tau_1^*\pi_t^* + z_t^* \ .$$

with z_t^* and its volatility following processes analogous to equations (19–20) in which (for now) the shocks may be correlated across countries. Repeating the above calculations for the U.K. and then substituting the results into equations (14–17) we get

$$\begin{aligned}
i_t - i_t^* &= \varphi_z a_1 z_t - \varphi_z^* a_1^* z_t^* + \tau_1 a_2 v_t - \tau_1^* a_2^* v_t^* && (34)\\
q_t &= D - D^* + a_1\varphi_z z_t - a_1^*\varphi_z^* z_t^* + a_2\varphi_v v_t - a_2^*\varphi_v^* v_t^* && (35)\\
p_t &= -\frac{1}{2}\left(a_1^2 v_t - a_1^{*2} v_t^* + \sigma_v^2 a_2^2 - \sigma_v^{*2} a_2^{*2}\right) && (36)
\end{aligned}$$

where $D \equiv C + (\sigma_v a_2)^2/2$. It is easily verified that $p_t + q_t = i_t - i_t^*$.

Result 1: *Symmetry and* $\varphi_z = 0$

> If all foreign and domestic parameter values are the same and $\varphi_z = \varphi_z^* = 0$, then the UIP regression parameter (9) is:

$$\begin{aligned}
b = \frac{Cov(i_t - i_t^*\,,\, q_t)}{Var(i_t - i_t^*)} &= \frac{Cov(p_t+q_t, q_t)}{Var(p_t+q_t)} && (37)\\
&= \frac{\varphi_v}{\tau_1} && (38)
\end{aligned}$$

3.1 Discussion

The sign of $Cov(p_t, q_t)$ does not depend on φ_z. That is, $Cov(p_t, q_t)$ is essentially the covariance between the kernel's mean and its variance and, while v_t appears in both, z_t appears only in the mean. The assumption $\varphi_z = 0$ is therefore relatively innocuous in the sense that it has no effect on one of the two necessary conditions (10) and (11).

We require $\tau_1 > 1$ for the solution to make sense. Therefore, according to equation (38), $0 < b < 1$ unless $\varphi_v < 0$. The latter is implausible. Nevertheless, the UIP regression coefficient can be significantly less than unity and the joint distribution of exchange rates and interest rates will admit positive expected excess returns on a suitably-defined trading strategy.

We cannot, at this point, account for $b < 0$. But the model does deliver some insights into our basic question of how Taylor rules restrict inflation dynamics and, consequently, exchange rate dynamics. We summarize with several remarks.

Remark 1: *This is not just a relabeled affine model*

Inspection of the pricing kernel, equation (32), indicates that it is basically a log-linear function of two unobservable factors. Is what we are doing just a relabeling of the class of latent-factor affine models described in Backus, Foresi, and Telmer (2001)? The answer is no and the reason is that the Taylor rule imposes economically-meaningful restrictions on the model's coefficients.

To see this consider a pricing kernel of the form

$$-\log m_{t+1} \quad = \quad \alpha + \beta v_t + \gamma v_t^{1/2} \varepsilon_{t+1} \qquad (39)$$

where v_t is an arbitrary, positive stochastic process, and an analogous expression describes m^*_{t+1}. Backus, Foresi, and Telmer (2001) show that such a structure generates a UIP coefficient $b < 0$ if $\beta > 0$ and $\beta < \gamma^2/2$. The former condition implies that the mean and variance of negative the log kernel move in the same direction — this gives $Cov(p_t, q_t) < 0$ — and the latter implies that the variance is more volatile so that $Var(p_t) > Var(q_t)$.

Now compare equations (39) and (32). The Taylor rule imposes the restrictions that β can only be positive if φ_v is negative (because $a_2 < 0$ since $\tau_1 > 1$) and that $\beta = \sigma_v \varphi_v \gamma$. Moreover, both β and γ are restricted by value of the policy parameter τ_1. In words, the UIP evidence requires the mean and the variance of the pricing kernel to move in particular ways relative to each other. The Taylor rule and its implied inflation dynamics place binding restrictions on how this can happen. The unrestricted pricing kernel in equation (39) can account for $b < 0$ irrespective of the dynamics of v_t. Imposing the Taylor rule says that v_t must be negatively autocorrelated.

Remark 2: *Reason that negatively-correlated volatility is necessary for $b < 0$?*

First, note that $a_2 < 0$, so that an increase in volatility v_t decreases inflation π_t. Why? Suppose not. Suppose that v_t increases. Then, since $\tau_1 > 1$, the Taylor rule implies that the interest rate i_t must increase *by more* than inflation π_t. However this contradicts the stationarity of inflation which implies that the conditional mean must increase *by less* than the contemporaneous value. Hence $a_2 < 0$. A similar argument implies that $a_1 < 0$ from equation (24). The point is that the dynamics of Taylor-rule implied inflation, at least until we get the real interest rate involved in Section 4, are driven by the *muted response of the interest rate* to a shock, relative to that of the inflation rate.

Next, to understand why $\varphi_v < 0$ is necessary for $b < 0$, consider again an increase in volatility v_t. Since $a_2 < 0$, the U.S. interest rate i_t and the contemporaneous inflation rate π_t must decline. But for $b < 0$ USD must be expected to *depreciate*. This means that, although π_t decreases, $E_t \pi_{t+1}$ must increase. This means that volatility must be negatively autocorrelated.

Finally, consider the more plausible case of positively autocorrelated volatility, $0 < \varphi_v < 1$. Then $b < 1$ which is, at least, going in the right direction (*e.g.*, Backus, Foresi, and Telmer (2001) show that the vanilla Cox-Ingersoll-Ross model generates $b > 1$). The reasoning, again, derives from the 'muted response of the interest rate' behavior required by the Taylor rule. This implies that $Cov(p_t, q_t) > 0$ — thus violating Fama's condition (10) — which says that if inflation and expected inflation move in the same direction as the interest rate (because $\varphi_v > 0$), then so must the USD currency risk premium. The regression (8) can be written

$$q_t \quad = \quad c + b(p_t + q_t) - \text{forecast error} \quad ,$$

where 'forecast error' is defined as $s_{t+1} - s_t - q_t$. Since $Cov(p_t, q_t) > 0$, then $Var(p_t + q_t) > Var(q_t)$ and, therefore, $0 < b < 1$.

Even more starkly, consider the case of $\varphi_v = 0$ so that $b = 0$. Then the exchange rate is a random walk — *i.e.*, $q_t = 0$ so that $s_t = E_t s_{t+1}$ — and all variation in the interest rate differential is variation in the risk premium, p_t. Taylor rule inflation dynamics, therefore, say that for UIP to be a good approximation, changes in volatility must show up strongly in the conditional mean of inflation and that this can only happen if volatility is highly autocorrelated.

Remark 3: *Identification of policy parameters*

Cochrane (2007) provides examples where policy parameters like τ_1 are impossible to distinguish from the parameters of the unobservable shocks. Result 1 bears similarity to Cochrane's simplest example. We can estimate b from data but, if we can't estimate φ_v directly then there are many combinations of φ_v and τ_1 that are consistent with any estimate of b.

Identification in our special case, however, is possible because of the conditional variance term in the interest rate equation: $i_t = E_t\pi_{t+1} - Var_t\pi_{t+1}$. To see this note that, with $\varphi_z = 0$, the autocorrelation of the interest rate is φ_v and, therefore, φ_v is identified by observables. Moreover,

$$\frac{i_t}{E_t\pi_{t+1}} = \frac{\tau_1}{\varphi_v} \quad ,$$

which identifies τ_1 because the variables on the left side are observable.

The more general case of $\varphi_z \neq 0$ doesn't work out as cleanly, but it appears that the autocorrelation of inflation and the interest rate jointly identify φ_z and φ_v and the above ratio again identifies the policy parameter τ_1. These results are all special cases of those described in Backus and Zin (2008).

3.2 Asymmetric Taylor Rules

The series of affine models outlined in Backus, Foresi, and Telmer (2001) suggest that asymmetries between the foreign and domestic pricing kernels are likely to play a critical role in achieving $b < 0$. Their approach is purely statistical in nature. There are many parameters and few sources of guidance for which asymmetries are plausible and which are not. This section asks if foreign and domestic Taylor rule asymmetries are plausible candidates.

Suppose that foreign and domestic Taylor rules depend on the exchange rate in addition to domestic inflation and a policy shock:

$$\begin{aligned} i_t &= \tau + \tau_1\pi_t + z_t + \tau_3 d_t && (40) \\ i_t^* &= \tau^* + \tau_1^*\pi_t^* + z_t^* + \tau_3^* d_t && (41) \end{aligned}$$

where d_t is the contemporaneous USD depreciation rate, $d_t = \log(S_t/S_{t-1})$. The asymmetry that we'll impose is that $\tau_3 = 0$ so that the Fed does not react to the depreciation rate whereas the Bank of England does. Foreign central banks reacting more to USD exchange rates seems plausible. It's also consistent with some empirical evidence in, for example, Clarida, Galí, and Gertler (1999), Engel and West (2006), and Eichenbaum and Evans (1995).

Assuming the same processes for the state variables as equations (19) and (20) (and their foreign counterparts), guess that the inflation solutions look like:

$$\begin{aligned} \pi_t &= a + a_1 z_t + a_2 z_t^* + a_3 v_t + a_4 v_t^* \equiv a + A^\top X_t \\ \pi_t^* &= a^* + a_1^* z_t + a_2^* z_t^* + a_3^* v_t + a_4^* v_t^* \equiv a^* + A^{*\top} X_t \end{aligned}$$

and collect the state variables into the vector

$$X_t^\top \equiv \left[z_t \; z_t^* \; v_t \; v_t^*\right]^\top \; .$$

Interest rates, from Euler equations with real interest rate = 0, must satisfy:

$$\begin{aligned} i_t &= C + B^\top X_t \\ i_t^* &= C^* + B^{*\top} X_t \end{aligned}$$

where,

$$\begin{aligned}
B^\top &\equiv \left[\; a_1\varphi_z \quad a_2\varphi_z^* \quad (a_3\varphi_v - \frac{a_1^2}{2}) \quad (a_4\varphi_v^* - \frac{a_2^2}{2}) \; \right] \\
C &\equiv a + a_3\theta_v(1-\varphi_v) + a_4\theta_v^*(1-\varphi_v^*) - \frac{1}{2}\left(a_3^2\sigma_v^2 + a_4^2\sigma_v^{*2}\right) \\
B^{*\top} &\equiv \left[\; a_1^*\varphi_z \quad a_2^*\varphi_z^* \quad (a_3^*\varphi_v - \frac{a_1^{*2}}{2}) \quad (a_4^*\varphi_v^* - \frac{a_2^{*2}}{2}) \; \right] \\
C^* &\equiv a^* + a_3^*\theta_v(1-\varphi_v) + a_4^*\theta_v^*(1-\varphi_v^*) - \frac{1}{2}\left(a_3^{*2}\sigma_v^2 + a_4^{*2}\sigma_v^{*2}\right)
\end{aligned}$$

The solution for the a coefficients and the following result are provided in Appendix B.

Result 2: *Asymmetric reaction to exchange rates*

> If foreign and domestic Taylor rules are equations (40) and (41), with $\tau_3 = 0$ and all remaining foreign and domestic parameter values the same, then $b < 0$ if $\tau_3^* > \tau_1$.

Remark 4: *Pathological policy behavior?*

Interpreted literally, $\tau_3^* > 0$ means that the Bank of England reacts to an *appreciation* in GBP by increasing the British interest rate. However, at the same time, there exist sensible calibrations of the model in which $Cov(i_t^*, \log(S_t/S_{t-1})) > 0$. This makes the obvious point that the Taylor rule coefficients must be interpreted with caution since all the endogenous variables in the rule are responding to the same shocks.

3.3 McCallum's Model

McCallum (1994), equation (17), posits a policy rule of the form

$$i_t - i_t^* \;=\; \lambda\big(s_t - s_{t-1}\big) + \sigma\big(i_{t-1} - i_{t-1}^*\big) + \zeta_t \ , \tag{42}$$

where ζ_t is a policy shock. He also defines UIP to include an exogenous shock, ξ_t, so that

$$i_t - i_t^* = E_t\big(s_{t+1} - s_t\big) + \xi_t \ . \tag{43}$$

McCallum solves the implicit difference equation for $s_t - s_{t-1}$ and finds that it takes the form

$$s_t - s_{t-1} \;=\; -\sigma/\lambda\big(i_t - i_{t-1}\big) - \lambda^{-1}\zeta_t + \big(\lambda + \sigma\big)^{-1}\xi_t \tag{44}$$

He specifies values $\sigma = 0.8$ and $\lambda = 0.2$ — justified by the policy-makers desire to smooth interest rates and 'lean-into-the-wind' regarding exchange rates — which resolve the UIP puzzle by implying a regression coefficient from our equation (8) of $b = -4$. McCallum's insight was, recognizing the empirical evidence of a risk premium in the interest rate differential, to understand that the policy rule and the equilibrium exchange rate must respond to the same shock that drives the risk premium.

