Control of Corporate Decisions: Shareholders *vs.* Management[*]

by

Milton Harris[†]

University of Chicago

and

Artur Raviv[‡]

Northwestern University

---

[*] The authors are grateful to Eddie Dekel, Paolo Fulghieri (the Associate Editor), Kohei Kawamura, Gustavo Manso, Robert Novy-Marx, Yoram Weiss, Bilge Yilmaz, three referees, and seminar participants at the University of Chicago Booth School of Business, Northwestern University Kellogg School of Management, Rice University Jones School of Management, Tel Aviv University Berglas School of Economics, the Securities and Exchange Commission, University of Pennsylvania Wharton School, the 2008 Econometric Society Winter Meetings, the 2008 Utah Winter Finance Conference, and the 2008 Western Finance Association Annual Meetings for helpful comments and to the Center for Research in Security Prices at the University of Chicago Booth School of Business for financial support.

[†] Professor Harris, the corresponding author, is the Chicago Board of Trade Professor of Finance and Economics at the University of Chicago Booth School of Business. He may be contacted at milt@uchicago.edu, +1 (773) 702-2549, University of Chicago Booth School of Business, 5807 South Woodlawn Avenue, Chicago, IL 60637, USA.

[‡] Professor Raviv is the Alan E. Peterson Distinguished Professor of Finance at the Kellogg Graduate School of Management, Northwestern University. He may be contacted at a-raviv@kellogg.northwestern.edu, +1 (847) 491-8342, Kellogg School, Northwestern University, 2001 Sheridan Road, Evanston, IL 60208, USA.

Control of Corporate Decisions: Shareholders *vs.* Management

ABSTRACT

Activist shareholders have lately been attempting to assert themselves in a struggle with management and regulators over control of corporate decisions. These efforts have met with mixed success. Meanwhile, shareholders have been pressing for changes in the rules governing access to the corporate proxy process, especially in regard to nominating directors. The key issue which these events have brought to light is whether, in fact, shareholders will be better off with enhanced control over corporate decisions. Proponents of increased shareholder participation argue that such participation is needed to counter the agency problems associated with management decisions. Opponents counter that shareholders lack the requisite knowledge and expertise to make effective decisions or that shareholders may have incentives to make value-reducing decisions. In this paper, we investigate what determines the optimality of shareholder control, taking account of some of the above arguments, both pro and con. Our main contribution is to use formal modeling to uncover some factors overlooked in these arguments. For example, we show that the claims that shareholders should not have control over important decisions because they lack sufficient information to make an informed decision or because they have a non-value-maximizing agenda are flawed. On the other hand, it has been argued that, since shareholders have the "correct" objective (value maximization) and can always delegate the decision to management when they believe management will make a better decision, shareholders should control all major decisions. We show that this argument is also flawed.

JEL Classification Codes: G3, G34, G38

Keywords: corporate governance, shareholder democracy, direct shareholder participation, proxy process.

Control of Corporate Decisions: Shareholders *vs.* Management

Activist shareholders have lately been attempting to assert themselves in a struggle with management and regulators over control of corporate decisions. These efforts have met with mixed success. Carl Icahn, along with some hedge funds, attempted, without success, to force a breakup of Time Warner, even going so far as to commission a study by Lazard Frères that touted the benefits of such a breakup. Kirk Kerkorian succeeded in placing his representative on the board of General Motors in a bid to get GM to enter into an alliance with Nissan and Renault. Again, this attempt at wresting control of some corporate decisions failed. On the other hand, Nelson Peltz succeeded in getting himself and an ally elected to the board of H.J. Heinz Co. and in getting management to implement accelerated cost cuts and restructuring. Meanwhile, Jay Sidhu, longtime CEO of Sovereign Bancorp resigned under pressure from Ralph Whitworth of Relational Investors who also succeed in getting himself and another independent director elected to the board of Sovereign. These examples illustrate the recent trend toward shareholder activism and, despite some successes, the difficulty that these shareholders face in affecting corporate decisions under the current rules.

