

# Signaling in Tender Offer Games\*

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September, 2009

## Abstract

We examine whether a bidder can use the terms of the tender offer to signal the post-takeover security benefits. As atomistic shareholders extract all the gains in security benefits, signaling equilibria are subject to a constraint that is absent from bilateral trade models. The buyer (bidder) must enjoy gains from trade that are excluded from bargaining (private benefits), but can nonetheless be relinquished and enable shareholders to draw inference about the security benefits. Restricted bids and cash-equity offers do not satisfy these requirements. Dilution, debt financing, probabilistic takeover outcomes and toeholds are all viable signals because they make bidder gains depend on the security benefits in a predictable manner. In all the signaling equilibria, lower-valued types must forgo a larger fraction of their private benefits and these signaling costs prevent some takeovers. When there is additional private information about the private benefits as in the case of two-dimensional bidder types, fully revealing equilibria cease to exist. This does not hold once bidders can offer not only cash or equity but also (more) elaborate contingent claims. Offers which include options avoid inefficiencies and implement the symmetric information outcome.

JEL Classifications: G32

Keywords: Signaling, Free-Rider Problem, Means of Payment, Restricted Bids, Two-dimensional Types

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\*We thank Patrick Bolton, François Desgeorges, Mike Fishman, Alan Morrison and seminar participants at Copenhagen University, the London School of Economics, NHH Bergen, Stockholm School of Economics, University of Amsterdam, the EFA Meeting in Bergen, and the ECGTN Conference in Barcelona for helpful comments. Financial support from the European Corporate Governance Training Network (ECGTN) and the Jan Wallander Foundation is gratefully acknowledged.

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# 1 Introduction

Presumably the best-known friction plaguing the market for corporate control is the free-rider problem (Grossman and Hart, 1980; Bradley, 1980): Small shareholders perceive their individual decision as negligible for the tender offer outcome, and hence do not tender unless the offer price matches at least the post-takeover share value. As a result, they extract all the gains in share value, which in turn may deter potential bidders.

Another friction which has arguably received less attention in the takeover literature is asymmetric information. In principle, the bidder as well as the target can possess relevant private information. Contrary to merger negotiations between two management teams, the information advantage in tender offers is likely to be one-sided. Since dispersed target shareholders do not actively monitor the firm, they seldom possess information not already impounded in the stock price. By contrast, the bidder has typically spent resources to identify the target and to devise post-takeover (restructuring) plans. To succeed, the bidder therefore has to credibly communicate that the offer price adequately compensates target shareholders. Otherwise, the offer will be rejected even though it may entail a takeover premium.

The interaction of information asymmetries and coordination failure makes tender offers distinct from the standard bilateral trade setting. One-sided asymmetric information does not affect the bilateral trade outcome when the informed party makes a take-it-or-leave-it offer and is able to match the other party's no-trade payoff. This is not true in tender offer games. Due to their free-riding behavior, uninformed shareholders have all the bargaining power, even though the informed party (bidder) moves first.

This paper explores how and when a bidder can signal her private information about the post-takeover security benefits to dispersed shareholders. To the best of our knowledge, this question has not yet been systematically analyzed. While it has been shown that separating equilibria can be constructed in tender offer games (Hirshleifer and Titman, 1990; Chowdhry and Jegadeesh, 1994), those equilibria require dispersed shareholders to randomize their tendering decisions in a "coordinated" way to produce specific takeover probabilities. The present paper derives the general principle lying beneath the existence of separating equilibria, thereby encompassing both previously identified and novel signaling devices.

Our analysis begins by considering a tender offer game with one-dimensional asymmetric information. Only the bidder knows the post-takeover security benefits. In addition, a successful bidder enjoys private benefits and cannot commit not to extract these benefits. In this setting, an impossibility result obtains: as shareholders extract all the gains in security benefits, the bidder cannot reveal her type through the offer terms. Thus, neither restricted

bids nor cash-equity offers are viable signals, which stands out against findings from bilateral merger models (e.g., Eckbo et al., 1990). The result exposes a fundamental conflict between incentive-compatibility and free-riding. The incentive-compatibility constraints require that high-valued bidders who have an incentive to mimic low-valued types earn information rents. However, the free-rider behavior precludes that these rents stem from gains in security benefits, as they are fully appropriated by the target shareholders. Thus, private benefits are a prerequisite for incentive-compatible revealing offers.<sup>1</sup>

But even if the takeover is associated with private benefits, they can only serve a signaling purpose if the bidder is able to commit to relinquish part of the private benefits. In addition, the bidder must abandon the private benefits in a manner that enables shareholders to infer the post-takeover security benefits. Hirshleifer and Titman (1990)'s probabilistic separating equilibrium showcases this principle. In their equilibrium, target shareholders randomize their tendering decision such that bids at lower prices fail with higher probability. The higher failure probability deters high-valued bidders from mimicking low-valued types. Crucially, the deterrence operates exclusively through the risk of forgoing private benefits. If bidders lack such benefits, or if high-valued bidders have substantially lower private benefits, the separating equilibrium breaks down.

A principal contribution of this paper is to identify this mechanism as a broad principle for the viability of signaling in tender offer games. Accordingly, signaling equilibria can be implemented through dilution, toeholds and debt-financing, even when tendering decisions are deterministic. These devices all allow the bidder to choose how much of the proceeds to divert or withhold from target shareholders, and the shareholders to infer the post-takeover security benefits. Specifically, the bidder can signal low security benefits by committing to dilute minority shareholders less, by choosing a smaller toehold, or by raising less debt finance. While each of these measures reduces her private benefits, they allow her to succeed at a lower price. The corresponding signaling equilibria therefore share the implication that the takeover price is positively correlated with the bidder's extraction of private gains (as exemplified by post-takeover dilution, toehold size, or takeover leverage). By contrast, in a tender offer game with symmetric information, such a positive correlation is not obtained as the bidder extracts as much private benefits as possible irrespective of her type.

In these signaling equilibria, lower-valued bidders must forgo more private benefits, either through failure or through reducing their level of private benefit extraction. As a result, the equilibrium outcomes typically exhibit inefficiencies at the "bottom": low-valued bidders

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<sup>1</sup>The result can also be cast in terms of signaling costs. A trustworthy signal must be costly for the bidder. For example, the bidder might *voluntarily* forgo gains in security benefits. Yet, the free-rider problem precludes this possibility by *forcing* the bidder to forgo all these gains.

are more prone to fail or do not even submit a bid. In particular, when takeover outcomes are deterministic, only bidders above a cut-off type make a bid in equilibrium, and a lower cut-off value implies more value-improving takeover activity.

Furthermore, the signaling equilibria are not robust to the introduction of additional private information. In a setting with two-dimensional bidder types, in which the bidder has additional information about her private benefits, signaling is no longer feasible. Essentially, relinquishing a given fraction of private benefits is no longer a viable signal because target shareholders are now unable to infer how costly such an offer is for the bidder.

Another finding is that bid restrictions, though either insufficient or redundant as a signal, promote takeover activity. This is because smaller transaction sizes mitigate the asymmetric information problem: With fewer traded shares, a bidder gains less from paying a price below the post-takeover share value. This reduces the incentives to mimic low-valued bidders, so that these bidders do not need to sacrifice as much private benefits to credibly reveal their type. Thus, more restricted bids translate into smaller signaling costs.

The positive impact of bid restrictions on takeover activity could be taken further if control did not require a majority stake. This suggests another interpretation of the asymmetric information problem in tender offer games: control transfers are impaired because control must be transferred along with *misvalued* cash flow rights. The appropriate solution is therefore to separate votes from cash flow rights. Indeed, we show that the use of non-voting shares or financial derivatives can generate signaling equilibria that completely eliminate the impact of asymmetric information. These financial instruments allow the bidder to buy the target shares against cash, strip the shares of their votes, repartition the cash flow rights and reissue only those cash flow rights that she wants to shed. While the first two steps give the bidder control, the last two steps can be used to self-impose penalties for “lying” about the post-takeover security benefits. In particular, call options enable target shareholders to seek “damages” from the bidder ex post if the security benefits turn out to be higher than ex ante professed. This makes the bid price de facto contingent on the post-takeover security benefits, thereby overcoming the information asymmetry. In practice, derivative claims are indeed used in takeovers; for instance, acquirers sometimes provide target shareholders with upside participation in the form of earn-outs to reduce the payment at the closing of the deal. These earn-outs are triggered only when the target’s post-takeover performance exceeds a certain threshold (Gallant and Ross, 2009).

Derivatives allow to implement the symmetric information outcome because the tender offer game differs from most other signaling models in corporate finance. In tender offer games, the gains from trade are typically realized upon the transfer of control, as opposed to the transfer of cash flow rights. Thus, in the market for corporate control, company shares

represent a bundle of two “goods”, cash flow and voting rights. Separating these goods is beneficial when frictions in the trade of one impede the trade of the other.

The tender offer game with privately informed bidders is a corporate finance application of the informed principal problem (Maskin and Tirole, 1990), with the added feature that the uninformed party suffers from a collective action (free-rider) problem. Grossman and Hart (1981) and Shleifer and Vishny (1986) offer the first analyses of asymmetric information in tender offer games. Both papers focus exclusively on pooling equilibria. Hirshleifer and Titman (1990) and Chowdry and Jegadeesh (1994) study tender offer games in which takeover outcomes are probabilistic. As we demonstrate, their separating equilibria are applications of a general principle which does not rely on the probabilistic tender offer outcome.<sup>2</sup>

Several papers show that the choice of payment method can overcome asymmetric information problems in mergers (Hansen, 1987; Fishman, 1989; Eckbo et al., 1990; Berkovitch and Naranayan, 1990). Importantly, all of these papers consider bilateral merger negotiations and hence abstract from the free-rider problem. With the exception of Berkovitch and Naranayan, these papers consider two-sided asymmetric information settings in which target shareholders know more either about the share value under the incumbent manager or the takeover synergies. Thus, the settings differ from ours in precisely those aspects that are characteristic of tender offers. The same holds true for Brusco et al. (2007) and Ferreira et al. (2007) who study cash-equity offers in a mechanism design framework. The problem they explore becomes rather simple under our informational assumptions, and pure cash offers would always implement the full information outcome.

The paper proceeds as follows. The next section presents the basic model with non-transferable private benefits. It shows that this model has no signaling equilibria and explains the importance of the free-rider problem for this result. In addition, we demonstrate that neither bid restrictions nor cash-equity offers are viable signals in this setting. Section 3 shows why and how signaling equilibria can be implemented. We first develop the principle in an abstract setting and then apply it to familiar variants of the tender offer game. We also show that additional private information eradicates fully revealing equilibria. Section 4 demonstrates how the symmetric information outcome can be implemented through the use of derivatives. Section 5 discusses the empirical implications of our analysis. Concluding remarks are in Section 6, and mathematical proofs are in the Appendix.

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<sup>2</sup>Signaling can also serve the purpose of deterring potential rivals as in Fishman (1988) and Liu (2008), rather than reducing the information gap between bidder and target shareholders.

## 2 Non-Transferable Private Benefits

Our basic setting closely follows existing tender offer models with asymmetric information (Shleifer and Vishny, 1986; Hirshleifer and Titman, 1990). There is a widely held firm that faces a single potential acquirer, henceforth the bidder. If the bidder gains control, she can generate security benefits  $X$ . The bidder learns her type prior to making the tender offer, whereas target shareholders merely know that  $X$  is distributed on  $\mathcal{X} = [0, \bar{X}]$  according to the continuously differentiable density function  $g(X)$ . The cumulative distribution function is denoted by  $G(X)$ . If the takeover does not materialize, the incumbent manager remains in control. The incumbent generates security benefits  $X^I$  which are known to all shareholders and normalized to zero. Thus, we restrict attention to the case of value-improving bids.

In addition, control confers exogenous private benefits  $\Phi \geq 0$  on the bidder. The private benefits are only known to the bidder and for simplicity a deterministic function of her type.<sup>3</sup> Furthermore, the bidder cannot commit not to extract the private benefits once she is in control. As the private benefits accrue exclusively to the bidder, they are de facto non-transferable. Our specification of private benefits can accommodate various sources of bidder gains, such as dilution (Grossman and Hart, 1980) and toeholds (Shleifer and Vishny, 1986). Though, for the sake of notational simplicity, we subsequently assume that the bidder has no initial stake.

As the firm has a one share - one vote structure, a successful tender offer must attract at least 50 percent of the firm's shares. The tender offer is conditional, and therefore becomes void if less than 50 percent of the shares are tendered. In addition, the bidder can restrict the offer to a fraction  $r \in [0.5, 1]$  of the shares. For simplicity, we assume that there are no takeover costs. Hence, the benchmark (full information) outcome is that all takeovers succeed.<sup>4</sup>

The timing of the model is as follows. In stage 0, the bidder learns her type  $X$ . In stage 1, she then decides whether to make a take-it-or-leave-it, conditional, restricted tender offer in cash. (Alternative means of payment will be considered later.) If she does not make a bid, the game moves immediately to stage 3. Otherwise, she offers to purchase a fraction  $r$  of the outstanding shares at a price  $rP$ .