In this section we show that McCallum's result can be recast in terms of our pricing kernel model and a policy rule that targets the interest rate itself, not the interest rate differential. The key ingredient is a lagged interest rate in the policy rule:

$$i_t \;=\; \tau + \tau_1\pi_t + \tau_4 i_{t-1} + z_t \ , \tag{45}$$

where the processes for z_t and its volatility v_t are the same as above. Guess that the solution for endogenous inflation is:

$$\pi_t = a + a_1 z_t + a_2 v_t + a_i i_{t-1} \ , \tag{46}$$

Substitute equation (46) into the pricing kernel and compute the expectation:

$$i_t = \frac{1}{1 - a_i}\Big(C + a_1\varphi_z z_t + (a_2\varphi_v - a_1^2/2)v_t\Big) \ , \tag{47}$$

where

$$C \equiv a + a_2\theta_v(1 - \varphi_v) - (a_2\sigma_v)^2/2 \tag{48}$$

Match-up the coefficients with the Taylor rule and solve for the a_j parameters:

$$\begin{aligned} a &= \frac{C}{\tau_1+\tau_4} - \frac{\tau}{\tau_1} && (49)\\ a_1 &= \frac{\tau_1+\tau_4}{\tau_1(\varphi_z-\tau_1-\tau_4)} && (50)\\ a_2 &= \frac{(\tau_1+\tau_4)^2}{2\tau_1^2(\varphi_z-\tau_1-\tau_4)^2(\varphi_v-\tau_1-\tau_4)} && (51)\\ a_i &= -\frac{\tau_4}{\tau_1} && (52) \end{aligned}$$

It's useful to note that

$$a_2 = \frac{a_1^2}{2(\varphi_v-\tau_1-\tau_4)} \tag{53}$$

Inflation and the short rate are:

$$\begin{aligned} \pi_t &= \frac{C}{\tau_1+\tau_4} - \frac{\tau}{\tau_1} + \frac{\tau_1+\tau_4}{\tau_1(\varphi_z-\tau_1-\tau_4)} z_t + && (54)\\ &+ \frac{(\tau_1+\tau_4)^2}{2\tau_1^2(\varphi_z-\tau_1-\tau_4)^2(\varphi_v-\tau_1-\tau_4)} v_t - \frac{\tau_4}{\tau_1} i_{t-1} && (55)\\ i_t &= \frac{\tau_1}{\tau_1+\tau_4} C + \frac{\varphi_z}{\varphi_z-\tau_1-\tau_4} z_t + \frac{(\tau_1+\tau_4)^2}{2\tau_1(\varphi_z-\tau_1-\tau_4)^2(\varphi_v-\tau_1-\tau_4)} v_t && (56)\\ &= \frac{1}{1-a_i}\Big(C + \varphi_z a_1 z_t + (\tau_1+\tau_4) a_2 v_t\Big) && (57) \end{aligned}$$

The pricing kernel is

$$-\log m_{t+1} = D + \frac{a_1\varphi_z}{1-a_i} z_t + \frac{a_2\varphi_v - a_i a_1^2/2}{1-a_i} v_t + a_1 v_t^{1/2}\varepsilon_{t+1} + \sigma_v a_2 w_{t+1} \tag{58}$$

where

$$D \equiv \frac{C}{1-a_i} + (\sigma_v a_2)^2/2 \tag{59}$$

The GBP-denominated kernel and variables are denoted with asterisks. If we assume that all foreign and domestic parameter values are the same (*i.e.*, $\tau = \tau^*$), the interest-rate differential, the expected depreciation rate, q_t, and the risk premium, p_t, are:

$$\begin{aligned} i_t - i_t^* &= (1+\tau_1 a_1)(z_t - z_t^*) + \tau_1 a_2 (v_t - v_t^*) && (60)\\ q_t &= (a_1\varphi_z + a_3 1 + \tau_1 a_1)(z_t - z_t^*) + (a_2\varphi_v + \tau_1 a_2 a_3)(v_t - v_t^*) && (61)\\ p_t &= -\frac{1}{2} a_1^2 (v_t - v_t^*) && (62) \end{aligned}$$

It is easily verified that $p_t + q_t = i_t - i_t^*$.

If we set $\varphi_z = 0$, then the regression parameter is:

$$\begin{aligned} b &= \frac{Cov(i_t - i_t^*\,,\, q_t)}{Var(i_t - i_t^*)} \\ &= \frac{\varphi_v - \tau_4}{\tau_1} \end{aligned}$$

To see the similarity to McCallum's model define $\zeta \equiv z_t - z_t^*$, and subtract the U.K. Taylor rule from its U.S. counterpart in (45). Assuming symmetry, we get

$$\begin{aligned} i_t - i_t^* &= \tau_1(\pi_t - \pi_t^*) + \tau_4\big(i_t - i_t^*\big) + \zeta_t && (63) \\ &= \tau_1(s_t - s_{t-1}) + \tau_4\big(i_t - i_t^*\big) + \zeta_t\,, && (64) \end{aligned}$$

where the second equality follows from market completeness and our simple pricing kernel model. This is the same as McCallum's policy rule with $\tau_1 = \lambda$ and $\tau_4 = \sigma$. His UIP "shock" is the same as our $p_t = -a_1^2(v_t - v_t^*)/2$, with $\varphi_z = \varphi_v = 0$. With $\varphi_v = 0$ we get the same UIP regression coefficient, $-\tau_4/\tau_1$. McCallum's model is basically a two-country Taylor rule model with a lagged interest rate in the policy rule and no dynamics in the shocks. Allowing for autocorrelated volatility diminishes the model's ability to account for a substantially negative UIP coefficient, a feature that McCallum's approach does not recognize. A value of $b < 0$ can only be achieved if volatility is less autocorrelated that the value of the interest rate smoothing policy parameter.

3.4 Summary

The goal of this section has been to ascertain how the imposition of a monetary policy, interest rate rule restricts inflation dynamics and how these restrictions are manifest in the exchange rate. What have we learned?

A good context for understanding the answer is the Alvarez, Atkeson, and Kehoe (2008) (AAK) paper. The nuts and bolts of their argument goes as follows. With lognormality, the nominal interest is

$$i_t = -E_t\big(\log m_{t+1}\big) - Var_t\big(\log m_{t+1}\big)/2$$

AAK argue that if exchange rates follow a random walk then variation in the conditional mean term must be small.[3] Therefore (according to them), "almost

---

[3] *i.e.*, random walk exchange rates mean that $E_t \log(S_{t+1}/S_t) = 0$, and, from equation (15), $E_t \log(S_{t+1}/S_t) = -E_t(\log m_{t+1} - \log m_{t+1}^*)$. Random walk exchange rates, therefore, imply that the *difference* between the mean of the log kernels does not vary, not the mean of the log kernels themselves. More on this below.

everything we say about monetary policy is wrong." The idea is that, in many existing models, the monetary policy transmission mechanism works through its affect on the conditional mean of the nominal marginal rate of substitution, m_t. But if exchange rates imply that the conditional mean is essentially a constant — so that 'everything we say is wrong' — then the mechanism must instead be working through the conditional variance.

If one takes the UIP evidence seriously, this isn't quite right. The UIP puzzle *requires* variation in the conditional means (*i.e.*, it says that exchange rates *are not* a random walk).[4] Moreover, it also requires that this variation be negatively correlated with variation in the conditional variances, and that the latter be larger than the former. In terms of monetary policy the message is that the standard story — that a shock that increases the mean (of the marginal rate of substitution) *decreases* the interest rate — is wrong. The UIP evidence says that we need to get used to thinking about a shock that increases the mean as *increasing* the interest rate, the reason being that the same shock must *decrease* the variance, and by more than it increases the mean.

Now, to what we've learned. We've learned that symmetric monetary policies as represented by Taylor rules of the form (18) can't deliver inflation dynamics that, by themselves, satisfy these requirements. The reason is basically what we label the 'muted response of the short rate'. The evidence requires that the conditional mean of inflation move by more than its contemporaneous value. But the one clear restriction imposed by the Taylor rule — that the interest rate must move *less* than contemporaneous inflation because the interest rate must also be equal to the conditional mean future inflation — says that this can't happen (unless volatility is negatively autocorrelated). This depends heavily on the real interest rate being a constant, something we relax in the next section.

So, is the model doomed? The second thing that we've learned is that, no, it isn't. The reason is related to something else that AAK don't quite get right. Exchange rate behavior tells us something about the *difference* between the domestic and foreign pricing kernels, not necessarily something about their *levels*. The above logic, and AAK's logic, is about levels, not differences. Symmetry makes the distinction irrelevant, but with asymmetry it's important. What our asymmetric example delivers is (i) inflation dynamics that, in each currency, satisfies 'muted response of the short rate' behavior, and (ii) a *difference* in inflation dynamics that gets the *difference* in the mean and the variance of the kernels moving in the right direction.

---

[4] Of course, the variation in the forecast error for exchange rates dwarfs the variation in the conditional mean (*i.e.*, the R^2 from the Fama-regressions is very small). Monthly changes in exchange rates certainly exhibit 'near random walk' behavior, and for policy questions the distinction may be a second-order effect. This argument, however, does not affect our main point regarding the AAK paper: that exchange rates are all about *differences* between pricing kernels and its hard to draw definitive conclusions about their *levels*.

To see this, recall that $X_t^\top \equiv \begin{bmatrix} z_t & z_t^* & v_t & v_t^* \end{bmatrix}^\top$ and consider the foreign and domestic pricing kernels in the asymmetric model:

$$\begin{aligned} -\log m_{t+1} &= \text{constants} + a_1 \varphi_z z_t + a_3 \varphi_v v_t + a_1 v_t^{1/2} \varepsilon_{t+1} + a_3 \sigma_v w_{t+1} \\ -\log m_{t+1}^* &= \text{constants} + A^\top \Lambda X_t + V(X_t)^{1/2} \begin{bmatrix} \varepsilon_{t+1} & \varepsilon_{t+1}^* & w_{t+1} & w_{t+1}^* \end{bmatrix}^\top \end{aligned}$$

where Λ is a diagonal matrix of autoregressive coefficients, and $V(X_t)$ is a diagonal matrix of conditional standard deviations. The asymmetric restriction that $\tau_3 = 0$ and $\tau_3^* \neq 0$ effectively makes this a 'common factor model' with asymmetric loadings on the common factors. A number of recent papers, Lustig, Roussanov, and Verdelhan (2009) for example, have argued persuasively for such a specification. What we've developed is one economic interpretation of their statistical exercise.[5]

More explicitly, consider the *difference* in the mean and variance of the log kernels from the symmetric and asymmetric examples of Sections 3 and 3.2. For the symmetric case we have

$$\begin{aligned} p_t &= -\frac{1}{2} a_1^2 \left(v_t - v_t^*\right) \\ q_t &= a_2 \varphi_v \left(v_t - v_t^*\right) \end{aligned}$$

whereas for the asymmetric case we have

$$\begin{aligned} p_t &= -\frac{1}{2}\left(a_1^2 - a_1^{*2}\right) v_t + \frac{1}{2} a_4^* v_t^* \\ q_t &= \varphi_v \left(a_3 - a_3^*\right) v_t - a_4^* v_t^* \end{aligned}$$

where the a coefficients are functions of the model's parameters, outlined above and in more detail in the appendix. What's going on in the symmetric case is transparent. p_t and q_t can only be negatively correlated if $\varphi_v < 0$ (since $a_2 < 0$). The asymmetric case is more complex, but it turns out that what's critical is that $(a_3 - a_3^*) < 0$. This in turn depends on the difference $(\tau_1 - \tau_3^*)$ being negative. Overall, what the asymmetric Taylor rule does is that it introduces an asymmetry in how a common factor between m and m^* affect their conditional means. This asymmetry causes the common factor to show up in exchange rates, and it can also flip the sign and deliver $b < 0$ with the right combination of parameter values.

4 Real Exchange Rates and Taylor Rules

We now incorporate real exchange rate variability and an interaction between real exchange rates and endogenous inflation. There are no nominal frictions in the

---

[5] Note that if the conditional mean coefficients on z_t and v_t were the same across m and m^* then, contrary to AAK's assertion, monetary policy *could* affect the mean of the pricing kernel while still allowing for a random walk exchange rate. This is simply because z_t and v_t *would not appear* in the difference between the means of the two log kernels.

model and thus monetary policy has no impact on real variables. The model features real shocks only. These shocks directly affect the real pricing kernel and, through the Taylor rules, the inflation process and the depreciation rate.