On another front, shareholders have been pressing for changes in the rules governing access to the corporate proxy process, especially in regard to nominating directors. For example, the American Federation of State, County and Municipal Employees (AFSCME), as a stockholder in American International Group (AIG), proposed that AIG shareholders be allowed to vote on a measure to give them a greater voice in the selection of directors. When AIG succeeded in obtaining permission from the SEC to keep the measure off the proxy, AFSCME sued, eventually winning in an appeal. The SEC offered, for public comment, a possible rule change as a result of the court ruling, but eventually they decided not to change the rule.

The key issue that these events have brought to light is whether, in fact, shareholders will be better off with enhanced control over corporate decisions. Proponents of increased shareholder participation argue that such participation is needed to counter the agency problems associated with management decisions. The leading proponent, at least in the academy, is Lucian Bebchuk (2005). In this view, boards of directors do not exercise sufficient control over self-interested managers because they are typically hand-picked by management insiders who control the proxy process. Moreover, it has been argued that, since shareholders have the "correct" objective (value maximization) and can always delegate the decision to management when they believe management will make a better decision, shareholders should control all major decisions.[1] An array of legal scholars opposes Bebchuk's conclusion, offering several arguments such as that shareholders lack sufficient information to make an informed decision or that some shareholders may have a non-value-maximizing agenda.[2] We refer to this approach as the "information approach." This approach takes the information of the parties as given and investigates the effect of the information structure on the efficacy of particular corporate governance structures such as shareholder control of major decisions. Another literature that largely predates Bebchuk (2005) addresses a different set of objections to the idea of shareholder control of corporate decisions. This literature, which is surveyed in the next section, focuses on the potential for increased shareholder influence over decisions to weaken incentives for management to engage in various value-increasing activities, including information acquisition. We refer to this approach as the "incentive approach."[3]

---

[1] For example, Bebchuk (2005, pp. 881-882), claiming that shareholder ignorance is no excuse for denying shareholders control, states, "After balancing the considerations for and against deference [to management], rational shareholders might often conclude that deference would be best on an expected-value basis. Other times, however, they might reach the opposite conclusion. Although shareholders cannot be expected to get it right in every case, it is their money that is on the line, and they thus naturally have incentives to reach decisions that would best serve their interests."

[2] Some argue, for example that large institutional shareholders, who will drive the decisions if shareholders are given more power, may try to inflate the firm's stock price with short-term measures that actually reduce firm value. Another example is that shareholders with social, political or environmental agendas may dominate the decision-making process. See Bainbridge (2006), Lipton (2002), Stout (2006), and Strine (2006).

[3] The phrases "information approach" and "incentive approach" are not meant to suggest that the information approach excludes incentive considerations or the incentive approach excludes information issues. We choose these phrases as shorthand for referring to the two strands of literature, because, in the information approach,

In this paper, we investigate what determines the optimal control of corporate decisions. In particular, we take a normative perspective and ask which allocation of control (to shareholders or management) maximizes share value.[4]

To analyze this issue, we adopt the information approach as opposed to the incentive approach. This is partly because the incentive approach has been more thoroughly investigated, but mainly because we want to address more directly the recent debate based on the information approach. This is not to deny the importance of the incentive approach in general or of the effect of corporate governance on the incentives to acquire information in particular. Nor do we deny the importance of the effect of corporate governance on the costs of acquiring information. We believe, however, that the recent debate based on the information approach is best addressed by focusing on the validity of the arguments *given* the information of the parties. Our main contribution is to use formal modeling to uncover some factors overlooked in the information-approach arguments, both pro and con.

Our modeling strategy incorporates the following key features. First, we assume management's preferences result in their making decisions that are biased relative to value maximizing decisions. Second, we assume *both* parties have private information relevant to a decision and consider strategic communication of this private information. Third, we emphasize that control over a decision by a given party does not require that party actually to make the decision. The controlling party may make the decision itself but also may delegate the decision to the other party. Finally, to address the opponents of shareholder power in the information approach, we consider situations in which shareholders are *misinformed* and situations in which non-value-maximizing shareholders hijack control whenever shareholders are allocated control.