In stage 2, the target shareholders non-cooperatively decide whether to tender their shares. Shareholders are homogeneous and atomistic. In stage 3, the incumbent manager

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<sup>3</sup>We analyse the more general case with two-dimensional bidder types later in the paper (section 4.2).

<sup>4</sup>Like other tender offer models exploring the free-rider problem, we assume that the firm's outstanding shares of mass 1 are dispersed among an infinite number of shareholders whose individual holdings are both equal and indivisible. When either of these assumption is relaxed, the Grossman and Hart (1980) result that all the gains in security benefits go to the target shareholders becomes diluted (Holmström and Nalebuff, 1992).

remains in control if the fraction of tendered shares  $\beta$  is less than 50 percent. Otherwise, the bidder gains control and pays  $\beta P$  unless the offer is oversubscribed, in which case she pays  $rP$ , and tendering shareholders are randomly rationed. We henceforth refer to the basic model as the tender offer game with non-transferable private benefits.

If target shareholders could coordinate their tendering decisions, they would accept the offer whenever the price at least matches the security benefits under the incumbent manager. Thus, their reservation price would be independent of the bidder's type, and the bidder would succeed and appropriate the entire value improvement from the takeover.

However, as the shareholders are atomistic and decide non-cooperatively, their reservation price depends on the bidder's type. Each of them tenders at stage 2 only if the offered price at least matches the expected security benefits. Since shareholders condition their expectations on the offer terms  $(r, P)$ , a successful tender offer must satisfy the free-rider condition  $P \geq E(X|r, P)$ . We assume that shareholders do not play weakly dominated strategies. This eliminates failure as an equilibrium outcome when the free-rider condition is strictly satisfied.<sup>5</sup>

When the bid price exactly equals the expected post-takeover share value, the target shareholders are strictly indifferent between tendering and retaining their shares. That is, they are indifferent between these actions irrespective of their beliefs about the takeover outcome, so that the weak dominance criterion does not pin down a tendering strategy. The prevalent way of resolving the indeterminacy when  $P = E(X|r, P)$  is to assume that each shareholder tenders in this case, and hence the bid succeeds with certainty.<sup>6</sup> Alternatively, one may assume that strictly indifferent shareholders randomize, and that this leads to a probabilistic outcome.<sup>7</sup> Subsequently, we focus on deterministic outcomes and examine the conditions under which fully revealing equilibria exist in which (some) bidders signal their type through the chosen offer terms. The exception is section 3.1.5 where we consider probabilistic outcomes of the basic model with non-transferable private benefits. Finally, to keep focus on the feasibility of signaling, we abstract from pooling equilibrium outcomes

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<sup>5</sup>Given a bid is conditional, a shareholder who believes the bid to fail is indifferent between tendering and retaining. Imposing this belief on all shareholders and breaking the indifference in favour of retaining supports failure as an equilibrium, irrespective of the offered price (Burkart et al., 2006). To avoid co-existence of success and failure as equilibrium outcomes, it is typically assumed that shareholders tender when they are indifferent (e.g., Shleifer and Vishny, 1986). Contrary to our assumption, this precludes failure as the equilibrium outcome for a conditional bid, and hence the existence of an equilibrium when the free-rider condition is violated.

<sup>6</sup>A common motivation for this approach is that the bidder could sway the shareholders by raising the price infinitesimally. Although this argument holds under full information, it does not apply in the asymmetric information setting, as even small price increases affect shareholders' expectations about the post-takeover security benefits.

<sup>7</sup>Judd (1985) shows that a continuum of i.i.d. variables can generate a stochastic aggregate outcome.

which exist irrespective of the transferability of private benefits.<sup>8</sup>

## 2.1 Impossibility of Signaling

Under the assumption that each shareholder tenders in case she is strictly indifferent, all shares ( $\beta = 1$ ) are tendered in a successful takeover. Accordingly, a successful restricted bid is oversubscribed, and the bidder randomly selects the fraction  $r$  among all shareholders whose shares she purchases. The remaining  $1 - r$  shareholders cannot sell and become minority shareholders.

The bidder's expected profit from a bid  $(r, P)$  is

$$\Pi(r, P) = q(r, P) [\Phi(X) + r(X - P)]$$

where  $q(r, P)$  denotes the success probability which is equal to 1 for  $P \geq E(X | r, P)$  and 0 otherwise. In a fully revealing equilibrium, the offer terms must be distinct across types that make a (successful) bid. This requires that each equilibrium offer satisfies the free-rider condition,  $P(X) \geq X$ , and the bidder's incentive-compatibility constraint

$$\Phi(X) + r(X) [X - P(X)] \geq \Phi(X) + r(X - P)$$

for all  $r \in [0.5, 1]$  and  $P \in \mathbb{R}$ .

**Theorem 1** *In deterministic tender offer games with non-transferable private benefits, no fully revealing equilibrium exists.*

Given that  $P(X) \geq X$ , a truthful bidder at best breaks even on the purchased shares, and her expected profit cannot exceed  $\Phi(X)$ . However, each type offering her actual security benefits cannot be an equilibrium outcome. If a type  $x$  would succeed with an offer  $rx$ , any type  $X > x$  would mimic type  $x$  to acquire shares at a price below their true value  $X$ . This also holds if each type would choose a different bid restriction  $r(\cdot)$ . Type  $X$ 's profits are higher when buying  $r(x)$  shares at a discount compared to buying  $r(X)$  shares at their fair price whether  $r(x)$  is smaller or larger than  $r(X)$ . These arguments eliminate  $P(X) = X$  combined either with a common  $r$  or a type-contingent  $r(\cdot)$  as possible equilibria. They also rule out outcomes in which some types offer more than their true security benefits but less than the highest-valued type's security benefits. Successful offers with  $P(x) \in (x, \bar{X})$  would

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<sup>8</sup>Pooling outcomes are extensively studied in Shleifer and Vishny (1986), Marquez and Yilmaz (2005) and At et al. (2008).

be mimicked by bidders of type  $X > P(x)$ . Thus, a bidder can credibly signal her type only by offering a sufficiently large premium such that  $P \geq \bar{X}$ .

Revealing her type with an offer  $P \geq \bar{X}$  is, however, not an attractive option for the bidder.<sup>9</sup> She can instead make a bid  $P = \bar{X}$  and restrict it to  $r = 0.5$ , the minimum fraction required to gain control. The less costly offer  $(0.5, \bar{X})$  succeeds as it satisfies the free-rider condition for all types (and any possible shareholder beliefs).

The inexistence result extends to settings where the private benefits are not a deterministic function of the bidder's type, but follow some—possibly type-contingent—density function. Indeed, the constraints in the bidder's maximization problem are not affected by the non-transferable private benefits. They cancel out in the incentive-compatibility constraint and they are not part of the free-rider condition.

Also, note that letting bidders choose the fraction of shares that they acquire does not enable them to signal their type. The sole function of the bid restriction is to limit the fraction of shares the bidder purchases for cash. This makes restricted bids in this setting equivalent to bids in which target shareholders are in part compensated through equity. Indeed, it is immaterial whether the bidder makes a partial bid for cash only or acquires all shares in exchange for some cash and  $1 - r$  shares in the target firm under her control. Moreover, control requires that the partial bid is for at least half the shares or that the equity component does not exceed the cash component in the cash-equity offer. By virtue of this equivalence, any fully revealing equilibrium in cash-equity offers would also have to exist in restricted cash only offers.

**Proposition 1** *Introducing cash-equity offers into deterministic tender offer games with non-transferable private benefits does not make fully revealing equilibria feasible.*

Proposition 1 contrasts with results from bilateral merger models where cash-equity offers can reveal the bidder's type (Hansen, 1987; Berkovitch and Naranayan, 1990; Eckbo et al., 1990).<sup>10</sup> Our basic framework differs in two key respects. First, target shareholders have no private information. Instead, they face a collective action problem, i.e., are unable to

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<sup>9</sup>In fact, an incentive-compatible schedule  $\{(r(\cdot), P(\cdot))\}$  can be constructed which entails that lower-valued bidders offer higher (per-share) prices but purchase fewer shares. Bidders abstain from mimicking lower-valued types as they would forego a larger more valuable equity stake. Conversely, bidders do not mimic higher-valued types because they would have to pay a larger total amount for an equity stake that is worth less.

<sup>10</sup>Contrary to negotiated mergers, tender offers are usually cash offers. In fact, the mode of acquisition is one of the most important determinants of the payment method (e.g., Martin, 1996). The standard explanation focuses on regulatory delays associated with equity offers, i.e., the greater cost of using equity as a means of payment. Our analysis suggests that means of payment do not help to overcome asymmetric information problems in tender offers.

coordinate their individual tendering decisions.<sup>11</sup> Second, the takeover is not undertaken to combine assets from two firms but to replace the incumbent managers. How or whether the free-rider problem affects signaling equilibria is studied in the next section, while the role of bidder assets will be explored later in the paper (section 3.1.3).

## 2.2 Free-Riding and Information Rents

To illustrate the role of the free-rider condition, we digress to a modified setting in which the bidder is able to appropriate part of the security benefits. Abstracting from a specific extensive form, we assume that the bidder has bargaining power  $\omega \in (0, 1)$  such that shareholders, if fully informed, would tender at a price  $P = (1 - \omega)X$ . Accordingly, the bidder would under full information appropriate a value improvement  $\omega X$  on the purchased shares. Like the private benefits  $\Phi$ , these gains depend on the bidder type and a successful takeover. But unlike the private benefits, the gains are transferable. That is, the bidder can (commit to) leave part of  $\omega X$  to the shareholders. A second, purely simplifying modification is the absence of private benefits ( $\Phi = 0$ ).

Given shareholders do not observe the bidder's type, they condition their beliefs on the offer terms and tender only if  $P \geq (1 - \omega)E(X|r, P)$ . Consequently, some bidders may not succeed or realize less than the full information profit  $\omega X$ . That is, some bidders may have to offer more than  $(1 - \omega)X$  or set  $r < 1$  to signal their type, while others may find such signals too costly.

**Proposition 2** *In the tender offer game with bidder bargaining power  $\omega \in (0, 1]$ , a fully revealing equilibrium exists. All types above the cut-off type  $X^c(\omega) \in [0, \bar{X})$  make a bid, and higher types buy more shares at a higher price and make a larger profit.*

Incentive-compatibility requires that both the fraction of shares offered to buy  $r(\cdot)$  and the price  $P(\cdot)$  increase with the bidder type. Higher-valued bidders offer to buy more shares at higher prices. On the one hand, bidders refrain from mimicking lower-valued types because the increase in the profit margin is offset by the smaller fraction of shares that can be bought at lower prices. On the other hand, bidders do not mimic higher-valued types as buying more shares requires paying a higher price.<sup>12</sup> Lower-valued bidders credibly reveal their type not

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<sup>11</sup>In merger models, the shareholders' reservation price is typically the stand-alone value of the target firm that although unknown to the bidder does not depend on her type. An exception is Berkovitch and Narayanan (1990) where shareholders' outside option is to wait for a competing bid, and this option value depends on the quality of the initial bidder relative to potential competitors.

<sup>12</sup>Proposition 2 implies that the bidder faces an upward-sloping supply curve: a larger demand for shares reveals a higher valuation which in turn raises target shareholders' ask price. This is akin to the downward-sloping demand curve that a privately informed issuer meets when selling securities (e.g., DeMarzo and

only by bidding for fewer shares but also by purchasing them at lower relative discounts  $|P(X) - X|/X$ , thereby having to concede an increasing fraction of their full information profit  $\omega X$  to the target shareholders. As a consequence, there exists a cut-off type  $X^c$  who just breaks even, offering exactly  $P = X^c$ . Conversely, the highest-valued bidder reaps her full information gains  $\omega \bar{X}$ . She can purchase all shares at  $P = (1 - \omega) \bar{X}$  because target shareholders always tender for  $P \geq (1 - \omega) \bar{X}$  irrespective of their beliefs.

Due to the above equivalence result (Proposition 1), Proposition 2 can also be phrased in terms of cash-equity offers. In this interpretation, the equilibrium offer schedule entails that higher-valued bidders use more cash and less equity. This is the same result as in the bilateral merger models of Eckbo et al. (1990) and Berkovitch and Narayanan (1990), though there is a subtle difference. In our setting, the bidder wants to signal low rather than high security benefits. This shifts the emphasis from cash as a high-value signal to equity as a low-value signal. Equity is a credible signal for low-valued bidders because relinquishing equity is costlier for high-valued bidders.

The positive relation between equilibrium profits and bidder types in Proposition 2 is a common feature of adverse selection models: incentive-compatibility requires that the types who have incentives to mimic others earn information rents (e.g., Laffont and Martimort, 2002). In our setting, higher-valued bidders receive these rents, and incentive-compatibility dictates the rate at which equilibrium profits decrease. Given the slope, the profit levels are determined by the boundary condition  $\Pi^*(\bar{X}) = \omega \bar{X}$ . That is, the highest type's equilibrium profit determines at which type the incentive-compatible profit falls to 0.