Domestic and a foreign representative agents have Epstein and Zin (1989) (EZ) preferences described by the solution to the following recursive equations:

$$U_t = [(1-\beta)c_t^{\rho} + \beta\mu_t(U_{t+1})^{\rho}]^{1/\rho}$$

and

$$U_t^* = [(1-\beta^*)c_t^{*\rho^*} + \beta^*\mu_t^*(U_{t+1}^*)^{\rho^*}]^{1/\rho^*},$$

where β and β^* characterize impatience, ρ and ρ^* measure the preference for intertemporal substitution, and the certainty equivalents of random future utility are specified as

$$\mu_t(U_{t+1}) \equiv E_t[U_{t+1}^{\alpha}]^{1/\alpha}$$

and

$$\mu_t^*(U_{t+1}^*) \equiv E_t[U_{t+1}^{*\alpha^*}]^{1/\alpha^*},$$

where α and α^* measure static relative risk aversion (RRA). Both α and ρ are defined for values not greater than one. The relative magnitude of α and ρ determines whether agents prefer early resolution of risk ($\alpha < \rho$), late resolution of risk ($\alpha > \rho$), or are indifferent to the timing of resolution of risk ($\alpha = \rho$). The domestic marginal rate of intertemporal substitution is

$$n_{t+1} = \beta\left(\frac{c_{t+1}}{c_t}\right)^{\rho-1}\left(\frac{U_{t+1}}{\mu_t(U_{t+1})}\right)^{\alpha-\rho}. \tag{65}$$

An equivalent expression can be obtained for the foreign representative agent. Standard time and state separable utility corresponds to the case in which $\alpha = \rho$. The nominal pricing kernel from equation (12) now takes the form

$$m_{t+1} \quad = \quad n_{t+1}e^{-\pi_{t+1}} \quad , \tag{66}$$

and, with complete markets, the nominal depreciation rate is defined by $\log(s_{t+1}/s_t) = \log(m_{t+1}^*/m_{t+1})$. We are silent on the risk-sharing mechanism that supports the consumption allocations inherent in n_{t+1} and n_{t+1}^*. We simply exploit the fact that, if domestic and foreign representative agents have recursive preferences and consumption processes that match the data then, with complete markets, the ratios of nominal and real pricing kernels will equal nominal and real depreciation rates. Bansal and Shaliastovich (2008), Colacito and Croce (2008), Gavazzoni (2008) and others follow a similar approach.

We next proceed by further specifying n_t, followed by a Taylor rule and the solution for π_t.

4.1 Consumption Growth and the Real Pricing Kernel

We assume that we observe the equilibrium consumption allocations in each country and specify the dynamics of their growth rates as heteroskedastic processes with both country-specific and world shocks. Domestic and foreign consumption growth evolve according to

$$\begin{aligned} x_{t+1} &= (1-\varphi_x)\theta_x + \varphi_x x_t + \varsigma_w w_t^{1/2}\epsilon^x_{t+1} + \varsigma_v v_t^{1/2}\eta^x_{t+1}, \\ x^*_{t+1} &= (1-\varphi^*_x)\theta^*_x + \varphi^*_x x^*_t + \varsigma^*_w w_t^{1/2}\epsilon^{x^*}_{t+1} + \varsigma^*_v v_t^{*1/2}\eta^{x^*}_{t+1}, \end{aligned}$$

where the process for the world volatility w evolves according to

$$w_{t+1} = (1-\varphi_w)\theta_w + \varphi_w w_t + \sigma_w \epsilon^w_{t+1},$$

and the country-specific stochastic volatility processes v and v^* are

$$v_{t+1} = (1-\varphi_v)\theta_v + \varphi_v v_t + \sigma_v \epsilon^v_{t+1},$$

$$v^*_{t+1} = (1-\varphi^*_v)\theta^*_v + \varphi^*_v v^*_t + \sigma^*_v \epsilon^{v^*}_{t+1}.$$

The innovations are multivariate normal:

$$\begin{bmatrix} \epsilon^x_t \\ \epsilon^{x^*}_t \\ \eta^x_t \\ \eta^{x^*}_t \\ \epsilon^v_t \\ \epsilon^{v^*}_t \\ \epsilon^w_t \end{bmatrix} \sim N\left(\mathbf{0} \; ; \begin{bmatrix} 1 & & & & & & \\ \chi_{\epsilon^x} & 1 & & & & & \\ 0 & 0 & 1 & & & & \\ 0 & 0 & 0 & 1 & & & \\ 0 & 0 & 0 & 0 & 1 & & \\ 0 & 0 & 0 & 0 & \chi_{\epsilon^v} & 1 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \right)$$

Following Hansen, Heaton, and Li (2005), we linearize the logarithm of the real pricing kernel around zero. The result is

$$\begin{aligned} -\log n_{t+1} = \delta^r + \gamma^r_x x_t + \gamma^r_w w_t + \gamma^r_v v_t + \lambda^r_{x,w} w_t^{1/2}\epsilon^x_{t+1} + \lambda^r_{x,v} v_t^{1/2}\eta^x_{t+1} \\ + \lambda^r_w \sigma_w \epsilon^w_{t+1} + \lambda^r_v \sigma_v \epsilon^v_{t+1}, \end{aligned} \tag{67}$$

where

$$\delta^r = -\log\beta + (1-\rho)(1-\varphi_x)\theta_x + \frac{\alpha}{2}(\alpha-\rho)(\xi_w^2\sigma_w^2 + \xi_v^2\sigma_v^2)$$

$$\gamma^r_x = (1-\rho)\varphi_x; \quad \gamma^r_w = \tfrac{\alpha}{2}(\alpha-\rho)(\xi_x+1)^2\varsigma_w^2; \quad \gamma^r_v = \gamma^r_w\left(\frac{\varsigma_v}{\varsigma_w}\right)^2 \tag{68}$$

$$\begin{aligned} \lambda^r_{x,w} &= [(1-\alpha)-(\alpha-\rho)\xi_x]\varsigma_w; \quad & \lambda^r_{x,v} &= [(1-\alpha)-(\alpha-\rho)\xi_x]\varsigma_v \\ \lambda^r_w &= -(\alpha-\rho)\xi_w; & \lambda^r_v &= -(\alpha-\rho)\xi_v \end{aligned}$$

$$\xi_x = \left(\frac{\kappa}{1-\kappa\varphi_x}\right)\varphi_x; \quad \xi_w = \left(\tfrac{\kappa}{1-\kappa\varphi_w}\right)\left[\tfrac{\alpha}{2}\left(\tfrac{1}{1-\kappa\varphi_x}\right)^2 \varsigma_w^2\right]$$

$$\xi_v = \left(\frac{\kappa}{1-\kappa\varphi_v}\right)\left[\frac{\alpha}{2}\left(\frac{1}{1-\kappa\varphi_x}\right)^2 \varsigma_v^2\right]$$

where κ is a linearization coefficient. It's useful to note that the factor loadings and prices of risk are linked according to:

$$\frac{\gamma_w^r}{\gamma_v^r} = \left(\frac{\lambda_w^r}{\lambda_v^r}\right)^2 \quad ; \frac{\lambda_w^r}{\lambda_v^r} = \left(\frac{\xi_w}{\xi_v}\right)$$

Details for the derivation can be found in Appendix C. Following the affine term structure literature, we refer to $\gamma^r = [\gamma_x^r \ \gamma_w^r \ \gamma_v^r]'$ as real factor loadings and to $\lambda^r = [\lambda_{x,w}^r \ \lambda_{x,v}^r \ \lambda_w^r \ \lambda_v^r]'$ as real prices of risk.

The conditional mean of the real pricing kernel is equal to

$$E_t \log n_{t+1} = -\delta^r - \gamma_x^r x_t - \gamma_w^r w_t - \gamma_v^r v_t \tag{69}$$

and its conditional variance is

$$Var_t \log n_{t+1} = \left(\lambda_w^r \sigma_w\right)^2 + \left(\lambda_v^r \sigma_v\right)^2 + \left(\lambda_{x,w}^r\right)^2 w_t + \left(\lambda_{x,v}^r\right)^2 v_t \tag{70}$$

The conditional mean depends both on consumption growth and stochastic volatility, whereas the conditional variance is a linear function of current stochastic volatility only.

The real short rate is

$$\begin{aligned} r_t &\equiv -\log E_t(n_{t+1}) \\ &= \bar{r} + \gamma_x^r x_t + (\gamma_w^r - (\lambda_{x,w}^r)^2/2) w_t + (\gamma_v^r - (\lambda_{x,v}^r)^2/2) v_t \end{aligned}$$

where

$$\bar{r} = \delta^r - \frac{1}{2}\left((\lambda_w^r \sigma_w)^2 + (\lambda_v^r \sigma_v)^2\right).$$

Assuming symmetry, the expression for the expected real depreciation q_t^r, the forward premium $f_t^r - s_t^r$ and the risk premium p_t^r are:[6]

$$q_t^r = \gamma_x^r (x_t - x_t^*) + \gamma_v^r (v_t - v_t^*).$$

$$f_t^r - s_t^r = \gamma_x^r (x_t - x_t^*) + (\gamma_v^r - (\lambda_{x,v}^r)^2/2)(v_t - v_t^*),$$

$$p_t^r = -\frac{1}{2}(\lambda_{x,v}^r)^2 (v_t - v_t^*).$$

---

[6] Symmetry means that *all* the parameters are the same across countries, including $\varsigma_w = \varsigma_w^*$ and $\varsigma_v = \varsigma_v^*$, i.e. symmetry in the sensitivity of consumption growth to the world and country specific volatility. Similarly to the previous section, the model can be extended to allow for asymmetric loadings and asymmetric state variables.

Result 3: *Symmetry*

> If all foreign and domestic parameter values are the same, then the real UIP regression parameter, obtained by the regressing the real interest rate differential on the real depreciation rate is:
>
> $$\begin{aligned} b^r &= \frac{Cov(f_t^r - s_t^r, q_t^r)}{Var(f_t^r - s_t^r)} \\ &= \frac{(\gamma_x^r)^2\, Var(x_t - x_t^*) + \gamma_v^r(\gamma_v^r - (\lambda_{x,v}^r)^2/2)\, Var(v_t - v_t^*)}{(\gamma_x^r)^2\, Var(x_t - x_t^*) + (\gamma_v^r - (\lambda_{x,v}^r)^2/2)^2\, Var(v_t - v_t^*)} \\ &= 1 + \frac{(\gamma_v^r - (\lambda_{x,v}^r)^2/2)}{2}\left(\frac{(\lambda_{x,v}^r)^2}{(\gamma_v^r - (\lambda_{x,v}^r)^2/2)^2 + (\gamma_x^r)^2 R_{dxdv}}\right), \end{aligned}$$
>
> where $s_t^r = \log S_t^r$ and $f_t^r = \log F_t^r$ are the log real spot and forward exchange rate, respectively, and $R_{dxdv} = \frac{Var(x_t - x_t^*)}{Var(v_t - v_t^*)}$. Without the presence of both stochastic volatility in consumption growth and EZ preferences, b^r is equal to one and, in real terms, UIP holds identically.

The last expression for b^r makes it clear that for $\gamma_v^r - (\lambda_{x,v}^r)^2/2 < 0$, we get $b^r < 1$ and a negative UIP slope is possible whenever $|\gamma_v^r - (\lambda_{x,v}^r)^2/2|$ is large enough. Also, when consumption is not autocorrelated ($\varphi_x = 0$), the expression above simplifies to

$$b^r = \frac{\gamma_v^r}{\gamma_v^r - (\lambda_{x,v}^r)^2/2}.$$

4.2 Taylor Rule and Endogenous Inflation

Domestic monetary policy is described by a Taylor rule in which the short interest rate depends on contemporaneous inflation and output growth:

$$i_t = \bar{\tau} + \tau_\pi \pi_t + \tau_x x_t. \tag{71}$$

There is no policy shock. Given the level of output and inflation, the interest rate is uniquely pinned down by the choice of the Taylor rule parameters.