---

the key driver is communication or utilization of information rather than incentives and *vice versa* for the incentive approach.

[4] We will use the word "optimal" to mean value-maximizing throughout the paper, even in cases where some shareholders have other objectives (see section 5 for a discussion). We also assume that maximizing share value and maximizing firm value are equivalent. We revisit the issue of who should allocate control briefly in Sections 4 and 6.

In general, our analysis highlights the complicated interaction among control rights, who actually makes the decision, and the extent of communication between the parties. In particular, control and the delegation choice of the controlling party determine not only the decision maker but also, in part, how much information will be used to make the decision. The decision maker's own private information will be fully utilized, but the other party's information will be only partially communicated to the decision maker. The extent of this communication is limited by the importance of the managerial agency problem, as well as by the extent of non-value-maximizing behavior on the part of shareholders.[5] The result is that whether shareholder control is optimal depends on such characteristics of the decision as the extent of private information on both sides and the extent of agency problems (potentially on both sides).

Our analysis results in several surprising conclusions:

- Shareholders should *always* control decisions about which they have no private information.
- Shareholders should *not* control some decisions about which they have private information.
- Misinformed shareholders *should* control some decisions.
- Shareholders with non-value-maximizing agendas *should* control some decisions.

To gain some intuition for these results, we begin with the case in which shareholders have no private information. A naïve approach, often taken by opponents to greater shareholder participation, suggests that, in such cases, shareholders should not be in control. This approach, however, ignores the possibility that shareholders will delegate the decision to better-informed managers. We show that, in such situations, shareholders, recognizing that they have no information not already known to management, will delegate the decision to management if and only if management's private information is sufficiently valuable that it outweighs the cost due to the managerial agency problem. That is, shareholders delegate the decision to managers in precisely the correct situations to maximize share value. This raises the question of why, given that shareholders are fully aware of their information limitations

---

[5] This part of our model draws heavily on Crawford and Sobel (1982) and is similar to the models in Dessein (2002), and Harris and Raviv (2005, 2008a). See section 2 for a detailed comparison.

and their preferences are perfectly aligned with our criterion for optimality, they should not control all important corporate decisions as argued by some proponents of greater shareholder participation. The reason that shareholder control is not always optimal is that, when shareholders have private information, they will fail to delegate the decision to managers in some situations in which such delegation would increase share value. This stems from a commitment problem that we discuss in detail below.

Having considered shareholders who are fully aware of their limitations, we next consider the case when shareholders believe they have more information than they, in fact, do. This analysis addresses the criticisms that not only are shareholders ill-informed relative to management but also are overconfident in their ability to understand the issues involved in some decisions. We consider mainly the extreme case in which shareholders believe they have substantial private information but in reality have no private information at all. We show that even in this case, for some decisions, firm value is maximized if these decisions are controlled by shareholders. Shareholders' misperception introduces a bias like that of management. Optimal control trades off the cost due to management's bias when they are in control against the cost of shareholders' bias and the cost of imperfect communication of management's information when shareholders are in control. It is not hard to see that this tradeoff could go either way. Moreover, the communication cost is attenuated if the shareholders' misperception bias is in the same direction as management's bias, further strengthening the case for shareholder control.

Finally, we examine the case for shareholder control when some shareholders have agendas other than firm value maximization. It is often claimed that some shareholders want to use corporate resources to further a social or political agenda at the expense of profits. For example, some shareholders may want the firm to pursue environmentally friendly production techniques even though these are not legally required and are more costly than other techniques. Other examples include wealth redistribution (*e.g.*, to workers), support for certain political candidates, boycotts of products of certain firms or countries, etc.[6]

---

[6] See, for example, Agrawal (2007), which provides evidence that some union pension funds vote differently in shareholder elections in firms that employ members of that union than they do in elections in firms that do not employ members.