**Corollary 1** *As the bidder's bargaining power  $\omega$  approaches 0, the cut-off type  $X^c(\omega)$  converges to  $\bar{X}$ .*

Corollary 1 brings to light the impact that the free-riding behavior has on the feasibility of revealing offers when private benefits are non-transferable. The subset of bidders that can signal their type without incurring a loss (on the purchased shares) increases in the fraction of the share value improvement that each bidder would be able to appropriate under full information. In other words, as the target shareholders' free-riding behavior becomes more severe, the bidder's ability to signal her type gradually deteriorates. In the limit ( $\omega = 0$ ), no bidder type is able to make a profit on the purchased shares, and the separating equilibrium breaks down.

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Duffie, 1999). Though contrary to the informed seller setting, gains from trade materialize in tender offer games only if the bidder acquires a control stake. Therefore, trade collapses once the incentive-compatible supply of shares is less than 0.5.

It should be noted that separation fails even though the bidder’s objective function satisfies the single-crossing property.<sup>13</sup> The impact of the free-rider behavior on the (in)existence of signaling equilibria can be interpreted in two ways. From the perspective of lower-valued types, the free-rider condition eliminates the possibility of producing a costly signal. Given that target shareholders extract all the gains in security benefits, the bidder cannot surrender (part of) these gains to signal her type. From the perspective of higher-valued types, the free-rider condition wipes out information rents. A bidder who at best breaks even on truthfully purchased shares would always want to mimic a lower-valued type.

When bidders have no bargaining power ( $\omega = 0$ ) and no private benefits ( $\Phi = 0$ ), neither a fully revealing nor a pooling equilibrium exists, and trade virtually collapses as only the highest type makes a bid. It is worth comparing this outcome to Milgrom and Stokey (1982)’s no-trade theorem that asymmetrically informed but rational parties cannot agree to a transaction unless there are aggregate gains to be shared. In the tender offer game, such gains are present, since the takeover improves the target’s value. Still, the tender offer is from the bidder’s perspective equivalent to a trade without any aggregate gains, as they are entirely appropriated by the free-riding shareholders. That is, trade breaks down because, albeit the bidder makes a take-it-or-leave-it offer, shareholders have all the bargaining power.

### 3 Relinquishing Private Benefits

The preceding discussion suggests that key to signaling is the existence of bidder gains that are excluded from bargaining (private benefits) but can nonetheless be relinquished in a manner which allows inference about the security benefits. Subsequently, we show that this principle lies beneath the existence of fully revealing equilibria. We first derive it in a general setting and then implement it in well-known variants of the tender offer game in which the bidder can choose how much private gains she appropriates.

#### 3.1 Fully revealing equilibria

Suppose that the bidder can commit to transfer any fraction of her private benefits  $\Phi$  to the target shareholders, such that she retains a fraction  $\alpha \in [0, 1]$ . The tender offer is then a triple  $(r, \alpha, P)$ ; it specifies the fraction of shares offered to acquire, the retention rate of private benefits and the per share cash price. For a given offer, the bidder’s payoff from a successful takeover is

$$\Pi(r, \alpha, P; X) = \alpha\Phi(X) + r(X - P). \quad (1)$$

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<sup>13</sup>For each fixed  $(r, P)$ ,  $-\frac{\partial\Pi/\partial r}{\partial\Pi/\partial P}$  is strictly monotone in  $X$ .

If a signaling equilibrium exists, the equilibrium outcome can also be implemented as a direct (truth-telling) mechanism. Let  $\hat{X}$  denote a bidder's self-reported type. The bidder's problem can then be formulated as

$$\begin{aligned} \max_{\hat{X}} \Pi(\hat{X}; X) &= \alpha(\hat{X})\Phi(X) + r(\hat{X})[X - P(\hat{X})] \\ \text{subject to (a)} \quad &r(\hat{X}) \in [0.5, 1] \\ \text{(b)} \quad &\alpha(\hat{X}) \in [0, 1] \\ \text{(c)} \quad &P(\hat{X}) \geq 0 \\ \text{(d)} \quad &r(\hat{X})P(\hat{X}) + [1 - \alpha(\hat{X})]\Phi(\hat{X}) \geq r(\hat{X})\hat{X} \end{aligned} \tag{2}$$

for all  $\hat{X} \in \mathcal{X}$ , where (d) is the free-rider condition. Under a fully revealing offer schedule  $\{r(\cdot), \alpha(\cdot), P(\cdot)\}$ , the solution to this problem and hence to its first-order condition

$$r(\hat{X})P'(\hat{X}) = \alpha'(\hat{X})\Phi(X) + r'(\hat{X})[X - P(\hat{X})] \tag{3}$$

must be  $\hat{X} = X$  for all  $X \in \mathcal{X}$ .

**Theorem 2** *In tender offer games where bidders can commit to relinquish any fraction of their private benefits, a fully revealing equilibrium exists if  $\Phi(\cdot)$  is a non-decreasing function. All types above the cut-off type  $X^c \in [0, \bar{X})$  make a bid, and higher types relinquish a smaller fraction of their private benefits. The bid restriction is a redundant signal.*

Incentive-compatibility requires that both the fraction of private benefits retained  $\alpha(\cdot)$  and the price  $P(\cdot)$  increase with the bidder type. Higher-valued bidders offer to buy shares at higher prices but also retain more of their private benefits. Bidders do not mimic lower-valued types because the gains from paying the lower price are offset by the loss in private benefits. Conversely, bidders refrain from mimicking higher-valued types because the gain from retaining a larger fraction of the private benefits does not compensate for the higher price. Furthermore, since lower-valued types are endowed with (weakly) less private benefits, this signaling mechanism—relinquishing private benefits—can lead to an *interior* cut-off type  $X^c \in (0, \bar{X})$  whose retained private benefits  $\alpha(X^c)\Phi(X^c)$  just equal her total takeover premium  $r(X^c)[P(X^c) - X^c]$ . By contrast, the highest-valued type purchases her shares at  $P(\bar{X}) = \bar{X}$  and keeps her entire private benefits  $\Phi(\bar{X})$ , thus reaping her full information profit.

There exist multiple fully revealing equilibrium schedules. This is most evident from the first-order condition (3) which is a functional equation with two degrees of freedom: one can, for example, freely choose  $r(\cdot)$  and  $P(\cdot)$  and then determine  $\alpha(\cdot)$  to satisfy (3). A

natural choice seems to be  $r(X) = 1$  and  $P(X) = X$ . In that case, the bidder offers the full information price, and the takeover premium—i.e., the value of the tender offer over and above  $X$ —is equal to the forgone private benefits:  $[1 - \alpha(X)]\Phi(X)$ . This schedule most clearly shows that the signaling “cost” is ultimately paid out of the bidder’s private gains. However, the bidder can also transfer part of these gains by paying a higher price  $P(X) > X$ . This makes her offer less attractive to mimic so that she needs to relinquish less private benefits. Paying the shareholders through  $P(\cdot)$  or relinquishing part of  $\Phi(\cdot)$  are therefore substitutes, provided that lower-valued types transfer a sufficiently large fraction of their private gains.

A second source of multiplicity is the freedom of choice with respect to  $r(\cdot)$ . While the chosen  $r(\cdot)$  affects  $\alpha(\cdot)$  and  $P(\cdot)$  through (3), bid restrictions are a redundant signal in the sense that a fully revealing equilibrium can be supported even when the restrictions are uniform,  $r(X) = \bar{r}$ . The reason is that signaling is achieved by transferring private benefits, which does not rely on restricting the bid. Though, this is not to say that bid restrictions are irrelevant because they affect the efficiency of the tender offer outcome. We will come back to this point in Section 4.1.

Finally, the assumption that  $\Phi(\cdot)$  is non-decreasing ensures that relinquishing a given fraction of the private benefits is more costly for higher-valued types. This condition is sufficient but not necessary to obtain fully revealing equilibria. When the condition is violated, bidders may find it too expensive to separate themselves from higher-valued types so that only pooling equilibria are feasible.<sup>14</sup> Though there exist cases with non-monotonic  $\Phi(\cdot)$  in which fully revealing equilibria can nonetheless be supported. More importantly, however, the assumption that  $\Phi'(\cdot) \geq 0$  seems in general economically plausible. In particular, it is naturally satisfied in common variants of the tender offer game, as we subsequently show.

### 3.1.1 Dilution

In the first extension of the basic tender offer game, we let the successful bidder choose what fraction  $\phi$  of the firm’s total post-takeover value  $V \in \mathcal{V}$  to divert as private benefits. Also, dilution does not dissipate value, so that a successful bid generates security benefits  $X(V) = (1 - \phi)V$  and private benefits  $\Phi(V) = \phi V$ .

Diversion as a source of private benefits was first introduced by Grossman and Hart (1980), who assume a uniform dilution rate  $\phi$ . Burkart et al. (1998) endogenize the ex post dilution decision by assuming that diversion dissipates value, so that  $\phi$  is determined by the

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<sup>14</sup>For instance, when  $\Phi(\cdot)$  is strictly decreasing, no type  $X < \bar{X}$  is able to separate herself from the highest-valued type  $\bar{X}$  by paying a lower price and relinquishing a larger share of private benefits, as this offer—if it were preferred by type  $X$  over  $\bar{X}$ ’s offer—would be mimicked by  $\bar{X}$ .

bidder’s post-takeover equity stake. This implies that the bidder can de facto pre-commit to a dilution rate by choosing what fraction of target shares she acquires.

Here, we take a simpler approach and assume that the bidder can commit not to extract more than a fraction  $\phi \in [0, \bar{\phi}]$  of  $V$  independent of the bid restriction.<sup>15</sup> The upper bound  $\bar{\phi} < 1$  is an exogenous limit set by legal shareholder protection, business combination laws, or more broadly the corporate governance system.<sup>16</sup> In practice, a takeover activist or private equity fund may include self-imposed constraints in the restructuring plans or in the post-takeover governance structure (e.g., management team or board composition of a special purpose acquisition company) which are announced as part of the takeover proposal. Similarly, the business combination agreement may contain covenants that restrict certain types of post-takeover transactions that would be detrimental to minority shareholders.

The tender offer terms are now given by the triple  $(r, \phi, P)$ . If  $\phi$  were uniform across bidder types, the setting would be equivalent to the tender offer game with non-transferable private benefits, in which no separating equilibrium exists. However, as  $\phi$  is a choice variable, Theorem 2 can be implemented.

**Application 1** *Limiting dilution is a viable signal. All types above the cut-off type make a bid, and higher types extract a larger fraction of the firm value as private benefits.*

To reveal their type, lower-valued bidders relinquish a larger fraction of private benefits by choosing a lower extraction rate. Incentive-compatibility requires that both the extraction rate  $\phi(\cdot)$  and the bid price  $P(\cdot)$  be increasing in the bidder type. Bidders who dilute more must pay a higher price. Conversely, bidders who pay a lower price must “tie their hands”, i.e., commit to dilute less. It is as if bidders can buy a larger degree of post-takeover discretion (control) by paying a higher price. This positive relationship between dilution and bid premia reflects the signaling role of private benefits. Indeed, in the symmetric information settings of Grossman and Hart (1980) or Burkart et al. (1998), bidders have no incentive to voluntarily dilute less and choose the maximum extraction rate ( $\phi = \bar{\phi}$ ) irrespective of their type.

The cut-off type is decreasing in the overall scope for dilution which is determined by the quality of the corporate governance system. Better minority shareholder protection (lower  $\bar{\phi}$ -values) reduces the possibility for bidders to reveal their type by voluntarily constraining their ability to dilute minority shareholders. In parallel to Corollary 1,  $V^c(\bar{\phi})$  converges to

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<sup>15</sup>Since the bid restriction is a redundant signal, we could construct fully revealing equilibria where dilution depends on the bidder’s equity stake. To this end,  $r(\cdot)$  is set to pin down an increasing  $\phi(\cdot)$ . Given  $r(\cdot)$  and  $\phi(\cdot)$ ,  $P(\cdot)$  is then chosen to satisfy the first-order condition. That is, the validity of the principle carries over to the inefficient diversion setting of Burkart et al. (1998).

<sup>16</sup>Business combination laws prohibit certain kinds of transactions between a large shareholder and the firm usually for a period of three to five years after the shareholder’s stake passes a pre-specified threshold.

$\bar{V}$  as  $\bar{\phi}$  approaches 0, and the fully revealing equilibrium collapses in the limit. Finally, note that the bidder’s maximum private benefits  $\Phi(X) = \bar{\phi}X$ , which she would extract under symmetric information, are increasing in her type  $X$ . That is,  $\Phi(\cdot)$  is, by construction, a non-decreasing function.