Following the technique developed in the previous section, guess that the solution for endogenous inflation has the form

$$\pi_t = a + a_1 x_t + a_2 v_t + a_3 w_t \tag{72}$$

and substitute it into the Euler equation (2), compute the moments, and then solve for the a_j coefficients by matching up the result with the Taylor rule (71). This gives,

$$a_1 = \frac{\gamma_x^r - \tau_x}{\tau_\pi - \varphi_x}; \quad a_2 = \frac{\gamma_v^r - \lambda_{x,v}^2/2}{\tau_\pi - \varphi_v}; \quad a_3 = \frac{\gamma_w^r - \lambda_{x,w}^2/2}{\tau_\pi - \varphi_w}$$

$$a = \frac{1}{\tau_\pi - 1}(\delta^r - \bar{\tau} + a_1(1-\varphi_x)\theta_x + a_2(1-\varphi_v)\theta_v + a_3(1-\varphi_w)\theta_w - (\lambda_v\sigma_v)^2/2 - (\lambda_w\sigma_w)^2/2)$$

where

$$\delta = \delta^r + a + a_1(1-\varphi_x)\theta_x + a_2(1-\varphi_v)\theta_v + a_3(1-\varphi_w)\theta_w$$

$$\gamma_x = \gamma_x^r + a_1\varphi_x; \quad \gamma_v = \gamma_v^r + a_2\varphi_v; \quad \gamma_w = \gamma_w^r + a_3\varphi_w;$$

$$\begin{aligned} \lambda_{x,v} = \lambda_{x,v}^r + a_1\varsigma_v; &\quad \lambda_{x,w} = \lambda_{x,w}^r + a_1\varsigma_w; \\ \lambda_v = \lambda_v^r + a_2; &\quad \lambda_w = \lambda_w^r + a_3; \end{aligned}$$

The linearized nominal pricing kernel is

$$\begin{aligned} -\log m_{t+1} &= -\log n_{t+1} + \pi_{t+1} \\ &= \delta + \gamma_x x_t + \gamma_w w_t + \gamma_v v_t + \lambda_{x,w} w_t^{1/2}\epsilon_{t+1}^x + \lambda_{x,v} v_t^{1/2}\eta_{t+1}^x + \\ &+ \lambda_w\sigma_w\epsilon_{t+1}^w + \lambda_v\sigma_v\epsilon_{t+1}^v, \end{aligned}$$

The Taylor rule parameters, through their determination of the equilibrium inflation process, affect both the factor loadings on the real factors as well as their prices of risk. This would not be the case if the inflation process was exogenously specified.

The nominal short rate is

$$\begin{aligned} i_t &\equiv -\log E_t(m_{t+1}) \\ &= \bar{\iota} + \gamma_x x_t + (\gamma_v - \lambda_{x,v}^2/2)v_t + (\gamma_w - \lambda_{x,w}^2/2)w_t \end{aligned}$$

where

$$\bar{\iota} = \delta - \frac{1}{2}\left((\lambda_v\sigma_v)^2 + (\lambda_w\sigma_w)^2\right).$$

The nominal interest rate differential, the expected depreciation rate and the risk premium can be easily derived from equations (14–17). Assuming symmetry across countries, we have

$$q_t = \gamma_x(x_t - x_t^*) + \gamma_v(v_t - v_t^*),$$

$$f_t - s_t = \gamma_x(x_t - x_t^*) + (\gamma_v - \lambda_{x,v}^2/2)(v_t - v_t^*),$$

$$p_t = -\frac{1}{2}(\lambda_{x,v})^2(v_t - v_t^*).$$

Result 4:

If all foreign and domestic parameter values are the same, the nominal UIP slope coefficient is

$$\begin{aligned} b &= \frac{Cov(f_t - s_t, q_t)}{Var(f_t - s_t)} \\ &= \frac{\gamma_x^2\, Var(x_t - x_t^*) + \gamma_v(\gamma_v - \lambda_{x,v}^2/2)\, Var(v_t - v_t^*)}{\gamma_x^2\, Var(x_t - x_t^*) + (\gamma_v - \lambda_{x,v}^2/2)^2\, Var(v_t - v_t^*)} \\ &= 1 + \frac{(\gamma_v - \lambda_{x,v}^2/2)}{2} \left(\frac{\lambda_{x,v}^2}{(\gamma_v - \lambda_{x,v}^2/2)^2 + \gamma_x^2 R_{dxdv}} \right). \end{aligned}$$

As was the case for the real UIP slope coefficient, for $\gamma_v - (\lambda_{x,v})^2/2 < 0$, we get $b < 1$ and a negative UIP slope is possible whenever $|\gamma_v - (\lambda_{x,v})^2/2|$ is large enough. Also, when consumption is not autocorrelated ($\varphi_x = 0$), the expression above simplifies to

$$b = \frac{\gamma_v}{\gamma_v - (\lambda_{x,v})^2/2}.$$

4.3 Discussion

The results obtained in this section rely crucially on three ingredients: EZ preferences, stochastic volatility and the choice of the Taylor rule parameters. We analyze their impact on the UIP slope coefficient and risk premium here below.

Remark 5: *With EZ preferences, volatility is priced as a separate source of risk*

From the previous section, we learned that if we want to explain the UIP puzzle we need stochastic volatility. In the model with real exchange rate variability, stochastic volatility comes from consumption growth, in the form of a country specific factor and a world factor. With symmetric parameters, the world factor does not enter the expressions for the expected depreciation q_t, the forward premium $f_t - s_t$ and the risk premium p_t. Therefore, we focus on the role played by country specific volatility v_t. Notice from equation (68) that with standard expected utility ($\alpha = \rho$), both the real factor loading γ_v^r and the real price of risk λ_v^r collapse to zero. EZ preferences allow agents to receive a compensation for taking volatility risk, to which they would not be entitled with standard time-additive expected utility preferences. The contemporaneous presence of both stochastic volatility and EZ preferences is needed to explain the anomaly in real terms. Without stochastic volatility in the real pricing kernel, the real currency risk premium is constant and both of Fama's condition are violated. Without EZ preferences, stochastic volatility in consumption growth is not priced at all.

Remark 6: *The role of the Taylor parameters in the UIP slope coefficient*

Assume for simplicity that consumption growth is not autocorrelated, that is $\varphi_x = 0$. In this case, the real and nominal UIP slope coefficient simplify to

$$b^r = \frac{\gamma_v^r}{\gamma_v^r - (\lambda_{x,v}^r)^2/2} \quad ; \quad b = \frac{\gamma_v}{\gamma_v - (\lambda_{x,v})^2/2}. \tag{73}$$

Equation (73) and (68) make it clear that b^r, the *real* UIP slope coefficient, depends exclusively on the relative magnitude of risk aversion and intertemporal elasticity of substitution. Only when agents have preference for the early resolution of risk ($\alpha < \rho$), we get $b^r < 0$. This is in line with the findings of Bansal and Shaliastovich (2008) and Gavazzoni (2008).

When we look at the *nominal* UIP slope b, the Taylor rules parameters come into play. The result is that different values of τ_π and τ_x can emphasize, mute, or even reverse the impact of the timing of resolution of uncertainty on the UIP slope coefficient. Indeed, we can show that there exist sets of preference parameters and Taylor rule parameters for which the real interest rate differential covaries positively with the expected real depreciation rate, but for which their nominal counterparts covary negatively, thus delivering results consistent with the UIP puzzle. To see this, notice from equation (73) that $b^r < 0$ when γ_v^r and $\gamma_v^r - (\lambda_{x,v}^r)^2/2$ have opposite sign. Clearly, this is possible only with a positive numerator and a negative denominator. But, with a careful choice of the τ parameters we can get the required $b < 0$, even when the denominator of the *real* UIP slope, $\gamma_v^r - (\lambda_{x,v}^r)^2/2$, is positive.

This feature is peculiar to our model with Taylor rules and endogenous inflation. Simply appending an exogenous inflation process to the real part of the model would not create this endogenous interaction between real and nominal variables but simply add a state variable to the problem. What we show is that there exists policy parameters consistent with the UIP evidence. This is far from saying that monetary policy is causing the carry trade behavior of interest rates, but suggest the existence of a non trivial connection between policy parameters, inflation and exchange rate behavior.

Remark 7: *The role of persistence in country specific volatility*

Similarly to the purely nominal examples of section 3, the persistence of country specific volatility φ_v plays a crucial role in the determination of the sign of the UIP slope. Too see this, consider again for simplicity the case in which consumption is not autocorrelated. In this case, as we have seen above, the *real* UIP slope coefficient b^r depends exclusively on the preference parameters. Instead, even when the consumption channel is shut down, the *nominal* UIP slope coefficient b

is also affected by the parameters driving the country specific stochastic volatility process, in particular by its autocorrelation φ_v. This is a direct consequence of endogenizing inflation and deriving the GHPZ monetary policy consistent pricing kernel. From equation (73) we have,

$$\begin{aligned} b &= \frac{\gamma_v}{\gamma_v - \lambda_{x,v}^2/2} \\ &= \frac{\gamma_v^r + a_2\varphi_v}{(\gamma_v^r + a_2\varphi_v) - \frac{\varsigma_v^2}{2}((1-\alpha)+a_1)^2}. \end{aligned}$$

Again, we need the numerator to be positive and the denominator to be negative. Notice that, when $\varphi_v = 0$, the numerator of b, the factor loading of stochastic volatility, is unaffected by monetary policy, i.e. $\gamma_v^r = \gamma_v$. Instead, when $\varphi_v \neq 0$, we have to take care of φ_v, which enters the formula for the factor loading of volatility, γ_v, both directly and indirectly, through its effect on a_2. In our main calibration, we will have $a_2 < 0$, and therefore $\gamma_v < \gamma_v^r$ whenever $\varphi_v > 0$.

In general, it is therefore harder to get the numerator of b to be positive. For given Taylor rule parameters, as the persistence of volatility increases, it gets harder to obtain a negative UIP slope coefficient. On the other hand, as pointed out in the previous remark, for given processes followed by the state variables, a careful choice of Taylor parameters can deliver the required Fama conditions.

4.4 Calibration

Ideally, we would like the model to reproduce a negative nominal UIP slope coefficient, together with persistent and large interest rate differentials, a near random walk behavior of nominal (and real) exchange rate, not so volatile inflation processes, and nominal and real exchange rate closely tied to each other. Also, we would like to have sensible consumption, both within and across countries. As we have shown, our task is complicated by the lower number of free parameters we have due to the fact that we incorporate Taylor rules for interest rate behavior. Consequently, when comparing our model's fit to the one of a model with exogenous inflation, we necessarily do worse. On the positive side, we can benefit from the lower dimensionality of the problem, which facilitates the economic interpretation of the parameters and has the potential to alleviate their identification issues.

We calibrate the real side of the model to reproduce the mean, variance and autocorrelation of the consumption growth process specified in Bansal and Shaliastovich (2008) and study the anomaly for U.S. monthly data. The parameters for the baseline calibration are shown in table 1. In particular, the monthly average consumption growth is equal to 0.15%, with a monthly unconditional volatility of 0.8%. We set the first order autocorrelation in consumption growth equal to 0.

The parameters of country specific volatility and world volatility, together with he sensitivity of consumption growth to these state variables, are chosen to match the unconditional volatility and the autocorrelation in consumption growth. The persistence of country specific volatility is equal to 0, while the prersistence of the world volatility factor is set at 0.997.

The cross-country correlation in consumption growth is around 0.35. This value is consistent with Brandt, Cochrane, and Santa-Clara (2006) which report correlation coefficients in the range of 0.24 and 0.42 for consumption growth between the U.S. and other industrialized countries. Nominal interest rate data are taken from Bansal and Shaliastovich (2008) and inflation data are taken from Gallmeyer, Hollifield, Palomino, and Zin (2007). The discount factor β is set equal to 0.9988 and the country specific volatility processes v_t are assumed to be uncorrelated. This captures the intuition that, in the short run, economies with the same intrinsic features can be hit by uncorrelated volatility shocks. The level of relative risk aversion and intertemporal elasticity of substitution are set equal to 1.5. This implies that agents in the economy have a preference for the early resolution of risk.

Table 2 reports the results. By construction, we match the mean, autocorrelation and volatility of consumption growth and the average level of inflation and nominal interest rate. The implied autocorrelation of the nominal interest rate is high (99%) and close to what we observe in the data. The implied inflation process has an annualized mean of 4.25%. The implied nominal UIP slope coefficient is equal to -0.23 and the volatility of the nominal depreciation rate is 20.34%, which is close to what we observe in the data. Recall that Fama's conditions are satisfied when the risk premium is more variable than the expected depreciation rate and covaries negatively with it. To illustrate this, figure 1 shows a simulated path for the currency risk premium p and the expected depreciation rate q.

Given that we shut down the consumption growth channel by setting its persistence equal to zero, the volatility of the real short rate is very low. However, as discussed in the previous section, an accurate choice of Taylor rule parameters can amplify the variability of nominal variables allowing us to obtain values for $Var(i_t)$ and $Var(\pi_t)$ which are in line with the data. Figure 2 and 3 show simulated paths for the nominal domestic interest rate and inflation, i and π_t, and a comparison between the domestic and foreign inflation rate, π and π^*. As a consequence of the complete symmetry of the model, these variables have very similar processes across countries. Again, this limitation can be overcome by allowing for asymmetric loadings.

The Taylor rule sensitivity to consumption growth is small and positive. A 1% increase in consumption growth implies a 1 basis point increase in the nominal interest rate. The Taylor rule sensitivity to inflation, τ_π, is equal to 1.004. The interpretetion of this parameter has to take into account that inflation is an endogenous variable to the problem and reacts to the same shocks affecting the interest rate.

Consistently with the data, the real and the nominal exchange rate are very highly correlated. Also, they look very close to being a random walk. This can be seen from the usual variance decomposition formula

$$Var(s_{t+1} - s_t) = E\,Var_t(s_{t+1} - s_t) + Var E_t(s_{t+1} - s_t). \tag{74}$$

The first term of equation (74), which is often referred as the unexplained component of the unconditional variance, accounts for 99.9% of the volatility of the depreciation rate. Moreover, the persistence of the depreciation rate is very low. Figure 4 and 5 show simulated paths for the nominal exchange rate and the nominal depreciation rate.