Thus, similar to management, decisions made by non-value maximizing shareholders entail an agency cost. Even though we continue to assume that the objective is share value maximization (more on this in section 5) and that the non-value-maximizing shareholders control any decision assigned to the shareholders, we find that shareholders should control some decisions. In particular, it is optimal for shareholders to control decisions for which non-value-maximizing shareholders' bias is smaller than that of management or in the opposite direction and management's private information is sufficiently unimportant relative to the net bias (management's bias net of shareholders' bias). If shareholders' bias is in the same direction as and greater than that of management, management control is optimal. Of course, when management's information is sufficiently important that shareholders delegate the decision to them regardless of shareholders' information, control is irrelevant.

The next section discusses the relationship of our paper to the "incentive approach" literature on corporate governance. Section 2 presents our model. In section 3, we analyze the "base case" in which shareholders want to maximize the value of their shares and accurately assess the extent of their private information. The case in which shareholders believe they have better information than they actually have is analyzed in section 4. We consider shareholders with non-value-maximizing objectives in section 5. Section 6 concludes.

1 Relation to the "Incentive Approach" Literature

There is a vast literature on corporate governance. Clearly we cannot do justice to this literature here.[7] Instead, we focus on papers that address the key issue of this paper, namely the optimality of shareholder control, using the "incentive approach." This literature focuses on the effect of the allocation of control on the incentives of various parties to engage in value-increasing activities, e.g., invest in firm-specific capital, to acquire information, etc. In this approach, the cost of shareholder control is that it

[7] For an excellent survey of the literature on corporate governance, see Becht, Bolton and Röell (2003). Hermalin and Weisbach (2003) and Adams, Hermalin and Weisbach (2008) provide thorough surveys of the literature on boards of directors.

reduces the incentives of management.[8] For example, if shareholders control the decision of whether to retain or dismiss the CEO, the CEO has less incentive to invest in firm-specific human capital.[9] In contrast, our approach emphasizes the effect of the allocation of control on the extent to which exogenous private information is used in making decisions, taking account of the possibilities of communication of information and delegation of decision-making authority.

We further classify the incentive approach literature into two branches, namely papers that explicitly consider the *allocation of control* over a decision as we do, versus those that address only the potential costs of shareholder control. There are two important papers in the former branch, Aghion and Tirole (1997) and Almazen and Suarez (2003), which we review first. We then consider several papers in the second branch. In reviewing this literature, it is important to recall that, in our terminology, the party in control of a decision is the party who has the right to decide who actually makes the decision, i.e., whether to make the decision itself or to *delegate* the decision to another party.

The paper in this literature that is, perhaps, closest to ours is Aghion and Tirole (1997), hereafter referred to as AT, which is primarily concerned with the allocation of "real" and "formal" authority. "Formal" authority is the authority to make the decision, while "real" authority refers to the party who effectively makes the decision. The AT model includes a principal and an agent who must choose a project from a set of projects with unobserved characteristics. Both can exert effort to learn the characteristics of all the projects. The agent's preferences over projects differ from those of the principal. If the principal has formal authority, and she learns the projects' characteristics (regardless of whether the agent also learns them), the principal will choose her most preferred project. If only the agent learns about the projects, the principal will prefer to accept the project recommended by the agent to choosing a project at random herself. In this case, the principal has formal authority, but the agent has real authority. If neither learns about the projects, the two parties prefer to choose no project than to choose one at

[8] Shleifer and Vishny (1997) discusses this idea in a survey of corporate governance.