### 3.1.2 Debt finance

Müller and Panunzi (2004) show that a leveraged tender offer, in which the takeover bid is partly financed through debt, can implement a takeover outcome that is quite similar to the outcome implemented by dilution. Initially, the bidder sets up a shell company that issues debt backed by claims on the target’s assets. She then makes a tender offer for the target shares and, if the bid is successful, merges the acquired firm with the shell company. The fact that the combined firm is indebted lowers its share value, which in turn lowers the bid price at which target shareholders are willing to tender their shares. Such a “bootstrap acquisition” allows the bidder to acquire the target at a lower price, and hence to appropriate part of the takeover gains.

Given that leverage functions as a source of “private benefits”, Theorem 2 suggests that it can also serve as a signaling device. To illustrate this possibility in the simplest fashion, consider the modified type space  $\mathcal{X}' = [\underline{X}, \bar{X}]$  where  $\underline{X} > 0$ . In addition, suppose that the bidder can raise debt up to  $D \in [0, \bar{D}]$  where  $\bar{D} < \underline{X}$  is an exogenously imposed limit. Thus, the tender offer terms are now given by the triple  $(r, D, P)$ .

**Application 2** *Debt financing is a viable signal. All types above the cut-off type make a bid, and higher types raise more debt.*

Application 2 showcases the general principle with less leverage corresponding to relinquishing more private benefits. Bidders do not mimic lower-valued types because the gains from purchasing the shares at a discount are offset by the decrease in leverage. Conversely, bidders do not mimic higher-valued types because the higher leverage is offset by the larger premium they would have to pay for the shares. In equilibrium, both  $D(\cdot)$  and  $P(\cdot)$  are increasing in the bidder type, i.e., more expensive takeovers go together with a larger amount of debt finance. The maximum possible amount of debt  $\Phi(X) = \bar{D}$ , which bidders would raise under symmetric information, is constant across bidder types. That is,  $\Phi(\cdot)$  is again a non-decreasing function. Similarly to before, the cut-off type is decreasing in  $\bar{D}$ , and takeover activity—absent other signaling devices—vanishes as  $\bar{D}$  approaches 0, i.e., as debt finance “dries” up.<sup>17</sup>

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<sup>17</sup>A thorough analysis of leverage as a signaling device in tender offers should, of course, relax the constraint

### 3.1.3 Bidder assets

The canonical tender offer game abstracts from bidder assets other than cash. When the bidder owns assets  $A > 0$ , the bidder could use claims to such assets to pay target shareholders, and the willingness to do so might reveal her type. Theorem 2 suggests that the viability of such signals requires that the bidder assets appreciate in value as a result of the takeover, and that this appreciation is non-negatively correlated with the value improvement in the target firm. In addition, these “synergy” gains must be exclusionary in that they do not accrue to non-tendering target shareholders and, as such, do not affect their reservation price (free-rider condition). In this sense, they are equivalent to private benefits.

Suppose that the bidder owns cash and a firm that generates security benefits  $Z + \lambda X$  if the bid succeeds and  $\lambda X$  otherwise. Both the value  $Z \geq 0$  and the parameter  $\lambda > 0$  are commonly known and the same for all types. If the bidder is successful, she combines the two firms in a holding company  $H$ . Target shareholders are offered a cash price  $C(\beta)$  and  $s(\beta)$  shares in the holding company, where  $\beta$  denotes the fraction of target shares tendered. Furthermore, target shareholders are cash-constrained, and the bidder is unwilling to relinquish majority control of the holding company. These assumptions impose two restrictions on the set of admissible offers, a *cash* constraint  $C(\beta) \geq 0$  and a *control* constraint  $s(\beta) \in [0, 0.5]$ .

**Application 3** *Sharing exclusionary synergy gains is a viable signal. All types above the cut-off type make a merger bid, and lower types pay more in equity of the merged company.*

The cash price  $C$  and the fraction of shares  $s$  that target shareholders receive in equilibrium are inversely related. Bidders who give target shareholders a larger fraction of the post-merger equity pay a smaller amount of cash. Even though  $\lambda X$  is perfectly correlated with  $X$ , the mere fact that the bidder assets are informative about the post-takeover share value is neither necessary nor sufficient to obtain a fully revealing equilibrium. The key is that the bidder assets appreciate in value as a result of the takeover, i.e., that the bidder enjoys the exclusionary gains  $Z$ . Relinquishing (more) post-merger equity is a means of sharing (more of) these “private benefits”. Accordingly, the cut-off type is decreasing in  $Z$ , and takeover activity—again, absent other signaling devices—vanishes as  $Z$  approaches 0. Note also that the value of the gains which the bidder would retain under symmetric information is  $\Phi(X) = Z + \lambda X$  and increasing in her type.

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$\bar{D} < \underline{X}$  and, more importantly, account for insolvency risks and financial distress costs, as in the more general framework of Müller and Panunzi (2004). We simply want to point out that bidders can use leverage as a means to reveal private information about the post-takeover share value to free-riding shareholders. Osano (2009) also analyzes the role of leverage in a tender offer game with private information but primarily focuses on pooling equilibria. In fact, the signaling incentives we describe here do not arise under his model assumptions.

The cash-equity equilibrium offers in Application 3 are similar to those found in the aforementioned means-of-payment literature. Though contrary to bilateral merger models, tender offer games with bidder assets do not require two-sided asymmetric information to generate a role for cash-equity offers. It is enough that the bidder has private information about the post-takeover value improvement in the target. As we will show in section 3.2, this signaling equilibrium collapses when the bidder has additional private information about the value of the her assets.

### 3.1.4 Toehold acquisition

Another source of bidder gains, first studied by Shleifer and Vishny (1986), are equity stakes that the bidder acquires prior to the tender offer (toeholds). Suppose that the bidder can purchase up to a fraction  $\bar{t}$  of the target shares in the open market—for simplicity, at the price of  $P = X^I$ —before she must make her takeover intentions public. The upper bound  $\bar{t}$  represents a mandatory disclosure, or mandatory bid, rule that prevents the bidder from acquiring an even larger pre-bid stake in the target.

**Application 4** *Toeholds are a viable signal. All types above the cut-off type make a bid, and higher types acquire larger toeholds.*

The signaling potential of endogenous toeholds has already been analyzed within a probabilistic tender offer game by Chowdhry and Jegadeesh (1990). (We turn to the probabilistic tender offer game below). Our analysis shows that toeholds as a signal are one implementation of the general principle, and that they do not rely upon probabilistic outcomes.

### 3.1.5 Probabilistic outcomes

Here we revisit the tender offer game with non-transferable private benefits. Contrary to before, we assume that strictly indifferent shareholders randomize their tendering decision, and that this results in an uncertain outcome. A prerequisite for a probabilistic outcome is that the offered price exactly matches the expected security benefits. Otherwise, shareholders either always or never tender. Given an offer  $P = E(X|r, P)$ , the success probability  $q(r, P)$  can lie anywhere in  $[0, 1]$ , and the expected fraction of acquired shares  $\gamma(r, P)$  can lie anywhere in  $[0.5, r]$ . The bidder’s expected profit from a bid  $(r, P)$  is therefore

$$\Pi(r, P) = q(r, P) [\Phi(X) + \gamma(r, P) (X - P)].$$

**Application 5** *In the probabilistic tender offer game with non-transferable private benefits, a fully revealing equilibrium exists if  $\Phi(\cdot)$  is a non-decreasing function. All types make a bid, and higher-valued types are more likely to succeed.*

In equilibrium, a lower-valued bidder pays a smaller price but her bid is less likely to succeed. The higher failure rate protects her bid from being mimicked by higher-valued types. Importantly, this deterrence effect *exclusively* operates through the risk of losing *private* benefits. In fact, if  $\Phi(\cdot) = 0$  or even if merely  $\Phi(\bar{X}) = 0$ , the signaling equilibrium breaks down.

Revealing bids with probabilistic outcomes are but another illustration of Theorem 2. The specific feature is that bidders do not signal their type through conceding private benefits *to the shareholders*, but rather through “burning” private benefits by way of failure. Common to the other applications, the outcome is inefficient: While all types actually make a bid in equilibrium, bids do not always succeed. Furthermore, the bid restriction remains a redundant signal and there are multiple feasible equilibrium schedules. Hirshleifer and Titman (1990) select a schedule where all types restrict their bid as much as possible ( $r = 0.5$ ).

### 3.2 Another Impossibility of Signaling

Theorem 2 and its various applications demonstrate how crucial private benefits are for the bidder’s ability to signal. However, their existence is merely a necessary but not a sufficient condition. The bidder must also be able to relinquish private benefits in a manner which is *informative*. More specifically, the target shareholders must be able to evaluate how costly a given signal is to the bidder, or else the signal may not be sufficiently credible.

To examine the informativeness of private benefits, we abandon the assumption that the bidder’s security benefits perfectly predict her private benefits (i.e., that  $\Phi$  is a deterministic function of  $X$ ). Instead, we assume that the bidder types are *two-dimensional*,  $(X, \Phi)$ , and continuously distributed on  $[\underline{X}, \bar{X}] \times [\underline{\Phi}, \bar{\Phi}]$ . The bidder is informed about both dimensions of her type. In contrast, the target shareholders neither know how much a particular bidder will improve the share value nor how much she values control. The setting is otherwise the same as in Theorem 2. For expositional convenience, we characterize the tender offer terms by the triple  $(r, \alpha, C)$  where  $C \equiv rP$ .

**Theorem 3** *In tender offer games with two-dimensional bidder types, no fully revealing equilibrium exists even when bidders can commit to relinquish any fraction of their private benefits.*

Signaling breaks down in the two-dimensional case because the private information about  $\Phi$  undermines the “credibility” of the retention rate  $\alpha$  (i.e., the relinquishing of private benefits) as a signal. Since  $\Phi$  is not a deterministic function of  $X$ , target shareholders cannot infer from the offer  $(r, \alpha, C)$  how costly it is for a bidder of (any) type to concede  $1 - \alpha$  of her private benefits. The uncertain relation between  $\Phi$  and  $X$  “jams” the signal. Theorem 3 thus weakens the case for fully revealing equilibria that rely on the use of the bidder’s private benefits as a means of revealing information about the security benefits.<sup>18</sup>

Whether private benefits  $\Phi$  are (un)informative about the security benefits  $X$  depends on the specific setting. In the case of dilution, toeholds or leverage, the relation between private benefits and security benefits is relatively well-defined by the “extraction” technology. Hence, we would argue that  $\Phi$  being a deterministic function of  $X$  seems a reasonable approximation. By contrast, it seems more plausible that bidder assets are at least to some extent unrelated to the target assets, and that the bidder has additional private information about the value of her assets. Theorem 3 thus challenges the robustness of the signaling equilibrium in mergers (Application 3), reinforcing the conclusion from Proposition 1 that cash-equity offers appear to be a less effective signaling device in tender offers than in bilateral negotiations.

## 4 Separating Votes and Cash Flows

In the above signaling equilibria, lower-valued bidders must relinquish more private benefits. As a result, the equilibrium outcomes typically exhibit inefficiencies at the “bottom”: low-valued bidders are more likely to fail or do not even submit a bid. In particular, when takeover outcomes are deterministic, only bidders above a cut-off type make a bid in equilibrium, and a lower cut-off value implies more value-improving takeover activity. This raises the question to what extent the tender offer can be shaped to improve the efficiency of the equilibrium outcome.

### 4.1 Efficiency of Restricted Bids

We address this issue in the framework with one-dimensional bidder types and transferable private benefits, i.e., the setting of Theorem 2 and its various applications. While the bid restriction  $r$  is a redundant signal in this setting, it affects the cut-off type and hence takeover activity.

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<sup>18</sup>Pooling equilibria also exist in the case of two-dimensional bidder types (Burkart and Lee, 2009).

**Theorem 4** *In tender offer games where bidders can commit to relinquish any fraction of their private benefits and  $\Phi(\cdot)$  is a non-decreasing function, restricting bids promotes (more) takeover activity.*

Bid restrictions promote takeover activity because smaller transaction sizes mitigate the asymmetric information problem: With fewer traded shares, a bidder gains less (in total) from paying a price below the post-takeover share value. This reduces the incentives to mimic low-valued bidders, so that these types do not need to sacrifice as much private benefits to credibly reveal low security benefits. Thus, more restricted bids translate into smaller signaling costs. This argument implies that the most efficient equilibrium is achieved when all types restrict their bid as far as possible ( $r = 0.5$ ). Like Theorem 2, this result applies to the different variants of the tender offer games studied in section 3.1.

In essence, Theorem 4 says that efficiency is *decreasing* in the fraction of shares traded.<sup>19</sup> Thus, unlike in other trade models with asymmetric information, quantity rationing is not an inefficient outcome but a means of mitigating the trade inefficiency. This counterintuitive result obtains because the gains from trade in tender offers are contingent on the transfer of votes, rather than the transfer of cash flow rights as in security issues.<sup>20</sup>

## 4.2 Dual-Class Offers

A share (trade) represents a (trade of a) bundle of two goods with potentially distinct values. This defining feature of tender offer games suggests an alternative perspective on the asymmetric information problem: value-improving control transfers are impeded because control must be transferred in conjunction with potentially misvalued cash flow rights. Hence, efficiency would be further improved if the bidder could gain majority control without acquiring a majority stake, i.e., if she could separate votes and cash flows to prevent the informational frictions of the cash flow trade from spilling over into the vote trade.<sup>21</sup>

Perhaps the most straightforward way to unbundle control and ownership is to make a dual-class security-exchange offer. The bidder offers to exchange each of the target's voting shares against a non-voting share. Shareholders accept the bid as it preserves their fraction of the cash flow rights. By construction, the bidder pays exactly the post-takeover security benefits to gain control. This replicates the full information outcome without revealing the bidder's type.