5 Conclusions

How is monetary policy related to the UIP puzzle? Ever since we've known about the apparent profitability of the currency carry trade people have speculated about a lurking role played by monetary policy. The story is that, for some reason, central banks find themselves on the short side of the trade, borrowing high yielding currencies to fund investments in low yielding currencies. In certain cases this has seemed almost obvious. It's well known, for instance, that in recent years the Reserve Bank of India has been accumulating USD reserves and, at the same time, sterilizing the impact on the domestic money supply through contractionary open-market operations. Since Indian interest rates have been relatively high, this policy basically *defines* what it means to be on the short side of the carry trade. This leads one to ask if carry trade losses are in some sense a *cost* of implementing Indian monetary policy? If so, is this a good policy? Is there some sense in which it is *causing* the exchange rate behavior associated with the carry trade?

Our paper's questions, while related, are less ambitious than these speculations about India. What we've shown goes as follows. It is almost a tautology that we can represent exchange rates as ratios of nominal pricing kernels in different currency *units*:

$$\frac{S_{t+1}}{S_t} = \frac{n^*_{t+1}\exp(-\pi^*_{t+1})}{n_{t+1}\exp(-\pi_{t+1})} \ .$$

It is less a tautology (but not too far off) that we can write down sensible stochastic processes for these four variables that are consistent with the carry trade evidence.[7] Previous work has shown that such processes have many parameters that are difficult to identify with sample moments of data. Our paper shows two things.

---

[7] Citations ... Backus-Foresi-Telmer, Bansal's paper(s), Saa Requeo, Lustig-Verdelhan, etc.

First, that by incorporating a Taylor rule for interest rate behavior we reduce the number of parameters. Doing so is sure to deteriorate the model's fit. But the benefit is lower dimensionality and parameters that are economically interpretable. Second, we've shown that some specifications of Taylor rules work and others don't. This seems helpful in and of itself. It also shows that there exist policy rules which, when combined with sensible pricing kernels, are *consistent* with the carry trade evidence. This is a far cry from saying that policy is *causing* carry trade behavior in interest rates and exchange rates, but it does suggest a connection that we find intriguing. In our models, for instance, there exist changes in the policy parameters, τ_1 and τ_3, under which the carry trade profits go away.

Finally, it's worth noting that India, of course, is much more the exception than the rule. Most central banks — especially if we limit ourselves to those from OECD countries — don't have such explicit, foreign-currency related policies. However, many countries *do* use nominal interest rate targeting to implement domestic policy and, therefore, we can think about central banks and the carry trade in a *consolidated* sense. For example, in early 2004 the U.K. less U.S. interest rate differential was around 3%. If we could somehow measure the changes in the Fed's and the Bank of England's balance sheets that were required to implement and sustain these policies, and if we consolidated what we find onto a single balance sheet, would we conclude that these two central banks were short the carry trade?

References

Alvarez, Fernando, Andrew Atkeson, and Patrick J. Kehoe, 2007, Time-varying risk, interest rates, and exchange rates in general equilibrium, Working paper 371, Federal Reserve Bank of Minneapolis.

———, 2008, If exchange rates are random walks, then almost everything we say about monetary policy is wrong, *Federal Reserve Bank of Minneapolis Quarterly Review* 32, 2–9.

Ang, Andrew, Sen Dong, and Monika Piazzesi, 2007, No-arbitrage taylor rules, Unpublished manuscript, Columbia University.

Backus, David K., Silverio Foresi, and Christopher I. Telmer, 2001, Affine term structure models and the forward premium anomaly, *Journal of Finance* 56, 279–304.

Backus, David K., Allan W. Gregory, and Christopher I. Telmer, 1993, Accounting for forward rates in markets for foreign currency, *Journal of Finance* 48, 1887–1908.

Backus, David K., and Stanley E. Zin, 2008, Notes on identification of taylor rules, Unpublished notes.

Bansal, Ravi, and Ivan Shaliastovich, 2008, A long-run risks explanation of predictability puzzles in bond and currency markets, Working Paper.

Bekaert, Geert, 1994, Exchange rate volatility and deviations from unbiasedness in a cash-in-advance model, *Journal of International Economics* 36, 29–52.

Brandt, Michael W., John H. Cochrane, and Pedro Santa-Clara, 2006, International risk sharing is better than you think, or exchange rates are too smooth, *Journal of Monetary Economics* 53, 671–698.

Burnside, Craig, Martin Eichenbaum, Isaac Kleshchelski, and Sergio Rebelo, 2006, The returns to currency speculation, Working Paper, Northwestern University.

Canova, Fabioi, and Jane Marrinan, 1993, Profits, risk and uncertainty in foreign exchange markets, *Journal of Monetary Economics* 32, 259–286.

Clarida, Richard, Jordi Galí, and Mark Gertler, 1999, The science of monetary policy: A new keynesian perspective, *Journal of Economic Literature* XXXVII, 1661–1707.

Cochrane, John H., 2007, Inflation determination with taylor rules: A critical review, Working paper, University of Chicago.

Colacito, Ricardo, and Mariano Massimiliano Croce, 2008, Risks for the long run and the real exchange rate, Unpublished manuscript, University of North Carolina.

Dutton, John, 1993, Real and monetary shocks and risk premia in forward markets for foreign exchange, *Journal of Money, Credit and Banking* 25, 731–754.

Eichenbaum, Martin S., and Charles L. Evans, 1995, Some empirical evidence on the effects of monetary policy shocks on exchange rates, *Quarterly Journal of Economics* 110, 975–1009.

Engel, Charles, and Kenneth D. West, 2006, Taylor rules and the deutschmark-dollar real exchange rate, *Journal of Money, Credit and Banking* 38, 1175–1194.

Epstein, L. G., and Stanley Zin, 1989, Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937–969.

Fama, Eugene F, 1984, Forward and spot exchange rates, *Journal of Monetary Economics* 14, 319–38.

Gallmeyer, Michael F., Burton Hollifield, Francisco Palomino, and Stanley E. Zin, 2007, Arbitrage-free bond pricing with dynamic macroeconomic models, Working paper, Carnegie Mellon University.

Gavazzoni, Federico, 2008, Uncovered interest rate parity puzzle: An explanation based on recursive utility and stochastic volatility, Working Paper.

Grilli, Vittorio, and Nouriel Roubini, 1992, Liquidity and exchange rates, *Journal of International Economics* 32, 339–352.

Hansen, Lars P., John C. Heaton, and Nan Li, 2005, Consumption strikes back?: Measuring long-run risk, NBER Working Paper No. 11476.

Lucas, Robert E, 1982, Interest rates and currency prices in a two-country world, *Journal of Monetary Economics* 10, 335–60.

Lustig, Hanno N., Nikolai L. Roussanov, and Adrien Verdelhan, 2009, Common risk factors in currency markets, Unpublished manuscript, UCLA.

Macklem, Tiff, 1991, Forward exchange rates and risk premiums in artificial economies, *Journal of International Money and Finance* 10, 365–391.

Marshall, David, 1992, Phd thesis,, Carnegie Mellon University.

McCallum, Bennett T., 1994, A reconsideration of the uncovered interest rate parity relationship, *Journal of Monetary Economics* 33, 105–132.

Schlagenhauf, Don E., and Jeffrey M. Wrase, 1995, Liquidity and real activity in a simple open economy model, *Journal of Monetary Economics* 35, 431–461.

Appendix A

Symmetric Model

The short rate must satisfy both the Euler equation and the Taylor rule:

$$\begin{aligned} i_t &= -\log E_t\, m_{t+1} && \text{(A1)} \\ i_t &= \tau + \tau_1 \pi_t + z_t \ \ , && \text{(A2)} \end{aligned}$$

where the processes for z_t and its volatility v_t are

$$\begin{aligned} z_t &= \varphi_z z_{t-1} + v_{t-1}^{1/2} \varepsilon_t && \text{(A3)} \\ v_t &= \theta_v(1-\varphi_v) + \varphi_v v_{t-1} + \sigma_v w_t && \text{(A4)} \end{aligned}$$

where ε_t and w_t are *i.i.d.* standard normal. Given that $m_{t+1} = n_{t+1} P_t / P_{t+1}$ and $\pi_{t+1} = \log(P_{t+1}/P_t)$, set the real pricing kernel to a constant and guess that the solution for endogenous inflation is:

$$\pi_t = a + a_1 z_t + a_2 v_t \ \ , \tag{A5}$$

Substitute equation (A5) into the pricing kernel and compute the expectation in equation (A1):

$$i_t = C + a_1 \varphi_z z_t + (a_2 \varphi_v - a_1^2/2) v_t \ \ , \tag{A6}$$

where

$$C \equiv -n + a + a_2 \theta_v (1-\varphi_v) - (a_2 \sigma_v)^2/2 \tag{A7}$$

Match-up the coefficients with the Taylor rule and solve for the a_i parameters:

$$\begin{aligned} a &= \frac{C-\tau}{\tau_1} && \text{(A8)} \\ a_1 &= \frac{1}{\varphi_z - \tau_1} && \text{(A9)} \\ a_2 &= \frac{1}{2(\varphi_z-\tau_1)^2(\varphi_v-\tau_1)} && \text{(A10)} \end{aligned}$$

It's useful to note that

$$a_2 = \frac{a_1^2}{2(\varphi_v - \tau_1)} \ \ .$$

Note that this is the same as saying that

$$\frac{\partial i_t}{\partial v_t} = \tau_1 \frac{\partial \pi_t}{\partial v_t} = \frac{\partial E_t \pi_{t+1}}{\partial v_t} - \frac{1}{2} \frac{\partial\, Var_t \pi_{t+1}}{\partial v_t}$$

Similarly, $a_1 = 1/(\varphi_z - \tau_1)$ is the same as saying that

$$\frac{\partial i_t}{\partial z_t} = \tau_1 \frac{\partial \pi_t}{\partial z_t} + 1 = \frac{\partial E_t \pi_{t+1}}{\partial z_t} - \frac{1}{2}\frac{\partial\, Var_t \pi_{t+1}}{\partial z_t} \quad .$$

Both of these things are kind of trivial. They just say that the effect of a shock on the Taylor rule equation must be consistent with the effect on the Euler equation. Note also that

$$C = \frac{\tau_1}{\tau_1 - 1}\Big(- n - \frac{\tau}{\tau_1} + a_2\theta_v(1-\varphi_v) - (a_2\sigma_v)^2/2\Big)$$

Inflation and the short rate are:

$$\begin{aligned}
\pi_t &= \frac{C-\tau}{\tau_1} + \frac{1}{\varphi_z - \tau_1} z_t + \frac{1}{2(\varphi_z-\tau_1)^2(\varphi_v-\tau_1)} v_t && \text{(A11)}\\
i_t &= C + \frac{\varphi_z}{\varphi_z - \tau_1}\, z_t + \frac{\tau_1}{2(\varphi_z-\tau_1)^2(\varphi_v-\tau_1)}\, v_t && \text{(A12)}\\
&= C + \varphi_z a_1 z_t + \tau_1 a_2 v_t && \text{(A13)}
\end{aligned}$$

The pricing kernel is

$$\begin{aligned}
-\log m_{t+1} &= C + (\sigma_v a_2)^2/2 + a_1\varphi_z z_t + a_2\varphi_v v_t + a_1 v_t^{1/2}\varepsilon_{t+1} + \sigma_v a_2 w_{t+1}\\
&= D + \frac{1}{\varphi_z-\tau_1}\varphi_z z_t + \frac{\varphi_v}{2(\varphi_z-\tau_1)^2(\varphi_v-\tau_1)} v_t && \text{(A14)}\\
&\quad + \frac{1}{\varphi_z-\tau_1} v_t^{1/2}\varepsilon_{t+1} + \frac{\sigma_v}{2(\varphi_z-\tau_1)^2(\varphi_v-\tau_1)} w_{t+1}
\end{aligned}$$

where

$$D \equiv C + (\sigma_v a_2)^2/2 \qquad \text{(A15)}$$

The GBP-denominated kernel and variables are denoted with asterisks. The interest-rate differential, the expected depreciation rate, q_t, and the risk premium, p_t, are:

$$\begin{aligned}
i_t - i_t^* &= \varphi_z a_1 z_t - \varphi_z^* a_1^* z_t^* + \tau_1 a_2 v_t - \tau_1^* a_2^* v_t^* && \text{(A16)}\\
q_t &= D - D^* + a_1\varphi_z z_t - a_1^*\varphi_z^* z_t^* + a_2\varphi_v v_t - a_2^*\varphi_v^* v_t^* && \text{(A17)}\\
p_t &= -\frac{1}{2}\left(a_1^2 v_t - a_1^{*2} v_t^* + \sigma_v^2 a_2^2 - \sigma_v^{*2} a_2^{*2}\right) && \text{(A18)}
\end{aligned}$$

It is easily verified that $p_t + q_t = i_t - i_t^*$.