[9] This example is from Almazan and Suarez (2003) which we discuss in more detail, along with other examples of this approach, below.

random. If the agent has formal authority, the same description applies with the roles of the parties reversed. Clearly, not having formal authority reduces one's benefit from learning about the projects, since he or she may not get to use the information. Consequently, the party without formal authority will expend less effort to obtain information, other things equal.[10]

To compare the results of AT with those of the current model, we interpret "formal authority" as what we call control. Giving "real authority" corresponds in our model to delegating the decision, since in both models, this results in the non-controlling party's most preferred outcome. We also identify AT's principal with our shareholders and AT's agent with our management. Using our terminology, the allocation of control in AT is determined by its effect on the incentives of the parties to produce information or by the relative importance of the decision to the two parties. In the current paper, the allocation of control is determined by the relative importance of the private information of the parties and the extent to which the private information of the controlling party distorts that party's delegation decision. Moreover, in AT, delegation is determined by who has information: the controlling party will delegate to the other if and only if the other party has information and the controlling party does not. In our paper, both parties (generally) have information, and delegation is determined by the *realization* of the information of the controlling party. Thus, in the current paper, shareholders sometimes delegate when they have information, while in AT, shareholders never delegate if they have information. Finally, in AT, the actual decision (project choice) is determined by who turns out to be the decision maker and whether or not she has information. In our paper, the actual decision is also determined in part by who turns out to be the decision maker but, in addition, depends on the decision-maker's own private information, the other party's private information and what that party communicates to the decision maker about this information. In our paper, the information communicated is different from the

[10] The idea that, when a principal has too much information, incentives for the agent are reduced is also considered by Crémer (1995). In this paper, the principal can commit to carry out a threat to fire the agent only by not acquiring information about the agent's ability.

information itself. The extent of this difference depends on the importance of the two parties' private information and the extent of their differences in preferences.

Some comparisons of specific results of the two papers are also possible.

- In the current paper, if shareholders have no private information, shareholder control is always optimal (strictly in some cases). In AT, this case would correspond to the marginal cost of effort for shareholders being sufficiently high that their equilibrium effort is zero even when they are in control. In AT, control is irrelevant in this case, since, regardless of who is in control, managers will always have real authority.
- If managers have little private information, again, shareholder control is strictly optimal in our model. In AT, however, this case would correspond to the marginal cost of effort for managers being sufficiently high that their equilibrium effort is zero even when they are in control. Therefore, control is irrelevant in this case, since, regardless of who is in control, shareholders will always have real authority.
- In the current paper, if shareholders are misinformed, it is still sometimes strictly optimal for them to have control. Strictly speaking, AT does not consider misinformed agents, but consider a modification of their model in which shareholders receive a signal with positive probability (which may depend on their effort) which they interpret as the characteristics of the projects but which is, in fact, pure noise. In this case, it is never optimal for shareholders to have control.
- If management's preferences become better aligned with shareholders', management control becomes more attractive for some parameter values in our model. In AT, better alignment of preferences always results in shareholder-control becoming more attractive.[11]

[11] By the phrase "management control becomes more attractive," we mean that management control becomes strictly optimal for some parameter values where it was suboptimal initially. In AT, preferences become more similar when the fraction of his private benefits the manager receives if the shareholders' preferred project is chosen increases. As mentioned in the text, AT's result is that shareholder control becomes more attractive in that case (AT, p. 15).

Almazan and Suarez (2003) considers the issue of who should control the decision to retain, or not, the incumbent CEO.[12] Allocation of control over the replacement decision is a device that determines the cost of inducing the incumbent CEO to invest in firm-specific human capital. The optimal allocation depends on which allocation best uses the available incentive devices, including incentive pay, severance pay, replacement and renegotiation. In particular, if the incumbent's control benefits are low and the probability of finding a substantially better replacement is high, it is optimal for the CEO to control his own replacement. In the opposite case, board control of the replacement decision is optimal. Obviously, Almazan and Suarez (2003) addresses control of one, specific decision, whereas the current paper addresses control over a wide range of decisions, and we focus on efficient use of private information as opposed to incentives.