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<sup>19</sup>At et al. (2008) show that Theorem 4 also holds in pooling equilibrium outcomes.

<sup>20</sup>This is not true if the bidder's incentives to improve the target value vary with her post-takeover stake. Separating cash flow and voting rights can in this case be beneficial even in the absence of asymmetric information (Burkart et al., 1998).

<sup>21</sup>For this reason it can be both socially and privately optimal for target shareholders to adopt a dual-class share structure (At et al., 2008).

Despite resolving the asymmetric information problem, the dual-class offer is problematic because it leaves all cash flow rights with the shareholders. That is, the bidder has no equity interest in the firm after the takeover. On the one hand, this makes the offer equivalent to a simple replacement of management, which begs the question why a takeover is necessary in the first place. On the other hand, it makes the offer prone to abuse by value-decreasing bidders (or "fly-by-night" operators), since it does not require the bidder to put up any cash (Bebchuk and Hart, 2001). Cash payments put at least some (lower) bounds on the bidder's quality.

### 4.3 Options

Value-increasing bids fail as bidders are not—or only to a limited extent—willing to pay for cash flows which they know do not to exist. To resolve this problem, dual-class offers or other extreme solutions, which leave the bidder with no equity interest in the firm ex post, are not necessary. Instead, the solution is to let the bidder acquire the target in exchange for cash and a set of securities which leave the “non-existing” cash flows to target shareholders. Such a trade can be implemented with call options. It merely requires that every type  $X \in \mathcal{X}$  purchases a target share in exchange for cash  $P(X) = X$  and a (cash-settled) call option with an exercise price of  $S(X) = X$ .

**Proposition 3** *Offers with call options allow to implement the full information outcome.*

If a type  $X$  would succeed with an offer  $x$  where  $X > x$ , she would pay a cash price  $x$  for shares that are worth  $X$ . However, ex post she would not capitalize on this gain, as the target shareholders would exercise their options once the actual value improvement becomes known. Conversely, the low-valued type does not mimic the high-valued type because she would pay  $X$  for shares that are worth  $x$ . Thus, the offer schedule is incentive-compatible. Moreover, every bidder succeeds, irrespective of how many shares she acquires or whether she enjoys any private benefits.

Financial engineering enables the bidder (i) to trade economic ownership void of voting rights and (ii) to issue contingent claims. The first step of the transaction consists of acquiring the target shares and stripping them of their votes. In the second step, the bidder re-issues the cash flow rights, restructured into claims that punish her for “lying” about the security benefits. The call options which are executed when the post-takeover security benefits are higher than professed penalize the pretense of low security benefits—ex post when the true value is observed. This makes the offer equivalent to the simplest solution to the asymmetric information problem: a bid price which is contingent on the post-takeover share value.

The above offer transfers, cash aside, only future claims but no actual future cash flows to target shareholders. This is an artefact of the assumption that the post-takeover security benefits  $X$  are deterministic (i.e., perfectly known by the bidder). Yet, the main intuition carries over to a setting with stochastic cash flows. Let  $X \in [0, \infty]$  be a random variable. Suppose that there are  $n$  bidder types  $\theta \in \Theta \equiv \{1, 2, \dots, n\}$ , each knowing the probability density function  $f_\theta(X)$  of her post-takeover cash flows. In addition, we assume that the family of densities  $\{f_\theta(X)\}_{\theta \in \Theta}$  satisfies the strong monotone likelihood ratio property (SMLRP). That is, for all  $\theta' > \theta$ ,  $f_{\theta'}(X)/f_\theta(X)$  is strictly increasing.

To construct a fully revealing equilibrium, we allow the bidder to pay cash and issue bonds and barrier options.<sup>22</sup> The barrier option used to construct the equilibrium is a (cash-settled) “knock-in” (or “up-and-in”) call option. This is a latent call option with an exercise price of  $S$  that becomes activated only once the security benefits  $X$  exceeds some “trigger” level  $T > S$ . When they are exercised, these options dilute the value of the firm’s equity. Hence, the combination of bonds and knock-in options can alternatively be interpreted as a reduced-form implementation of convertible bonds. To simplify the exposition, we further assume that  $\Phi = 0$ .

**Proposition 4** *In the tender offer game with stochastic post-takeover security benefits, a fully revealing equilibrium exists if SMLRP holds. All bidder types make a bid and purchase the target shares for a combination of cash, bonds and knock-in call options.*

The security design solution in Proposition 4 offers both upside and downside “protection” to the sellers. The security issuer—here, the bidder—primarily wants to signal a high value. To this end, she issues knock-in options that are designed to transfer some high cash flow realizations to the target shareholders. However, the use of knock-in options makes the bidder prone to being mimicked by (even) lower-valued types. Because this casts doubt on the value of the offered options, she must include bonds to separate herself from lower-valued types. Thus, the bidder’s need to separate from lower-valued types through offering downside protection *derives endogenously* from the bidder’s *primary* intention to separate from higher-valued types through offering upside protection.<sup>23</sup>

A recent strand of literature emphasizes the potential adverse effects of using contingent claims to unbundle control and ownership (e.g., Hu and Black, 2006, 2008). This literature

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<sup>22</sup>A barrier option is a derivative where the option to exercise depends on the (price of the) underlying crossing or reaching a given barrier level.

<sup>23</sup>It is instructive to compare this outcome to the security design solutions in Myers and Majluf (1984) and Duffie and DeMarzo (1999). There, the issuer is (only) concerned with signaling a high value and therefore issues debt-like securities that are truncated from above. Here, in tender offer games, the bidder wants to signal a low value, which is why she (also) issues securities that are truncated from below.

contends that re-allocating control void of economic ownership can lead to inefficient decision-making (“empty voting”). For example, acquiring control but leaving the “upside” to the target shareholders could undermine the bidder’s incentives to increase the share value. In a moral hazard setting, where the bidder’s post-takeover incentives depend on the nature of the economic interest that she acquires in the firm, it would therefore seem both socially and privately suboptimal for the bidder to pay target shareholders with knock-in options.

Proposition 4 thus suggests that adverse selection and moral hazard do not yield similar predictions in tender offer games. Note that this is not the case in external financing models where moral hazard and adverse selection both predict that control and ownership should be bundled and that, if they need to be separated, the controlling party should retain the “upside”.<sup>24</sup> In tender offer games, *unbundling*—rather than *bundling*—control and ownership can improve the efficiency of the control allocation. Moreover, conceding the “upside” to outside shareholders can be optimal for the controlling shareholder.

Key to the benefit of unbundling is that the two counterparties, the bidder and the target shareholders, value cash flow rights and voting rights differently. As a result, asymmetric information about one set of rights can impede the (efficient) transfer of the other set of rights. Such situations are by no means confined to tender offers. It can arise in any transaction involving bundles of goods, in which the buyer and the seller not only value the goods differently but also have different levels of information about their quality. Adopting this a view for cash flow and voting rights can lead to new insights for corporate governance and security design.

## 5 Empirical Implications

Due to the free-rider problem, signaling in tender offers is rather different from signaling in bilateral merger negotiations. This is most evident in our finding that cash-equity offers are much less likely to provide the signaling benefits that they are associated with in the merger literature (Corollary 1 and Proposition 3). Empirically, this suggests that

**P1** The use of all-cash or cash-equity offers should be less pronounced in (hostile) tender offers than in negotiated takeovers.

**P2** The proportion of cash used as a means of payment should be less strongly correlated with the takeover gains in tender offers than in negotiated takeovers.

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<sup>24</sup>See e.g., Jensen and Meckling (1976), Leland and Pyle (1977), Myers and Majluf (1984), Innes (1990), Duffie and DeMarzo (1999).

The basic problem in models of bilateral merger negotiations under asymmetric information is a lemon’s problem, namely that informed parties are prone to *overstate* the value of their assets as e.g., in Myers and Majluf (1984). In contrast, tender offers with privately informed bidders are an example of a “smart buyer” problem, in which the informed bidder has an incentive to *understate* the value of the asset. One way for the bidder to credibly reveal a lower value is to share more of her private gains from trade, so long as the willingness to do so allows the uninformed seller(s) to infer the true asset value (Theorem 2). Common to such signaling equilibria is the prediction that takeover premia should be positively correlated with the fraction of the takeover surplus that the bidder extracts as private benefits (hereafter, relative private benefits). By comparison, in the absence of asymmetric information, bidders would appropriate the maximum possible amount of private benefits, in which case bid premia and the bidder’s relative private benefits would either be unrelated (in the case of dilution or toeholds) or inversely related (in the case of leverage).

Since private benefits are difficult to measure directly, empirical tests of the relationship between takeover premia and private benefits may have to rely on proxies for private benefit extraction. For such tests, the preceding analysis suggests that

**P3** Across tender offers, takeover premia should be positively correlated with fewer voluntary governance restrictions for the bidder, greater amounts of bidder debt in bootstrap acquisitions, or larger toehold sizes.

A potential caveat is that these predictions may empirically be of second-order importance, as the choice of governance, toeholds or debt may primarily be driven by other factors, such as free cash flow problems (Jensen, 1986) or aggressive bidding strategies (Chowdhry and Nanda, 1993; Bulow et al., 1999). For instance, if the bidder’s most important concern is to deter potential rival bidders, the presence of toeholds might coincide with lower takeover premia, contrary to the above prediction (see e.g., Betton et al., 2008).

There exist only few empirical studies regarding the role of debt finance in takeovers. Schlingemann (2004) finds no significant relationship between the amount of debt financing before a takeover announcement and bidder gains. In contrast, Martynova and Renneboog (2009) find that debt financing is associated with positive valuation effects, i.e., represents a positive signal to investors.<sup>25</sup> For a more in-depth analysis of the effects of takeover leverage, however, one should discriminate between tender offers and negotiated mergers, contested and uncontested tender offers, and bootstrap and non-bootstrap acquisitions.

Another way for bidders to signal their true value is to include contingent claims in their tender offer (Propositions 3 and 4). The distinctive feature of this security design solution is

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<sup>25</sup>See also the paper by Toffanin (2005) cited in Betton et al. (2008).

that, in order for the bidder to placate target shareholders' fears of being short-changed, the tender offer includes claims that become effective if (and only if) the post-takeover target value turns out to be higher than professed at the time of the offer. One example for such claims are convertible bonds. Another example are *earnouts* which trigger payments to the sellers only if—some time after the deal has been closed—certain performance targets are achieved. Our analysis thus suggests that

**P4** Bidders avoid overpaying for the target by offering option-like claims, such as convertible bonds or earnouts, that pay off in case the target's post-takeover performance exceeds a certain threshold.

The following quote from a practitioner's memo about special purpose acquisition companies (SPACs) supports the notion that bidders may provide upside participation in the form of earnouts to reduce the payment at the closing of a deal:<sup>26</sup>

Using earnouts, the SPAC is able to pay a lesser guaranteed amount to the sellers of the target business on the closing date and will have to pay the additional amounts only if the operating business meets or exceeds expectations. Although the sellers receive a smaller payment on the closing date, they have the chance to receive significantly more than they otherwise would have if they were being paid solely for the current value of the business. Earnout arrangements can therefore benefit both parties and increase the likelihood of the transaction being approved.

There are a few academic studies on the use of derivative securities in control transactions. In his textbook, Gaughan (2002) describes a wide range of derivatives used in mergers and acquisitions, including claims that provide sellers with both downside *and* upside protection. Officer (2004) studies the use of collars in mergers. Datar et al. (2001) and Ragozzino and Reuer (2009) study earnouts, and Finnerty and Yan (2009) study convertibles. All of these papers report evidence suggesting that acquirers use derivatives to mitigate asymmetric information problems. Though, the working hypotheses almost purely focus on lemon's problems (sell-side private information) and ignore potential smart buyer problems (buy-side private information).

## 6 Conclusion

This paper analyzes tender offers in which a single bidder is better informed about the post-takeover share value than dispersed target shareholders. Two key features of the tender

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<sup>26</sup>The quote is taken from Gallant and Ross (2009).

offer process render this situation very different from standard bilateral trade models. First, free-riding shareholders have full bargaining power over the value improvement in the target shares, even though the better informed bidder makes a take-it-or-leave-it offer. Second, the parties in a tender offer bargain both over control (voting rights) and over ownership (cash flow rights) in the target firm. That is, unlike other signaling models in finance, a share (trade) represents a (trade of a) bundle of two goods with potentially distinct values.