If we assume that all foreign and domestic parameter values are the same (*i.e.*, $\tau = \tau^*$) and if we set $\varphi_z = 0$, then the regression parameter is:

$$\begin{aligned}
b &= \frac{Cov(i_t - i_t^*\, ,\, q_t)}{Var(i_t - i_t^*)}\\
&= \frac{\varphi_v}{\tau_1}
\end{aligned}$$

Appendix B
Asymmetric Taylor Rule

Taylor rules

$$\begin{aligned} i_t &= \tau + \tau_1 \pi_t + z_t + \tau_3 d_t \\ i_t^* &= \tau^* + \tau_1^* \pi_t^* + z_t^* + \tau_3^* d_t \\ d_t &\equiv \log(S_t / S_{t-1}) = \pi_t - \pi_t^* \end{aligned}$$

State variables,

$$\begin{aligned} z_t &= \varphi_z z_{t-1} + v_{t-1}^{1/2} \varepsilon_t && \text{(B1)} \\ v_t &= \theta_v (1 - \varphi_v) + \varphi_v v_{t-1} + \sigma_v w_t && \text{(B2)} \end{aligned}$$

and the associated foreign-country processes with asterisks and with all shocks *i.i.d.*. Collect them in the state vector, X_t:

$$X_t \equiv \begin{bmatrix} z_t \; z_t^* \; v_t \; v_t^* \end{bmatrix}^\top$$

Inflation solutions:

$$\begin{aligned} \pi_t &= a + a_1 z_t + a_2 z_t^* + a_3 v_t + a_4 v_t^* \equiv a + A^\top X_t \\ \pi_t^* &= a^* + a_1^* z_t + a_2^* z_t^* + a_3^* v_t + a_4^* v_t^* \equiv a^* + A^{*\top} X_t \end{aligned}$$

Interest rates, from Euler equations with real interest rate = 0:

$$\begin{aligned} i_t &= C + B^\top X_t \\ i_t^* &= C^* + B^{*\top} X_t \end{aligned}$$

where,

$$\begin{aligned} B^\top &\equiv \left[\; a_1 \varphi_z \quad a_2 \varphi_z^* \quad (a_3 \varphi_v - \frac{a_1^2}{2}) \quad (a_4 \varphi_v^* - \frac{a_2^2}{2}) \; \right] \\ C &\equiv a + a_3 \theta_v (1 - \varphi_v) + a_4 \theta_v^* (1 - \varphi_v^*) - \frac{1}{2} \big(a_3^2 \sigma_v^2 + a_4^2 \sigma_v^{*2} \big) \\ B^{*\top} &\equiv \left[\; a_1^* \varphi_z \quad a_2^* \varphi_z^* \quad (a_3^* \varphi_v - \frac{a_1^{*2}}{2}) \quad (a_4^* \varphi_v^* - \frac{a_2^{*2}}{2}) \; \right] \\ C^* &\equiv a^* + a_3^* \theta_v (1 - \varphi_v) + a_4^* \theta_v^* (1 - \varphi_v^*) - \frac{1}{2} \big(a_3^{*2} \sigma_v^2 + a_4^{*2} \sigma_v^{*2} \big) \end{aligned}$$

Taylor rules become:

$$\begin{aligned} i_t &= \tau + \tau_1 (a + A^\top X_t) + z_t + \tau_3 \big(a + A^\top X_t - a^* - A^{*\top} X_t \big) \\ &= \tau + \tau_1 a + \tau_3 (a - a^*) + \big(\tau_1 A^\top + \iota_z^\top + \tau_3 [A^\top - A^{*\top}] \big) X_t \\ i_t^* &= \tau^* + \tau_1^* (a^* + A^{*\top} X_t) + z_t^* + \tau_3^* \big(a + A^\top X_t - a^* - A^{*\top} X_t \big) \\ &= \tau^* + \tau_1^* a^* + \tau_3^* (a - a^*) + \big(\tau_1^* A^{*\top} + \iota_z^{*\top} + \tau_3^* [A^\top - A^{*\top}] \big) X_t \end{aligned}$$

where $\iota_z^\top \equiv [\,1\ \ 0\ \ 0\ \ 0\,]$ and $\iota_z^{*\top} \equiv [\,0\ \ 1\ \ 0\ \ 0\,]$. Matching-up the coefficients means

$$\begin{aligned}
C &= \tau + \tau_1 a + \tau_3(a - a^*) \\
C^* &= \tau^* + \tau_1^* a^* + \tau_3^*(a - a^*) \\
B &= \tau_1 A^\top + \iota_z^\top + \tau_3(A^\top - A^{*\top}) \\
B^* &= \tau_1^* A^{*\top} + \iota_z^{*\top} + \tau_3^*(A^\top - A^{*\top})
\end{aligned}$$

To solve for the constants (the first two equations):

$$\begin{bmatrix} 1-\tau_1-\tau_3 & \tau_3 \\ -\tau_3^* & 1-\tau_1^*+\tau_3^* \end{bmatrix} \begin{bmatrix} a \\ a^* \end{bmatrix} = \begin{bmatrix} \tau - stuff \\ \tau^* - stuff^* \end{bmatrix}$$

where $stuff$ and $stuff^*$ are everything on the LHS of the solutions for C and C^*, except the first terms, a and a^*.

The B equations are eight equations in eight unknowns, A and A^*. Conditional on these, the C equations are two-in-two, a and a^*. The B equations can be broken into 4 blocks of 2. It's useful to write them out because you can see where the singularity lies.

$$\begin{bmatrix} (\tau_1+\tau_3-\varphi_z) & -\tau_3 \\ \tau_3^* & (\tau_1^*-\tau_3^*-\varphi_z) \end{bmatrix} \begin{bmatrix} a_1 \\ a_1^* \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} (\tau_1+\tau_3-\varphi_z^*) & -\tau_3 \\ \tau_3^* & (\tau_1^*-\tau_3^*-\varphi_z^*) \end{bmatrix} \begin{bmatrix} a_2 \\ a_2^* \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} (\tau_1+\tau_3-\varphi_v) & -\tau_3 \\ \tau_3^* & (\tau_1^*-\tau_3^*-\varphi_v) \end{bmatrix} \begin{bmatrix} a_3 \\ a_3^* \end{bmatrix} = \begin{bmatrix} -a_1^2/2 \\ -a_1^{*2}/2 \end{bmatrix}$$

$$\begin{bmatrix} (\tau_1+\tau_3-\varphi_v^*) & -\tau_3 \\ \tau_3^* & (\tau_1^*-\tau_3^*-\varphi_v^*) \end{bmatrix} \begin{bmatrix} a_4 \\ a_4^* \end{bmatrix} = \begin{bmatrix} -a_2^2/2 \\ -a_2^{*2}/2 \end{bmatrix}$$

Two singularities exist:

- *UIP holds exactly.* If $\tau_3 = 0$ (so that the Fed ignores the FX rate), $\varphi_v = \varphi_v^*$ and $\tau_1 = \tau_1^*$ (complete symmetry in parameters, save τ_3 and τ_3^*) then a singularity is $\tau_3^* = \tau_1 - \varphi_v$. As τ_3^* approaches this from below or above, the UIP coefficient goes to 1.0.
- *Anomaly resolved.* Similarly, if $\tau_3 = 0$, $\varphi_v = \varphi_v^*$ and $\tau_1 = \tau_1^*$ then a singularity is $\tau_3^* = \tau_1$. As τ_3^* approaches from *below*, the UIP coefficient goes to infinity. As τ_3^* approaches from *above*, it goes to *negative* infinity.

The latter condition is where the UIP regression coefficient changes sign. This says that we need $\tau_3^* > \tau_3$. This may seem pathological. It says that — if we interpret these coefficients as policy responses (which we shouldn't) — the ECB responds to an appreciation in EUR by increasing interest rates *more* than 1:1 (and more than the 'Taylor principle' magnitude of $\tau_1 > 1$).

Appendix C
Linearization for the Pricing Kernel

The log of the equilibrium domestic marginal rate of substitution in equation (65) is given by

$$\log(m^r_{t+1}) = \log\beta + (\rho - 1)x_{t+1} + (\alpha - \rho)[\log W_{t+1} - \log\mu_t(W_{t+1})],$$

where $x_{t+1} \equiv \log(c_{t+1}/c_t)$ is the log of the ratio of domestic observed consumption in $t+1$ relative to t and W_t is the value function. The first two terms are standard expected utility terms: the pure time preference parameter β and a consumption growth term times the inverse of the negative of the intertemporal elasticity of substitution. The third term in the pricing kernel is a new term coming from EZ preferences.

We work on a linearized version of the real pricing kernel, following the findings of Hansen, Heaton, and Li (2005). In particular, I focus on the the value function of each representative agent, scaled by the observed equilibrium consumption level

$$\begin{aligned} W_t/c_t &= [(1-\beta) + \beta(\mu_t(W_{t+1})/c_t)^\rho]^{1/\rho} \\ &= \left[(1-\beta) + \beta\mu_t\left(\frac{W_{t+1}}{c_{t+1}} \times \frac{c_{t+1}}{c_t}\right)^\rho\right]^{1/\rho}, \end{aligned}$$

where I use the linear homogeneity of μ_t. In logs,

$$w_t = \rho^{-1}\log[(1-\beta) + \beta\exp(\rho u_t)],$$

where $w_t = \log(W_t/c_t)$ and $u_t \equiv \log(\mu_t(\exp(w_{t+1} + x_{t+1})))$. Taking a linear approximation of the right-hand side as a function of u_t around the point $\bar{m}$, I get

$$\begin{aligned} w_t &\approx \rho^{-1}\log[(1-\beta) + \beta\exp(\rho\bar{m})] + \left[\frac{\beta\exp(\rho\bar{m})}{1-\beta+\beta\exp(\rho\bar{m})}\right](u_t - \bar{m}) \\ &\equiv \bar{\kappa} + \kappa u_t \end{aligned}$$

where $\kappa < 1$. Approximating around $\bar{m} = 0$, results in $\bar{\kappa} = 0$ and $\kappa = \beta$, and for the general case of $\rho = 0$, the "log aggregator", the linear approximation is exact with $\bar{\kappa} = 1 - \beta$ and $\kappa = \beta$.

Given the state variables of the economy, x, v and w, and the log-linear structure of the model, we conjecture a solution for the value function of the form,

$$z_t = \bar{\xi} + \xi_x x_t + \xi_v v_t + \xi_w w_t,$$

where $\bar{\xi}$, ξ_x, ξ_v and ξ_w are constants to be determined. Therefore

$$z_{t+1} + x_{t+1} = \bar{\xi} + (\xi_x + 1)x_{t+1} + \xi_v v_{t+1} + \xi_w w_{t+1}$$

and, using the properties of lognormal random variables, u_t can be expressed as

$$\begin{aligned} u_t &\equiv \log(\mu_t(\exp(z_{t+1}+x_{t+1}))) \\ &= \log(E_t[\exp(z_{t+1}+x_{t+1})^{\alpha}]^{\frac{1}{\alpha}})] \\ &= E_t[z_{t+1}+x_{t+1}]+\frac{\alpha}{2}\mathrm{Var}_t[z_{t+1}+x_{t+1}]. \end{aligned}$$

Using the above expression, we solve for the value-function parameters by matching coefficients

$$\begin{aligned} \xi_x &= \kappa(\xi_x+1)\varphi_x \\ \Rightarrow \xi_x &= \left(\frac{\kappa}{1-\kappa\phi_x}\right)\varphi_x \\ \xi_v &= \kappa[\xi_v\varphi_v+\frac{\alpha}{2}(\xi_x+1)^2\varsigma_v^2] \\ \Rightarrow \xi_v &= \left(\frac{\kappa}{1-\kappa\varphi_v}\right)\left[\frac{\alpha}{2}\left(\frac{1}{1-\kappa\varphi_x}\right)^2\varsigma_v^2\right] \\ \xi_w &= \kappa[\xi_w\varphi_w+\frac{\alpha}{2}(\xi_x+1)^2\varsigma_w^2] \\ \Rightarrow \xi_w &= \left(\frac{\kappa}{1-\kappa\varphi_w}\right)\left[\frac{\alpha}{2}\left(\frac{1}{1-\kappa\varphi_x}\right)^2\varsigma_w^2\right] \\ \bar{\xi} &= \frac{\bar{\kappa}}{1-\kappa}+\frac{\kappa}{1-\kappa}\left[(\xi_x+1)(1-\varphi_x)\theta_x+\xi_v(1-\varphi_v)\theta_v+\xi_w(1-\varphi_w)\theta_w+\frac{\alpha}{2}\xi_v^2\sigma_v^2+\frac{\alpha}{2}\xi_w^2\sigma_w^2\right]. \end{aligned}$$

The solution allows us to simplify the term $[\log W_{t+1}-\log\mu_t(W_{t+1})]$ in the pricing kernel in equation (5):

$$\begin{aligned} \log W_{t+1}-\log\mu_t(W_{t+1}) &= z_{t+1}+x_{t+1}-\log\mu_t(\exp(z_{t+1}+x_{t+1})) \\ &= (\xi_x+1)[x_{t+1}-E_tx_{t+1}]+\xi_v[v_{t+1}-E_tv_{t+1}]+\xi_w[w_{t+1}-E_tw_{t+1}] \\ &\quad -\frac{\alpha}{2}(\xi_x+1)^2\mathrm{Var}_t[x_{t+1}]-\frac{\alpha}{2}\xi_v^2\mathrm{Var}_t[v_{t+1}]-\frac{\alpha}{2}\xi_w^2\mathrm{Var}_t[w_{t+1}] \\ &= (\xi_x+1)(\varsigma_v v_t^{1/2}\eta_{t+1}^x+\varsigma_w w_t^{1/2}\epsilon_{t+1}^x)+\xi_v\sigma_v\epsilon_{t+1}^v+\xi_w\sigma_w\epsilon_{t+1}^w \\ &- \frac{\alpha}{2}(\xi_x+1)^2(\varsigma_v^2v_t+\varsigma_w^2w_t)-\frac{\alpha}{2}(\xi_v^2\sigma_v^2+\xi_w^2\sigma_w^2). \end{aligned}$$

Equation (67) follows by collecting terms.