The second branch of the literature assumes shareholders control the decision and asks what are the potential costs of this allocation of control. Burkhart, Gromb and Panunzi (1997), hereafter BGP, emphasizes the benefits to be had if the controlling party can commit *ex ante* to choices that are *ex post* suboptimal for them. In particular, BGP considers low ownership concentration as a commitment device. We also emphasize a commitment problem but assume no device is available to resolve it. BGP uses a model similar to that of AT in which the principal is the shareholders, and the agent is management. The paper argues that when ownership is highly concentrated, shareholders have stronger incentives to acquire information. As in AT, this results in a lower probability that management will be able to use its information and thus reduces incentives for management to acquire information. More diffuse outside ownership acts as a commitment not to preempt management's decisions. As in AT, shareholders in the model of BGP never delegate if they have information, while in the current paper, shareholders sometimes delegate when they have information.

---

[12] Hermalin and Weisbach (1998) also model a board's decision about whether to retain or replace the CEO. When the CEO's past performance is good, the board voluntarily reduces its independence as a way to commit not to obtain information that might result in dismissal of the CEO. A number of interesting results are obtained regarding CEO compensation and turnover and the degree of independence of the board.

Adams and Ferreira (2007) assumes the firm's board of directors controls the decision but considers the optimal degree of independence of the board. Shareholders can choose more or less independent boards. The more independent the board, the less likely it is that the board will delegate the decision to the CEO who values having the right to make the decision. One advantage of not delegating to the CEO (i.e., having a more independent board) is that the CEO is biased so will choose the "wrong" decision from the point of view of shareholders. Another potential advantage of not delegating to the CEO is that it allows full use of private information that the board may have about the optimal decision.[13] The disadvantage of not delegating to the CEO is that it makes it more costly for the CEO to share information with the board. This information is a "key" that allows the board to obtain private information about the optimal decision. Thus, a friendly board (low independence) that has high probability of delegating the decision to the CEO gives the CEO an incentive to reveal the key by lowering his cost of doing so. If one equates "more independent board" with "more shareholder control," one can interpret the Adams-Ferreira results as showing that the optimal degree of shareholder control may be less than the maximal degree, because shareholder control reduces the incentive for CEOs to share information with the board.

Myers (2000), like BGP, also argues that ownership concentration can reduce value by reducing incentives for management. Unlike in BGP, in Myers (2000), the incentive at issue is not to acquire information but, instead, to expend effort. In the Myers model, once effort is expended by management, their unique human capital is no longer required to generate the cash flows associated with the project. If ownership by outside shareholders is too concentrated, outsiders will expropriate any rents that management could otherwise capture. Expecting this, management will not expend effort in the prior period, reducing all future cash flows. Selling the firm to a diffuse set of shareholders is assumed to

[13] Adams and Ferreira (2007) shares with the current paper two key elements. First, both models assume management's preferences for decisions are biased relative to shareholders (indeed both models employ quadratic loss functions). Second, in both models, communication between shareholders and management results in loss of information as modeled in Crawford and Sobel (1982).

reduce the rents that these shareholders can expropriate from management, providing an incentive to management to expend the required effort.

While the papers discussed above consider incentives for gathering information, for expending effort, or for sharing information, Shleifer and Summers (1988) focus on incentives for investing in firm-specific capital. Although the main focus of Shleifer and Summers is on the potentially detrimental effects of hostile takeovers, one can interpret their argument as highlighting a problem with shareholder control. By making hostile takeovers too easy, shareholder control may lead to inefficiently low investment by managers and suppliers in firm-specific capital (human and physical).

The literature described in this section focuses on the distortion of incentives as the cost of shareholder control while we stress the importance of managerial private information and shareholders' inability to delegate efficiently when they also have private information.[14] As discussed above, the two approaches lead to somewhat different conclusions.