We demonstrate that these differences lead to constraints as well as solutions that are absent in bilateral trade models. Because the bidder is forced to concede all gains in share value to the shareholders, she cannot signal her type by *voluntarily* giving up such gains. Neither restricted bids nor cash-equity offers are therefore viable signals in tender offers. Instead, the bidder must enjoy private benefits that are not only excluded from bargaining but can also be forgone in a manner which allows inference about the post-takeover share value. This is never possible when the bidder has additional private information about the private benefits, as in the case of two-dimensional bidder types. In the one-dimensional case, relinquishing private benefits is a viable signal because they depend on the security benefits in a predictable manner. Dilution, debt financing or toeholds can serve this purpose. The underlying principle in all cases is the same: the bidder must forgo (more) private benefits to signal a low(er) type. Unfortunately, some low-value bidders may find it too costly to signal their type even if the takeover would be efficient.

Such inefficiencies can be overcome if the bidder can include derivatives in the tender offer terms. Derivatives allow the bidder to separate cash flow rights from voting rights. This separation prevents that the information problems in the trade of cash flow rights spill over into, and thereby impair, the trade of voting rights. As a result, control can be transferred efficiently irrespective of any potential disagreement between the bidder and the target shareholders about the value of the post-takeover cash flow rights.

Our analysis has implications for the design of takeover bids. For instance, it suggests that derivatives as a means of payment are an effective signaling device in takeovers, while cash-equity offers should play a less prominent role in tender offers than in negotiated mergers. Furthermore, acquiring firms may signal their quality through self-imposed restrictions on post-takeover decisions or through the amount of takeover leverage. The main theoretical contribution of this paper is to study how the interaction of asymmetric information and collective action problems, in a specific market setting, may bear on the optimal design of a trade contract. We believe that there are situations other than tender offers in which such interactions are potentially important as e.g., renegotiations with dispersed bondholders.

# Proofs

## Proof of Proposition 1

Proposition 1 follows from the equivalence of mixed offers and restricted cash-only offers which the subsequent lemma establishes. Consider a bid for  $r$  shares that offers a cash price  $C$  and  $t$  shares in the post-takeover firm.

**Lemma 1** *Under full information, the restricted mixed offer  $(r, C, t)$  and the restricted cash-only offer  $(r^{co}, C^{co})$  with  $C^{co} = C$  and  $r^{co} = r - t$  are payoff-equivalent.*

**Proof.** To succeed, the mixed offer must satisfy the free-rider condition  $C + tX \geq rX$ , or equivalently

$$C/r + (t/r) X \geq X. \quad (4)$$

Given the condition is satisfied, all shareholders tender, and the bidder's payoff is

$$\Phi(X) + r [X - (C/r + (t/r) X)]. \quad (5)$$

Rearranging the free-rider condition (4) to

$$C \geq (r - t) X$$

and the bidder's payoff (5) to

$$\Phi(X) + (r - t) X - C$$

shows that the restricted cash-only offer  $(r^{co}, C^{co})$  with  $C^{co} = C$  and  $r^{co} = r - t$  is payoff-equivalent for any  $X$ . ■

Hence, if a fully revealing equilibrium in mixed offers were to exist, a fully revealing equilibrium in cash-only offers would also exist. As Theorem 1 rules out the latter, a mix of cash and equity is not a viable signal. ■

## Proof of Proposition 2

We first characterize properties of an incentive-compatible  $r$ - $P$ -schedule.

**Lemma 2** *In a fully revealing equilibrium,  $r(\cdot)$ ,  $P(\cdot)$  and  $\Pi(\cdot)$  must be increasing.*

**Proof.** Without loss of generality, consider an arbitrary pair of types,  $X$  and  $x$  with  $X > x$ . A fully revealing schedule  $\{(r(\cdot), P(\cdot))\}$  must satisfy the non-mimicking constraints

$$r(X) [X - P(X)] \geq r(x) [X - P(x)] \quad \text{for } (x, X) \in \mathcal{X}^2. \quad (6)$$

We show by contradiction that (6) requires  $r(X) > r(x)$ . The non-mimicking constraints for type  $X$  and  $x$  are

$$C(X) - r(X)X \leq C(x) - r(x)X \quad \text{and} \quad C(x) - r(x)x \leq C(X) - r(X)x \quad (7)$$

where  $C(\cdot) \equiv r(\cdot)P(\cdot)$ . For  $r(X) = r(x)$ , the inequalities hold jointly only if  $C(X) = C(x)$ , and hence  $P(X) = P(x)$ , in which case the two offers would be identical. For  $r(X) < r(x)$ , rewrite (7) as

$$C(x) \geq C(X) + [r(x) - r(X)]X \quad \text{and} \quad C(x) \leq C(X) + [r(x) - r(X)]x.$$

Since  $C(X) + [r(x) - r(X)]X > C(X) + [r(x) - r(X)]x$ , the constraints cannot hold jointly. Thus, the non-mimicking constraints are violated unless  $r(\cdot)$  is increasing.

Given  $r(X) > r(x)$ , condition (6) implies that the bid price and the bidder's profit must also be increasing in her type. To this end, we rewrite (7) as

$$r(X)[X - P(X)] \geq r(x)[X - P(x)] \quad \text{and} \quad \frac{r(X)}{r(x)}[x - P(X)] \leq x - P(x).$$

Given that  $r(X)/r(x) > 1$ , the second inequality implies  $P(X) > P(x)$ . Furthermore, as  $r(x)[X - P(x)] > r(x)[x - P(x)]$ , the first inequality implies  $r(X)[X - P(X)] \geq r(x)[x - P(x)]$ . Thus, higher types must pay higher prices and make higher profits. ■

**Local Optimality.** Lemma 2 states necessary conditions for incentive-compatibility. To derive a particular schedule, we assume that  $r(\cdot)$  and  $C(\cdot)$  are continuously differentiable functions, and rephrase the bidder's optimization problem as a direct truth-telling mechanism:

$$\max_{\hat{X} \in \mathcal{X}} \left\{ r(\hat{X})X - C(\hat{X}) \right\}. \quad (8)$$

In equilibrium, the first-order condition must hold at  $\hat{X} = X$ , i.e.

$$r'(X)X = C'(X). \quad (9)$$

**Quasi-concavity.** Condition (9) is sufficient to ensure incentive-compatibility if the objective function in (8) is quasi-concave (and out-of-equilibrium beliefs are suitably chosen). Substituting (9) into the derivative of the objective function gives

$$\left. \frac{\partial}{\partial \hat{X}} \left[ r(\hat{X})X - C(\hat{X}) \right] \right|_{C'(\hat{X})=r'(\hat{X})\hat{X}} = r'(\hat{X})X - r'(\hat{X})\hat{X} = r'(\hat{X})(X - \hat{X}).$$

Given that  $r'(\cdot) > 0$  (Lemma 2), it follows that the derivative switches sign for all types  $X \in (0, \bar{X})$  once (from positive to negative), and the objective function is strictly quasi-concave.

**Cut-off type.** Condition (9) puts a constraint on how equilibrium profits  $\Pi^*(X) = r(X)X - C(X)$  vary across types. By the envelope theorem,

$$\frac{\partial \Pi^*(X)}{\partial X} = \underbrace{r'(X)X - C'(X)}_{=0} + r(X) = r(X). \quad (10)$$

That is, the marginal change in profits is given by the bid restriction  $r(X)$ .

Given that bidders have bargaining power  $\omega$ , shareholders always tender at the price  $P = (1 - \omega)\bar{X}$ . As type  $\bar{X}$  buys shares below their true value, she buys all shares and makes a profit  $\Pi^*(\bar{X}) = \omega\bar{X}$ . Since profits decrease at the rate  $r(X)$  (condition (10)), the cut-off type  $X^c$ , making zero profits, is defined by

$$\int_{X^c}^{\bar{X}} r(u) du = \omega\bar{X}. \quad (11)$$

**Out-of-equilibrium beliefs.** The proposed schedule can be supported as a signaling equilibrium with out-of-equilibrium beliefs that any deviation comes from the highest-valued bidder type  $\bar{X}$ . Under these beliefs, the target shareholders do not tender their shares in response to a deviation bid  $(\tilde{r}, \tilde{P})$  unless  $\tilde{P} \geq (1 - \omega)\bar{X} = P(\bar{X})$ . Consider two cases. (i) For bidder types  $X \in [P(\bar{X}), \bar{X}]$ , the deviation bid  $(\tilde{r}, P(\bar{X}))$  would yield a positive profit. Yet, it is dominated by the  $(r(\bar{X}), P(\bar{X})) = (1, P(\bar{X}))$ , the equilibrium bid of the highest type, which we know to be mimicking-proof. Hence, by implication, the deviation is unattractive to these types. (ii) For bidder types  $X \in [0, P(\bar{X})]$ , the deviation bid would yield a loss and is therefore unattractive to these types. ■

## Proof of Corollary 1

From the definition of the cut-off type (equation (11)), it follows that  $\partial X^c / \partial \omega < 0$  and  $\lim_{\omega \rightarrow 0} X^c = \bar{X}$ . ■

## Proof of Theorem 2

Given that  $\Phi'(\cdot) \geq 0$ , there exists a schedule  $\{\alpha(\cdot), r(\cdot), P(\cdot)\}$  with  $\alpha' > 0$ ,  $r' \geq 0$  and  $P' > 0$  that can be supported as a fully revealing equilibrium.

**Quasi-concavity.** Suppose that the proposed schedule satisfies (3) for all  $X \in \mathcal{X}$ . This

schedule then satisfies quasi-concavity of the objective function. Specifically, we show that

$$\partial\Pi/\partial\hat{X} = \alpha'(\hat{X})\Phi(X) + r'(\hat{X})[X - P(\hat{X})] - r(\hat{X})P'(\hat{X}) \quad (12)$$

is non-negative when  $\hat{X} \leq X$  and non-positive when  $X \geq \hat{X}$ .

Condition (3) implies that

$$r(\hat{X})P'(\hat{X}) = \alpha'(\hat{X})\Phi(\hat{X}) + r'(\hat{X})[\hat{X} - P(\hat{X})].$$

Substituting the right-hand side into (12) and rearranging yields

$$\partial\Pi/\partial\hat{X} = \alpha'(\hat{X}) \left[ \Phi(X) - \Phi(\hat{X}) \right] + r'(\hat{X}) \left[ X - \hat{X} \right]. \quad (13)$$

The assumption  $\Phi'(\cdot) \geq 0$  implies that  $\Phi(X) \geq \Phi(\hat{X})$  when  $\hat{X} \leq X$  and that  $\Phi(X) \leq \Phi(\hat{X})$  when  $\hat{X} \geq X$ . Given that  $\alpha' > 0$  and  $r' \geq 0$ , it follows that

$$\partial\Pi/\partial\hat{X} \text{ is } \begin{cases} \text{non-negative} & \text{for } \hat{X} < X \\ 0 & \text{for } \hat{X} = X \\ \text{non-positive} & \text{for } \hat{X} > X \end{cases} .$$

Thus, the proposed schedule makes  $\Pi(\hat{X}; X)$  weakly quasi-concave for all  $X \in \mathcal{X}$ . This also holds for  $r'(\hat{X}) = 0$ , in which case all bidder types propose the same bid restriction.

**Local optimality.** Condition (3) is a functional equation for  $\alpha(\cdot)$ ,  $r(\cdot)$  and  $P(\cdot)$  with *two degrees of freedom*. To derive an example of an incentive-compatible schedule, we set  $r(\cdot) = 0.5$ . Then, condition (3) simplifies to

$$\alpha'(X) = \frac{P'(X)}{2\Phi(X)}.$$

Integrating on both sides over  $[X, \bar{X}]$  yields

$$\int_X^{\bar{X}} \alpha'(u) du = \int_X^{\bar{X}} \frac{P'(u)}{2\Phi(u)} du \quad \Leftrightarrow \quad \alpha(\bar{X}) - \alpha(X) = \int_X^{\bar{X}} \frac{P'(u)}{2\Phi(u)} du.$$

As the highest-valued type does not have to relinquish any private benefits [ $\alpha(\bar{X}) = 1$ ],

$$\alpha(X) = 1 - \int_X^{\bar{X}} \frac{P'(u)}{\Phi(u)} du. \quad (14)$$

One possible price schedule is  $P(X) = X$  in which case  $\alpha(X) = 1 - \int_X^{\bar{X}} [\Phi(u)]^{-1} du$ . As

shareholders receive  $P(X) + [1 - \alpha(X)] \Phi(X)$ , the free-rider condition is also satisfied.

**Cut-off type.** The condition (3) puts a constraint on how equilibrium profits vary across types in equilibrium. By the envelope theorem, we have that equilibrium profits must be increasing at the rate

$$\partial \Pi^* / \partial X = \alpha(X) \Phi'(X) + r(X),$$

for *any* schedule that satisfies (3).