Appendix D

Moment Conditions

- Consumption growth:[8]

$$\begin{aligned} E_t(x_{t+1}) = (1-\varphi_x)\theta_x + \varphi_x x_t, &\quad Var_t(x_{t+1}) = \varsigma_w^2 w_t + \varsigma_v^2 v_t, \\ E(x_{t+1}) = \theta_x, &\quad Var(x_{t+1}) = \tfrac{\varsigma_w^2\theta_w + \varsigma_v^2\theta_v}{1-\varphi_x^2}, \\ Corr(x_{t+1}, x_t) = \varphi_x, &\quad Cov(x_{t+1}, x_{t+1}^*) = \tfrac{\varsigma_w^2\theta_w\chi_{\epsilon^x}}{1-\varphi_x^2} \end{aligned}$$

$$Corr(x_{t+1}, x_{t+1}^*) = \frac{\varsigma_w^2\theta_w\chi_{\epsilon^x}}{\varsigma_w^2\theta_w + \varsigma_v^2\theta_v}$$

$$Var(x_{t+1} - x_{t+1}^*) = 2\frac{\varsigma_w^2\theta_w(1-\chi_{\epsilon^x}) + \varsigma_v^2\theta_v}{1-\varphi_x^2}$$

- World volatility:

$$E(w_{t+1}) = \theta_w; \quad Var(w_{t+1}) = \tfrac{\sigma_w^2}{1-\varphi_w^2}; \quad Corr(w_{t+1}, w_t) = \varphi_w;$$

- Country specific volatility:

$$E(v_{t+1}) = \theta_v; \quad Var(v_{t+1}) = \tfrac{\sigma_v^2}{1-\varphi_v^2}; \quad Corr(v_{t+1}, v_t) = \varphi_v$$

$$Var(v_{t+1} - v_{t+1}^*) = 2\frac{\sigma_v^2(1-\chi_{\epsilon^v})}{1-\varphi_v^2}$$

- Real pricing kernel:

$$\begin{aligned} E_t(\log n_{t+1}) &= -(\delta^r + \gamma_x^r x_t + \gamma_w^r w_t + \gamma_v^r v_t), \\ Var_t(\log n_{t+1}) &= (\lambda_w^r\sigma_w)^2 + (\lambda_v^r\sigma_v)^2 + (\lambda_{x,w}^r)^2 w_t + (\lambda_{x,v}^r)^2 v_t \end{aligned}$$

- Real risk free interest rate:

$$E(r_t) = \bar{r} + \gamma_x^r\theta_x + (\gamma_w^r - (\lambda_{x,w}^r)^2/2)\theta_w + (\gamma_v^r - (\lambda_{x,v}^r)^2/2)\theta_v$$

$$\begin{aligned} Var(r_t) &= (\gamma_x^r)^2\, Var(x_t) + (\gamma_w^r - (\lambda_{x,w}^r)^2/2)^2\, Var(w_t) + \\ &+ (\gamma_v^r - (\lambda_{x,v}^r)^2/2)^2\, Var(v_t) \end{aligned}$$

---

[8] We assume symmetry for cross-country moments.

$$\begin{aligned} Corr(r_{t+1}, r_t) &= 1 - (1-\varphi_x)(\gamma_x^r)^2 \frac{Var(x_t)}{Var(r_t)} \\ &- (1-\varphi_v)(\gamma_{x,v}^r - (\lambda_{x,v}^r)^2/2) \frac{Var(v_t)}{Var(r_t)} \\ &- (1-\varphi_w)(\gamma_{x,w}^r - (\lambda_{x,w}^r)^2/2) \frac{Var(w_t)}{Var(r_t)} \end{aligned}$$

- Real depreciation rate:

$$E_t(d_{t+1}^r) = q_t^r; \quad E(d_t^r) = 0$$

$$\begin{aligned} Var(d_t^r) &= 2[(\lambda_v^r \sigma_v^r)^2 (1-\chi_{\epsilon^v}) + (\lambda_{x,w}^r)^2 \theta_w (1-\chi_{\epsilon^x}) + (\lambda_{x,v}^r)^2 \theta_v] \\ &+ (\gamma_x^r)^2\, Var(x_t - x_t^*) + (\gamma_v^r)^2\, Var(v_t - v_t^*) \end{aligned}$$

- Inflation:

$$E(\pi_t) = a + a_1\theta_x + a_2\theta_v + a_3\theta_w, \quad Var(\pi_t) = a_1^2\, Var(x_t) + a_2^2\, Var(v_t) + a_3^2\, Var(w_t)$$

$$Corr(\pi_{t+1}, \pi_t) = 1 - (1-\varphi_x) a_1^2 \frac{Var(x_t)}{Var(\pi_t)} - (1-\varphi_v) a_2^2 \frac{Var(v_t)}{Var(\pi_t)} - (1-\varphi_w) a_3^2 \frac{Var(w_t)}{Var(\pi_t)}$$

$$corr(x_t, \pi_t) = a_1 \frac{Stdev(x_t)}{Stdev(\pi_t)}, \quad corr(x_{t+1}, \pi_t) = a_1 \varphi_x \frac{Stdev(x_t)}{Stdev(\pi_t)}$$

- Nominal interest rate:

$$E(i_t) = \bar{\iota} + \gamma_x \theta_x + (\gamma_v - (\lambda_{x,v})^2/2)\theta_v + (\gamma_w - (\lambda_{x,w})^2/2)\theta_v$$

$$\begin{aligned} Var(i_t) &= (\gamma_x)^2\, Var(x_t) + (\gamma_w^r - (\lambda_{x,w}^r)^2/2)^2\, Var(w_t) \\ &+ (\gamma_v - (\lambda_{x,v})^2/2)^2\, Var(v_t) \end{aligned}$$

$$\begin{aligned} Corr(i_{t+1}, i_t) &= 1 - (1-\varphi_x)\gamma_x^2 \frac{Var(x_t)}{Var(i_t)} \\ &- (1-\varphi_v)(\gamma_{x,v} - \lambda_{x,v}^2/2) \frac{Var(v_t)}{Var(i_t)} \\ &- (1-\varphi_w)(\gamma_{x,w} - \lambda_{x,w}^2/2) \frac{Var(w_t)}{Var(i_t)} \end{aligned}$$

$$corr(i_t, \pi_t) = \frac{a_1 \gamma_x\, Var(x) + a_2(\gamma_v - \lambda_{x,v}^2/2)\, Var(v_t) + a_3(\gamma_w - \lambda_{x,w}^2/2)\, Var(w_t)}{Stdev(i_t) Stdev(\pi_t)}$$

- Nominal depreciation rate:

$$E_t(d_{t+1}) = q_t; \quad E(d_t) = 0$$

$$\begin{aligned} Var(d_t) &= 2[(\lambda_v \sigma_v)^2 (1 - \chi_{\epsilon^v}) + \lambda_{x,w}^2 \theta_w (1 - \chi_{\epsilon^x}) + \lambda_{x,v}^2 \theta_v] \\ &+ \gamma_x^2 \, Var(x_t - x_t^*) + \gamma_v^2 \, Var(v_t - v_t^*) \end{aligned}$$

Table 1: Calibration parameters: Consumption growth, country specific and world stochastic volatility, and rate of time-preference

Parameter		Value
Time preference parameter	β	0.9988
Mean of consumption growth	θ_x	0.0015
Autocorrelation in consumption growth	ϕ_x	0
Mean of country specific volatility	θ_v	3.88e-3
Autocorrelation in stochastic volatility	ϕ_v	0
Volatility of country specific variance	σ_v	6.5e-4
Mean of world volatility	θ_w	4.58e-3
Autocorrelation in world volatility	ϕ_w	0.997
Volatility of world variance	σ_w	1.33e-4
Sensitivity of consumption growth to country specific volatility	ς_v	0.0050
Sensitivity of consumption growth to world volatility	ς_w	0.1180
Correlation of consumption innovation to world vol	χ_{ϵ^x}	1
Correlation of country specific volatility shocks	χ_{ϵ^v}	0
Taylor rule constant	$\bar{\tau}$	0.0021
Taylor rule sensitivity to inflation	τ_π	1.0040
Taylor rule sensitivity to consumption growth	τ_x	0.010

Table 2: Results

Moment	Data	Model
$E(i_t) \times 12$	6.82%	6.80%
$\sigma(i_t) \times 12$	3.76%	3.71%
$corr(i_t, i_{t-1})$	0.98	0.99
$E(\pi_t) \times 12$	4.24%	4.25%
$\sigma(\pi_t) \times 12$	3.66%	3.70%
$corr(\pi_t, \pi_{t-1})$	0.71	0.99
b^r	negative	-0.22
b	negative	-0.23
$\sigma(s_{t+1} - s_t) \times 12$	15.0%	20.3%

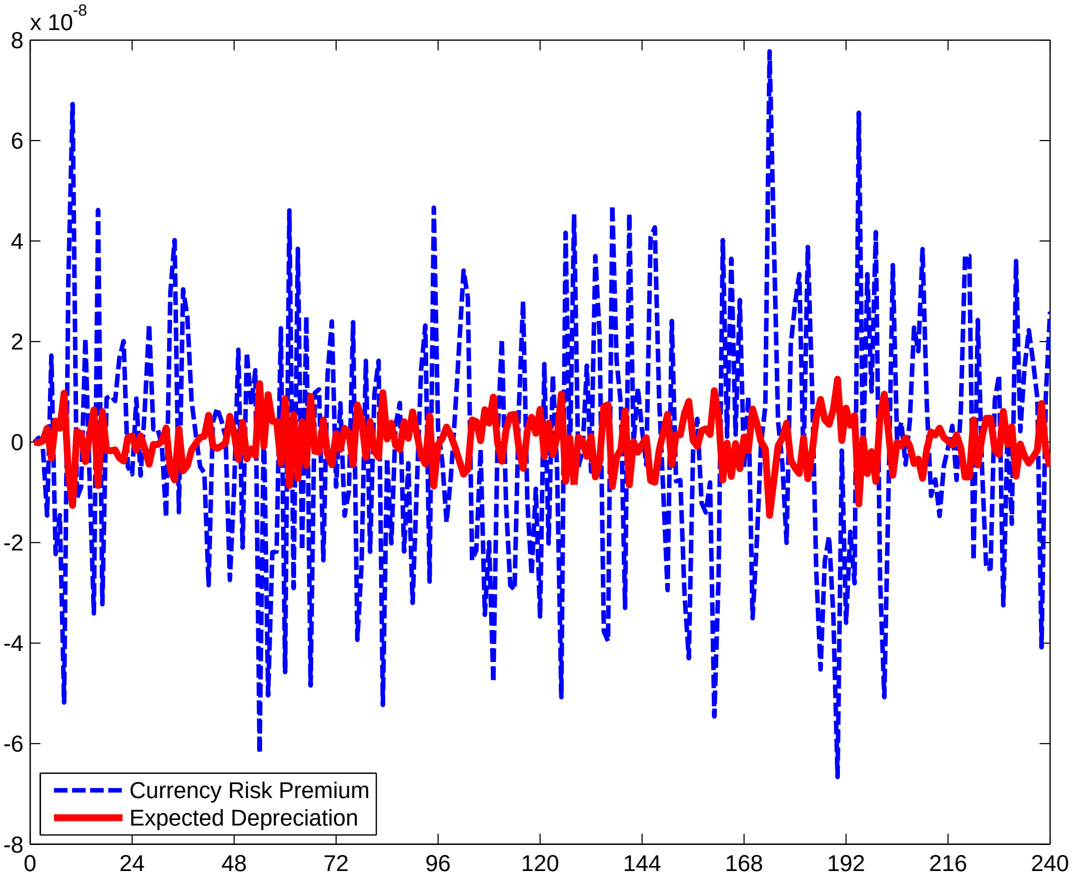

Figure 1: Currency Risk Premium and Expected Depreciation over 240 months.