Given an equilibrium exists, the cut-off type  $X^c$  is given by

$$\int_{X^c}^{\bar{X}} \{\alpha(u) \Phi'(u) + r(u)\} du = \Phi(\bar{X}). \quad (15)$$

Under the proposed equilibrium schedule, bidder types below  $X^c$  incur a loss under the proposed schedule. Hence, they prefer *not* making a bid over making the bid prescribed by the proposed schedule. The option of notmaking a bid does not undermine the non-mimicking constraints. Under the proposed schedule, the bidder prefers a loss-making offer to offers made by higher-valued types. A fortiori, she also prefers a zero-profit offer over the latter.

**Out-of-equilibrium beliefs.** The proposed schedule can be supported as a signaling equilibrium under the out-of-equilibrium beliefs that any deviation comes from the highest-valued bidder type,  $\bar{X}$ . Under these beliefs, the target shareholders do not tender their shares in response to a deviation bid  $(\tilde{r}, \tilde{\alpha}, \tilde{P})$  unless  $\tilde{P} \geq \bar{X}$ . Any such bid, however, is weakly dominated by  $(0.5, 1, \bar{X})$ , which is the equilibrium bid of the highest type. Since  $(0.5, 1, \bar{X})$  is mimicking-proof, any successful deviation bid is—by implication—unattractive under the proposed out-of-equilibrium beliefs.

**Bid restriction.** The above proves that the bid restriction  $r$  is a redundant signal. This follows because, as we have shown, the schedule

$$\{r^*(X), \alpha^*(X), P^*(X)\} = \begin{cases} \left\{ 0.5, 1 - \int_X^{\bar{X}} [\Phi(u)]^{-1} du, X \right\} & \text{for } X \in [X^c, \bar{X}] \\ \{0, 0, 0\} & \text{for } X \in [0, X^c] \end{cases}$$

can, among others, be supported as a fully revealing equilibrium.

By contrast, the private benefit retention rate  $\alpha$  and the price  $P$  are indispensable as signals. First, if  $\alpha$  is invariant across types, Theorem 1 applies. Second, a uniform price in a fully revealing equilibrium must satisfy  $P = \bar{X}$ . But then all bidder types  $X < \bar{X}$  prefer the offer  $(0.5, 1, \bar{X})$ , which always succeeds irrespective of shareholder beliefs, to any other offer with  $P = \bar{X}$ . Hence, they would pool. ■

### Application 1

Given bidder of type  $V \in \mathcal{V}$  can choose  $\phi \in [0, \bar{\phi}]$ , she solves

$$\max_{\hat{V}} \phi(\hat{V})V + r(\hat{V}) \left[ (1 - \phi(\hat{V})V) - P(\hat{V}) \right]$$

Set  $r(\hat{V}) = r$ , express  $\phi$  as  $\bar{\phi}\psi$  with  $\psi \in (0, 1)$ , and the objective function simplifies to

$$\begin{aligned} & \bar{\phi}\psi(\hat{V})V + r \left[ (1 - \bar{\phi}\psi(\hat{V})V) - P(\hat{V}) \right] \\ = & \bar{\phi}\psi(\hat{V})V + r \left[ (1 - \bar{\phi})V - P(\hat{V}) + (1 - \psi(\hat{V}))\bar{\phi}V \right] \\ = & \left[ \psi(\hat{V}) + r(1 - \psi(\hat{V})) \right] \bar{\phi}V + r \left[ (1 - \bar{\phi})V - P(\hat{V}) \right] \end{aligned}$$

This last expression is isomorphic to (1) when using the definitions  $X \equiv (1 - \bar{\phi})V$ ,  $\bar{\phi}V \equiv \Phi(X)$  and  $\alpha \equiv \left[ \psi(\hat{V}) + r(1 - \psi(\hat{V})) \right]$ . Note that  $\bar{\phi}V$  is increasing in the bidder's type. ■

### Application 2

Given bidder of type  $\mathcal{X}' \in [\underline{X}, \bar{X}]$  can choose  $D \in [0, \bar{D}]$ , she solves

$$\max D(\hat{X}) + r(\hat{X}) \left[ X - D(\hat{X}) - P(\hat{X}) \right]$$

Set  $r(\hat{X}) = r$ , express  $D$  as  $\bar{D}\psi$  and rewrite the objective function as

$$\begin{aligned} & \bar{D}\psi(\hat{X}) + r \left[ X - \bar{D}\psi(\hat{X}) - P(\hat{X}) \right] \\ = & \bar{D}\psi(\hat{X}) + r \left[ X - \bar{D} - P(\hat{X}) + (1 - \psi(\hat{X}))\bar{D} \right] \\ = & \left[ \psi(\hat{X}) + r(1 - \psi(\hat{X})) \right] \bar{D} + r \left[ X - \bar{D} - P(\hat{X}) \right] \end{aligned}$$

This last expression is isomorphic to (1) when using the definitions  $\bar{D} \equiv \Phi(X)$  and  $\alpha \equiv \left[ \psi(\hat{X}) + r(1 - \psi(\hat{X})) \right]$ . Note that  $\bar{D}$ , i.e.,  $\Phi(\cdot)$ , is constant across bidder types. ■

### Application 3

Denote the value of the bidder assets by  $A(\mathbb{I}, X) \equiv \mathbb{I}_{\beta \geq 0.5}Z + \lambda X$ . The indicator function  $\mathbb{I}_{\beta \geq 0.5}$  takes the value 1 if the bid succeeds and 0 otherwise. If the bid succeeds ( $\beta \geq 0.5$ ), the holding company is worth  $H(\beta, X) = A(1, X) + \beta X$ . Under full information, shareholders do not tender unless  $C(\beta) + s(\beta)H(\beta, X) \geq X$ . To ensure a successful merger ( $\beta = 1$ ), the

bidder must choose  $s(\beta)$  and  $C(\beta)$  such that

$$s(\beta) \geq \frac{X - C(\beta)}{Z + \beta X + \lambda X} \quad (16)$$

for all  $\beta \in [0.5, 1]$ . In this case, all shareholders tender their shares whenever they believe that more than half the shares are tendered, and the bidder must ultimately pay  $C(1)$  and  $s(1)$ . To simplify the exposition, we omit  $\beta$  and express the bidder's offer as a pair  $(s, C)$  which must satisfy the free-rider condition for  $\beta = 1$ , i.e.,  $s \geq (X - C)/(Z + X + \lambda X)$ .<sup>27</sup> Note that condition (16) violates neither the cash constraint nor the control constraint if  $C(\beta)$  is chosen sufficiently high.

For a given cash price  $C$  and equity component  $s$ , the bidder's profit from a successful merger is therefore

$$\Pi(X) = (1 - s)H(1, X) - C - \lambda X = (1 - s)Z + (1 - s) \left( X - \frac{s\lambda X + C}{1 - s} \right).$$

Now define  $\alpha \equiv 1 - s$ ,  $r = \alpha$ ,  $\Phi(X) = Z$ , and  $P = \frac{s\lambda X + C}{1 - s}$ . The bidder can use  $s$  to adjust  $\alpha$  and  $r$ , and she can use  $C$  to adjust  $P$ . From Theorem 2, it follows that  $\alpha$  must be increasing which in turn implies that  $s$  must be decreasing. The constraint  $\alpha = r$  results from the fact that the bidder merges the firms and pays the target shareholders with holding company shares.

In this setting, the cut-off type is not necessarily determined by the participation constraint ( $\Pi \geq 0$ ). As lower types issue more equity, they may also run either into the control constraint  $s(\cdot) \leq 0.5$  or into the cash constraint  $C(\cdot) \geq 0$ . The latter may occur because the bidder can in principle become a net *issuer*, rather than a net purchaser, of financial claims. The cash constraint is relevant for bidders for whom  $A$  is very large relative to  $X$ . Notwithstanding, the participation constraint becomes binding as  $Z$  decreases. In particular,  $Z = 0$  is equivalent to  $\Phi(X) = 0$ , and hence causes signaling breaks down. (It is straightforward to verify that using  $\lambda X$  (instead of  $Z$ ) as the synergy gains leads to similar results; in particular, signaling breaks down when  $\lambda = 0$ .) ■

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<sup>27</sup>Even without a contingent offer, there exists a self-fulfilling equilibrium in which the merger succeeds for  $(C, t)$  as long as it satisfies the free-rider condition for  $\beta = 1$ : If each shareholder believes that all other shareholders tender, she also tenders. Hence, one can alternatively focus on non-contingent offers, and select merger success as the equilibrium outcome whenever it is consistent with the free-rider condition.

#### Application 4

The bidder can choose  $t \in [0, \hat{t}]$  and solves

$$\max t(\hat{X})X + r_t(\hat{X})(1 - t(\hat{X})) \left[ X - P(\hat{X}) \right]$$

with  $r_t \in \left[ \frac{0.5-t}{1-t}, 1 \right]$ . Set  $r_t(\hat{X}) = r_t$ , express  $t$  as  $\bar{t}\psi$  and rewrite the objective function as

$$\begin{aligned} & \bar{t}\psi(\hat{X})X + r_t(1 - \bar{t}\psi(\hat{X})) \left[ X - P(\hat{X}) \right] \\ &= \left[ \psi(\hat{X}) + r_t(1 - \psi(\hat{X})) \right] \bar{t}X + r_t \left[ X - P(\hat{X}) \right] \end{aligned}$$

This last expression is isomorphic to (1) when using the definitions  $\bar{t}X \equiv \Phi(X)$  and  $\alpha \equiv \left[ \psi(\hat{X}) + r(1 - \psi(\hat{X})) \right]$ . Note that  $\bar{t}X$ , i.e.,  $\Phi(\cdot)$ , is increasing in the bidder's type. ■

#### Application 5

The objective function in the probabilistic tender offer game is:

$$\begin{aligned} \Pi(r, P) &= q(r, P) [\Phi(X) + \gamma(r, P)(X - P)] \\ &= \alpha\Phi(X) + \hat{r}(X - P) \end{aligned}$$

where  $\alpha \equiv q(r, P)$  and  $\hat{r} \equiv q(r, P)\gamma(r, P)$ . The last expression is isomorphic to (1), except that  $\hat{r}$  can take values below 0.5. Provided that  $\Phi(\cdot)$  is a non-decreasing function, Theorem 2 can thus be applied. ■

### Proof of Theorem 3

Consider the type  $(\bar{X}, \underline{\Phi})$  and an arbitrary type  $(X, \Phi) \neq (\bar{X}, \underline{\Phi})$ . In any fully revealing equilibrium, type  $(\bar{X}, \underline{\Phi})$  cannot be held to a profit lower than  $\underline{\Phi}$  because she can always succeed with the bid  $(r, 1, r\bar{X})$ . At the same time, she cannot earn more than  $\underline{\Phi}$  because of the free-rider condition. In order for type  $(\bar{X}, \underline{\Phi})$  not to mimic type  $(X, \Phi)$ , the latter type must make an offer  $(r, \alpha, C)$  which satisfies  $\underline{\Phi} \geq r\bar{X} + \alpha\underline{\Phi} - C$ , or equivalently

$$C \geq \underline{C} \equiv r\bar{X} - (1 - \alpha)\underline{\Phi}. \quad (17)$$

In addition, a truthful offer by  $(X, \Phi)$  must also yield a higher profit than the "out-of-equilibrium" offer  $(0.5, 1, 0.5\bar{X})$  which succeeds irrespective of target shareholder beliefs.

That is, her offer  $(r, \alpha, C)$  must satisfy  $rX + \alpha\Phi - C \geq 0.5(X - \bar{X}) + \Phi$ , or equivalently

$$C \leq \bar{C} \equiv (r - 0.5)X + 0.5\bar{X} - (1 - \alpha)\Phi. \quad (18)$$

The constraints (17) and (18) are simultaneously satisfied if  $\bar{C} \geq \underline{C}$  holds. Straightforward manipulations yield  $(r - 0.5)(X - \bar{X}) \geq (1 - \alpha)(\Phi - \underline{\Phi})$ . This condition is violated, unless all types with  $X < \bar{X}$  make the pooling offer  $(0.5, 1, 0.5\bar{X})$ . ■

## Proof of Theorem 4

We show that every implementable level of takeover activity can also be implemented by a schedule with  $P(X) = X$  and  $r'(X) = 0$  (step 1) and that minimizing  $r$  promotes takeover activity (step 2).

**Step 1.** Consider an equilibrium schedule which implements a cut-off type  $X'$ . This type satisfies  $\Pi(X'; X') = \alpha(X')\Phi(X') + r(X')[X' - P(X')] = 0$ . Keeping the cut-off type constant, we maintain the zero-profit condition by setting  $\alpha(X') = 0$  and  $P(X') = X'$ .

This provided, the *same* cut-off type can also be characterized by

$$\begin{aligned} \Phi(\bar{X}) - \alpha(X')\Phi(X') &= \Phi(\bar{X}) \\ \alpha(\bar{X})\Phi(\bar{X}) - \alpha(X')\Phi(X') &= \Phi(\bar{X}) \\ [\alpha(u)\Phi(u)]_{X'}^{\bar{X}} &= \Phi(\bar{X}) \\ \int_{X'}^{\bar{X}} \{\alpha(u)\Phi'(u) + \alpha'(u)\Phi(u)\} du &= \Phi(\bar{X}) \end{aligned} \quad (19)$$

where we use the fact that  $\alpha(\bar{X}) = 1$ .

Next, recall that (15) characterizes a unique cut-off type  $X^c$  for any equilibrium schedule,

$$\int_{X^c}^{\bar{X}} \left\{ \alpha(\hat{X})\Phi'(\hat{X}) + r(\hat{X}) \right\} d\hat{X} = \Phi(\bar{X}), \quad (20)$$

and consider an equilibrium schedule which specifies  $P(X) = X$  and  $r(X) = r(X')$ . Under this schedule, the first-order condition (3) becomes  $r(X') = \alpha'(\hat{X})\Phi(\hat{X})$ . Substituting  $r(X') = r(\hat{X}) = \alpha'(\hat{X})\Phi(\hat{X})$ , equation (20) becomes equation (19), and  $X^c = X'$ .

**Step 2.** Given step 1, we can restrict our attention to schedules with  $P(X) = X$  and  $r(X) = r$ . Under these schedules, the bidder's equilibrium profit is

$$\Pi^*(X) = \alpha(X)\Phi(X).$$

When  $\alpha(\cdot)$  declines at a lesser rate, bidder profits deteriorate slower, as the type decreases. Using conditon (3),  $P'(X) = 1$  and  $r'(X) = 0$ , the slope of  $\alpha(\cdot)$  is

$$\alpha'(X) = \frac{r}{\Phi(X)}$$

and is increasing in  $r$  for any  $X \in \mathcal{X}$ . That is, bidder profits deteriorate at a lesser rate and the cut-off type is lower when  $r$  is smaller. ■

## Proof of Proposition 4

We first establish the following two auxiliary results.

**Lemma 3** *For all  $\theta' > \theta$ , there exists a unique  $X_\theta(\theta') \in (0, \infty)$  s.t.*

$$f_\theta(X) \begin{cases} > f_{\theta'}(X) & \text{for all } X < X_\theta(\theta') \\ < f_{\theta'}(X) & \text{for all } X > X_\theta(\theta') \end{cases} .$$

**Proof.** By SMLRP, for all  $\theta' > \theta$ , there is a unique  $X_\theta(\theta') \in (0, \infty)$  s.t.

$$f_{\theta'}(X)/f_\theta(X) \begin{cases} < 1 & \text{for } X < X_\theta(\theta') \\ = 1 & \text{for } X \in \mathcal{X}(\theta, \theta') \\ > 1 & \text{for } X > X_\theta(\theta') \end{cases} .$$

Otherwise, if  $f_{\theta'}(X)/f_\theta(X)$  is either always larger or always smaller than 1, it cannot be that  $F_\theta(\infty) = F_{\theta'}(\infty)$ . This implies the result. ■

**Lemma 4** *For all  $\theta'' > \theta' > \theta$ ,  $X_{\theta'}(\theta'') \geq X_\theta(\theta')$ .*

**Proof.** Suppose to the contrary that

$$X_{\theta'}(\theta'') < X_\theta(\theta'). \tag{★}$$

By Lemma 3, it then follows that

$$\begin{aligned} (a) \quad & \text{For } X \in (0, X_{\theta'}(\theta'')): & \frac{f_{\theta''}(X)}{f_{\theta'}(X)} < 1 \text{ and } \frac{f_{\theta'}(X)}{f_\theta(X)} < 1 & \Rightarrow \frac{f_{\theta''}(X)}{f_\theta(X)} < 1 \\ (b) \quad & \text{For } X = X_{\theta'}(\theta''): & \frac{f_{\theta''}(X)}{f_{\theta'}(X)} = 1 \text{ and } \frac{f_{\theta'}(X)}{f_\theta(X)} < 1 & \Rightarrow \frac{f_{\theta''}(X)}{f_\theta(X)} < 1 \\ (c) \quad & \text{For } X \in (X_{\theta'}(\theta''), X_\theta(\theta')): & \frac{f_{\theta''}(X)}{f_{\theta'}(X)} > 1 \text{ and } \frac{f_{\theta'}(X)}{f_\theta(X)} < 1 & \Rightarrow \frac{f_{\theta''}(X)}{f_\theta(X)} \geq 1 \\ (d) \quad & \text{For } X = X_\theta(\theta'): & \frac{f_{\theta''}(X)}{f_{\theta'}(X)} > 1 \text{ and } \frac{f_{\theta'}(X)}{f_\theta(X)} = 1 & \Rightarrow \frac{f_{\theta''}(X)}{f_\theta(X)} > 1 \\ (e) \quad & \text{For } X \in (X_\theta(\theta'), \infty): & \frac{f_{\theta''}(X)}{f_{\theta'}(X)} > 1 \text{ and } \frac{f_{\theta'}(X)}{f_\theta(X)} > 1 & \Rightarrow \frac{f_{\theta''}(X)}{f_\theta(X)} > 1 \end{aligned}$$

Observe that (i)  $f_{\theta''}(X) = f_{\theta'}(X)$  for  $X = X_{\theta'}(\theta'')$  and (ii)  $f_{\theta'}(X) = f_{\theta}(X)$  for  $X = X_{\theta}(\theta')$ . SMLRP implies that  $f_{\theta''}(X)/f_{\theta}(X) \leq 1$  in case (c), and hence that (iii)  $f_{\theta''}(X) = f_{\theta}(X)$  for  $X = X_{\theta}(\theta')$ . Points (ii) and (iii) together imply that (iv)  $f_{\theta''}(X) = f_{\theta'}(X)$  for  $X = X_{\theta}(\theta')$ . Given that  $f_{\theta''}(X) > f_{\theta'}(X)$  in case (c), points (iv) and (i) can only be reconciled with SMLRP if  $X_{\theta'}(\theta'') = X_{\theta}(\theta')$ . However, this contradicts inequality (★). ■

**Main proof.** The proof proceeds in two steps. In the first step, we compare adjacent types and analyze local incentive-compatibility. In the second step, we show that an offer which is locally mimicking-proof is also globally mimicking-proof.

**Local incentive-compatibility.** Consider a type  $\theta$  who, for each target share, offers a cash price  $P_{\theta}$ , a debt claim with face value  $D$ , and a (cash-settled) knock-in call option with exercise price  $S_{\theta}$  and trigger level  $T_{\theta}$ .

Absent private benefits, a fully efficient equilibrium requires that the bidder's cash price is weakly lower than the expected value of the cash flow rights that she acquires. At the same time, the free-rider condition requires that the cash price is weakly higher than the expected value of the transferred cash flow rights. Both constraints can only be satisfied simultaneously if they are both binding:

$$P_{\theta} = \int_D^{T_{\theta}} (X - D) f_{\theta}(X) dX + \int_{T_{\theta}}^{\infty} (S_{\theta} - D)^+ f_{\theta}(X) dX.$$

Consequently, every truthful offer must yield zero bidder profits.

(i) The next higher type  $\theta + 1$  does not mimic  $\theta$  iff

$$-P_{\theta} + \int_D^{T_{\theta}} (X - D) f_{\theta+1}(X) dX + \int_{T_{\theta}}^{\infty} (S_{\theta} - D)^+ f_{\theta+1}(X) dX \leq 0.$$

Substituting for  $P_{\theta}$ , the inequality can be written as

$$(S_{\theta} - D)^+ \int_{T_{\theta}}^{\infty} [f_{\theta+1}(X) - f_{\theta}(X)] dX \leq \int_D^{T_{\theta}} [f_{\theta}(X) - f_{\theta+1}(X)] (X - D) dX. \quad (21)$$

Set  $T_{\theta} = X_{\theta}(\theta + 1)$ . By Lemma 3, both integrals are then *strictly* positive for any  $D < T_{\theta} = X_{\theta}(\theta + 1)$ , in which case there exists a  $S_{\theta} > 0$  such that (21) is satisfied.

(ii) Analogously, the next lower type  $\theta - 1$  does not mimic  $\theta$  iff

$$(S_{\theta} - D)^+ \int_{T_{\theta}}^{\infty} [f_{\theta-1}(X) - f_{\theta}(X)] dX \leq \int_D^{T_{\theta}} [f_{\theta}(X) - f_{\theta-1}(X)] (X - D) dX. \quad (22)$$

Set  $D = X_{\theta-1}(\theta)$ . By Lemma 3, the right-hand side is then *strictly* positive. By Lemma 4,  $T_{\theta} = X_{\theta}(\theta + 1) \geq X_{\theta-1}(\theta)$  so that the left-hand side integral is *strictly* negative. So, (22)

holds.

**Global incentive-compatibility.** We now consider in turn types higher than  $\theta + 1$  and types lower than  $\theta - 1$ .

(i) Given  $T_\theta = X_\theta(\theta + 1)$  and  $D = X_{\theta-1}(\theta)$ , consider now the incentive-compatibility constraint of an arbitrary type  $\theta^+ > \theta + 1$  vis-à-vis type  $\theta$ :

$$[S_\theta - X_{\theta-1}(\theta)]^+ \int_{X_\theta(\theta+1)}^{\infty} [f_{\theta^+}(X) - f_\theta(X)] dX \leq \int_{X_{\theta-1}(\theta)}^{X_\theta(\theta+1)} [f_\theta(X) - f_{\theta^+}(X)] [X - X_{\theta-1}(\theta)] dX.$$

Defining  $\eta(X) \equiv f_{\theta+1}(X) - f_{\theta^+}(X)$ , write the inequality as

$$[S_\theta - X_{\theta-1}(\theta)]^+ \int_{X_\theta(\theta+1)}^{\infty} [f_{\theta+1}(X) - f_\theta(X) - \eta(X)] dX \leq \int_{X_{\theta-1}(\theta)}^{X_\theta(\theta+1)} [f_\theta(X) - f_{\theta+1}(X) + \eta(X)] [X - X_{\theta-1}(\theta)] dX. \quad (23)$$

By Lemma 4,  $X_{\theta+1}(\theta^+) \geq X_\theta(\theta + 1)$  so that  $\eta(X) > 0$  for all  $X < X_\theta(\theta + 1)$ . This implies that the right-hand side of (23) is larger than the right-hand side of (21), and hence *strictly* positive. Turning to the left-hand side, because

$$- \int_{X_\theta(\theta+1)}^{\infty} \eta(X) dX = \int_0^{X_\theta(\theta+1)} \eta(X) dX - \int_0^{\infty} \eta(X) dX = \int_0^{X_\theta(\theta+1)} \eta(X) dX > 0,$$

the integral on the left-hand side of (23) is larger than the integral on the left-hand side of (21), and hence *strictly* positive. We conclude that—for  $T_\theta = X_\theta(\theta + 1)$  and  $D = X_{\theta-1}(\theta)$ —there exists a *strictly* positive price,  $S_\theta > 0$ , such that no type  $\theta^+ > \theta$  mimics type  $\theta$ .

(ii) Given  $T_\theta = X_\theta(\theta + 1)$  and  $D = X_{\theta-1}(\theta)$ , consider now the incentive-compatibility constraint of an arbitrary type  $\theta^- < \theta - 1$  vis-à-vis type  $\theta$ :

$$[S_\theta - X_{\theta-1}(\theta)]^+ \int_{X_\theta(\theta+1)}^{\infty} [f_{\theta^-}(X) - f_\theta(X)] dX \leq \int_{X_{\theta-1}(\theta)}^{X_\theta(\theta+1)} [f_\theta(X) - f_{\theta^-}(X)] [X - X_{\theta-1}(\theta)] dX.$$

Defining  $\zeta(X) \equiv f_{\theta-1}(X) - f_{\theta-}(X)$ , write the inequality as

$$[S_{\theta} - X_{\theta-1}(\theta)]^+ \int_{X_{\theta}(\theta+1)}^{\infty} [f_{\theta-1}(X) - f_{\theta}(X) - \zeta(X)] dX \leq \int_{X_{\theta-1}(\theta)}^{X_{\theta}(\theta+1)} [f_{\theta}(X) - f_{\theta-1}(X) + \zeta(X)] [X - X_{\theta-1}(\theta)] dX. \quad (24)$$

By Lemma 4,  $X_{\theta-1}(\theta) \geq X_{\theta-}(\theta - 1)$  so that  $\zeta(X) > 0$  for all  $X > X_{\theta-1}(\theta)$ . This implies that the right-hand side of (24) is larger than the right-hand side of (22), and hence *strictly* positive. Turning to the left-hand side, again by Lemma 4,  $X_{\theta}(\theta+1) \geq X_{\theta-1}(\theta) \geq X_{\theta-}(\theta-1)$  so that  $\zeta(X) > 0$  for all  $X > X_{\theta}(\theta + 1)$ . This implies that the left-hand side integral of (24) is smaller than the left-hand side integral of (22), and hence *strictly* negative. So, (24) holds. We conclude that—for  $T_{\theta} = X_{\theta}(\theta + 1)$ ,  $D = X_{\theta-1}(\theta)$ , and  $S_{\theta} > 0$ —no type  $\theta^- < \theta$  mimics type  $\theta$ . ■

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