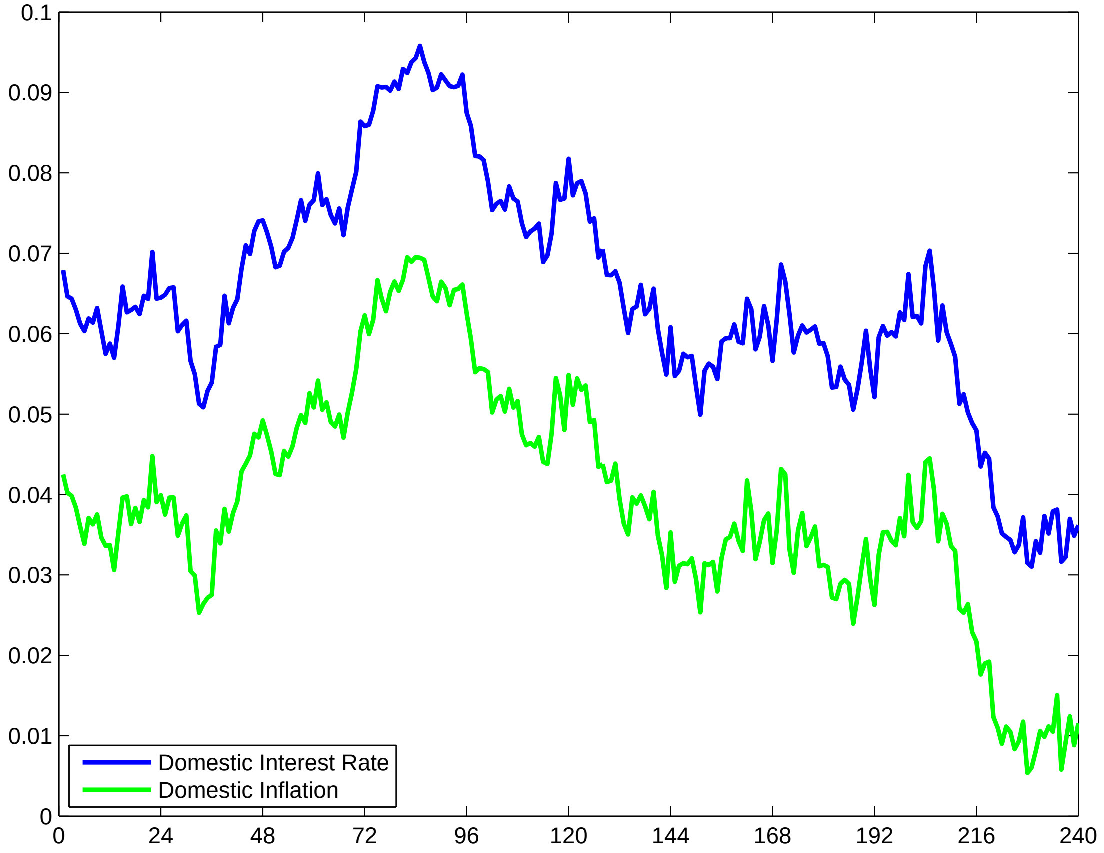

Figure 2: Domestic Nominal Interest Rate i and Domestic Inflation Rate π over 240 months.

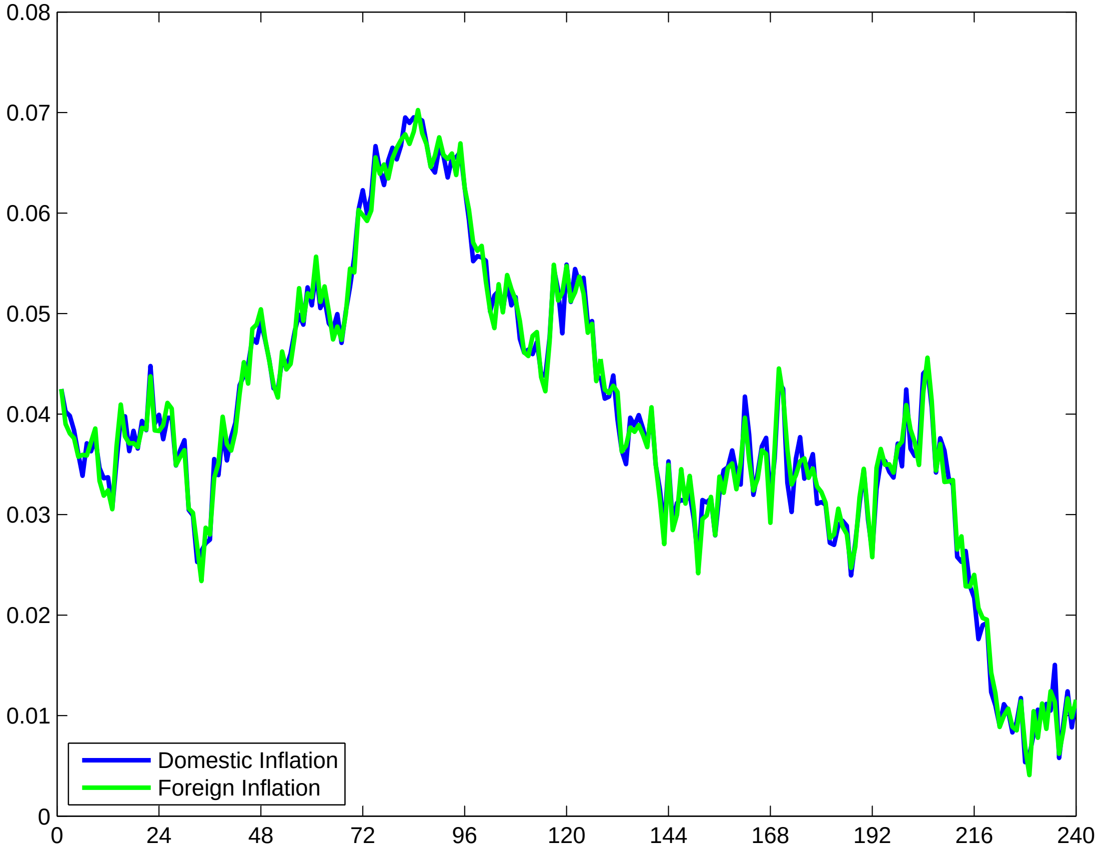

Figure 3: Domestic and Foreign Inflation over 240 months.

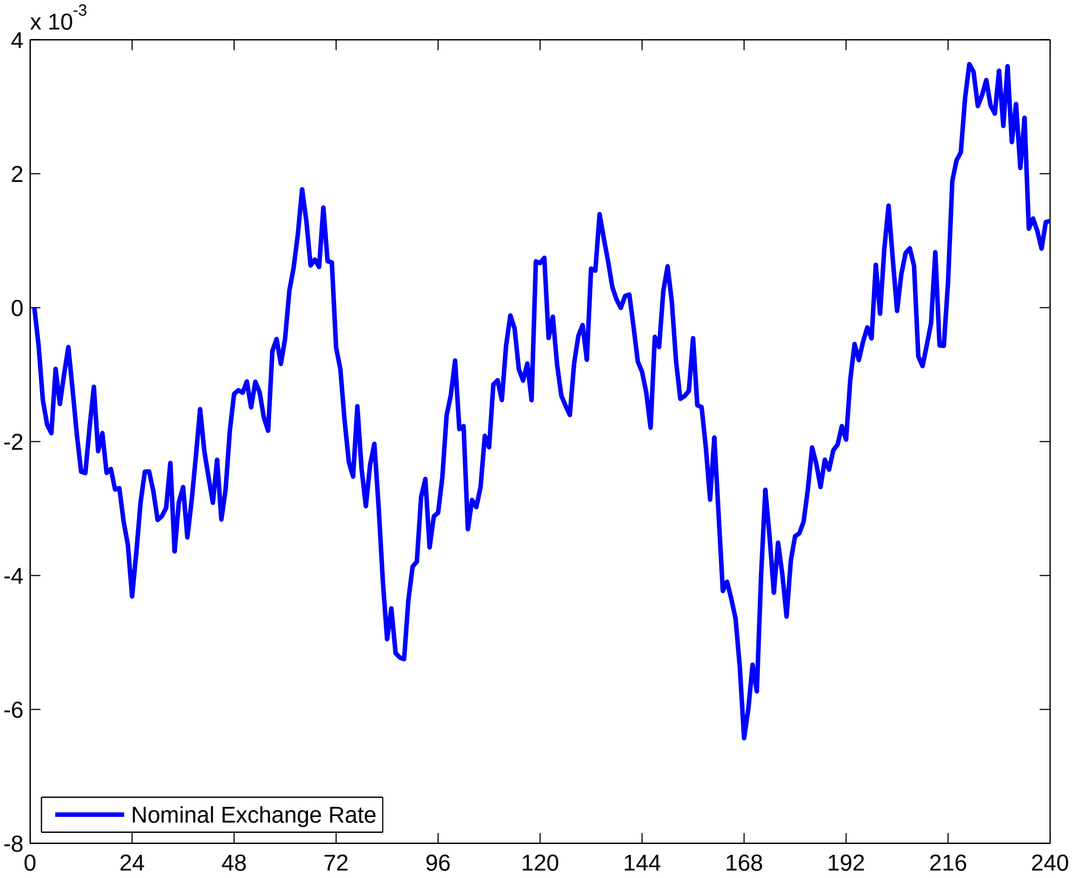

Figure 4: Nominal Exchange rate (level) over 240 months.

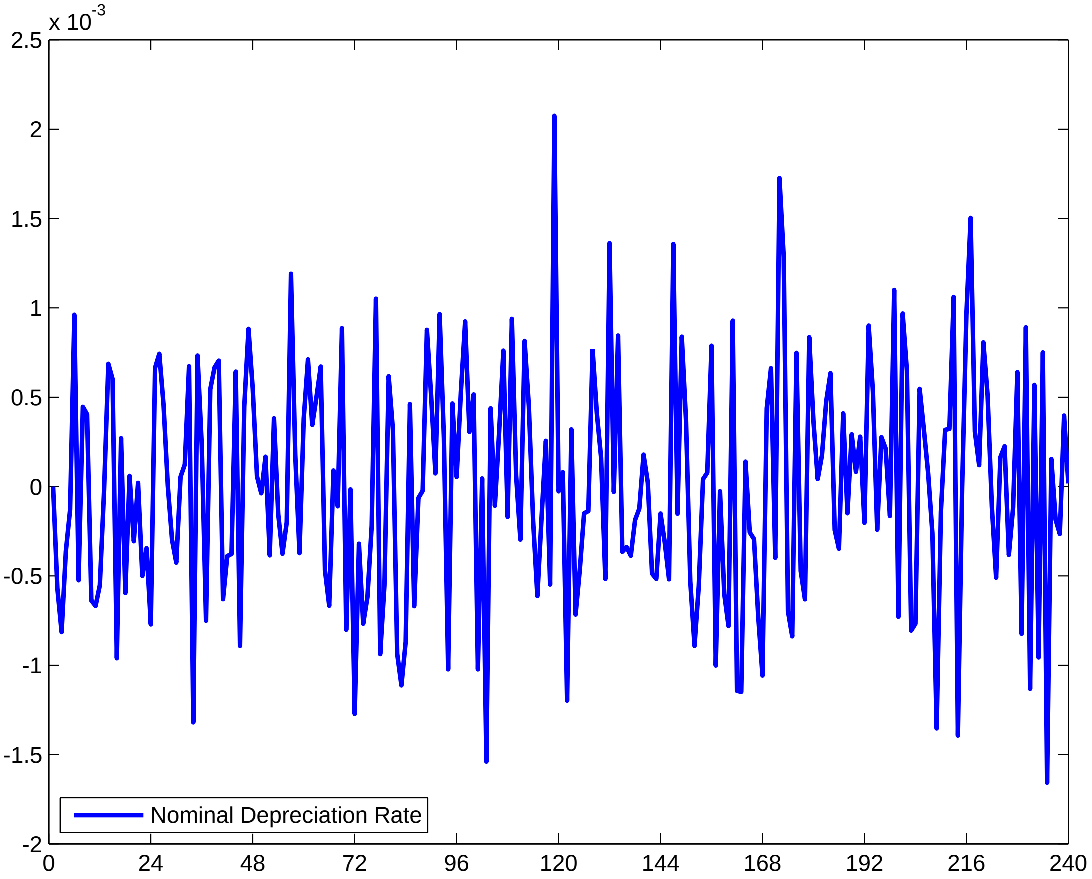

Figure 5: Nominal Depreciation rate over 240 months.