2 The Model

Our model is similar to those in Crawford and Sobel (1982), Dessein (2002), and Harris and Raviv (2005, 2008a). Crawford and Sobel (1982), hereafter CS, was the first model to explain the behavior of an agent with private information who may communicate this information to a principal who then takes a decision that affects the welfare of both parties. In the CS, so-called "cheap-talk model," the agent's communication has no direct effect on his welfare, unlike in signaling models. The communication does, however, have an indirect effect through the principal's decision. CS fully characterizes the equilibria of the resulting game and works out closed-form solutions for the uniform-quadratic case that we adopt here. Dessein introduces into the CS model the possibility that the principal,

[14] Many papers outside the corporate governance area use the idea that incentives can be adversely affected by the behavior of principals and that some commitment device that prevents the principal from engaging in such behavior can improve the allocation of resources. For example, in Rotemberg and Saloner (1994) the commitment device is a narrow business strategy. In Rotemberg and Saloner (2000), the commitment device is a CEO with a "vision" that biases the types of projects that are implemented. Obviously, these papers are not directly relevant to the issue at hand.

instead of taking the decision herself, delegates the decision to the agent. Harris and Raviv (2005) extends the model of Dessein by introducing private information on the part of the principal, as well as on the part of the agent, and by allowing the principal's delegation decision to depend on her private information. The CS, Dessein and Harris-Raviv (2005) models assume that the principal has control over the decision. Harris and Raviv (2008a) analyzes the issue of whether "insiders" or "outsiders" on a board of directors should control corporate decisions. It starts with the model of Harris and Raviv (2005) and introduces endogenous, costly information acquisition by outside board members to determine board size endogenously. The current model analyzes the control issue using a specialized version of the Harris-Raviv (2008a) model. By eliminating endogenous information acquisition and assuming that, while shareholders have private information, their private information is relatively unimportant (see below for a precise definition), we obtain more precise results, especially regarding delegation, for the base case in which shareholders are not misinformed and are value-maximizing. The specialization also allows us to extend the model to address the issues of misinformed and non-value-maximizing shareholders.

We consider various decisions that affect the value of a firm. We assume each such decision can be represented by a continuous variable denoted *s*. Some examples of the kinds of decisions we have in mind are the reservation price for sale of the firm or some of its assets, the reservation price for acquiring other firms, the size of a major investment, pricing, executive compensation, product mix, how much to grow using acquisitions, how much to grow using internal investment, how much cash to keep on hand, etc. The issue we analyze is who should optimally control a given decision, shareholders or management. Control of the decision allows the controlling group to make this decision itself, based on its own information and anything it might learn from the other group, or to delegate the decision to the other group. If the controlling party delegates the decision to the other party, then that party chooses *s*, based on its own information and anything they might learn from the controlling party, including anything they can infer from the decision to delegate. We assume that the board of directors is controlled by management insiders and hence always acts in their interests which diverge from the interests of shareholders, as will be explained below.[15]

We do not consider how conflicts among the members of either group, management or shareholders, are resolved, nor do we model the sharing of information among the members of a group. Instead, we model management and shareholders as if each behaves as a single agent with the preferences and information described below. In particular, we do not analyze voting by shareholders or other schemes for aggregating their preferences and information into decisions. Of course, this is an important consideration in deciding whether shareholder control is optimal. Our aim is simply to understand, assuming the difficult issue of preference and information aggregation can be successfully resolved, which decisions it makes sense for shareholders to control.[16]

As in CS and the literature that uses this model, we do not allow explicit contracts contingent on control, reports, decisions, or outcomes. With regard to contracts contingent on control, suppose our model implies that shareholder control maximizes share value for a given decision. We could allow management to accept a lower salary in return for shareholders transferring control to them. It is difficult to imagine, however, that, in any real-world situation, management's gain from obtaining control is greater than shareholders' loss from management control. On the other hand, if our model implies that management control maximizes share value, shareholders could not extract any compensation from management for allowing them control, since management knows that shareholders are better off with management control. In either case, allowing monetary transfers contingent on control would not affect the allocation of control.

---

[15] Obviously, if the interests of shareholders are perfectly represented by independent directors on the board whose information includes any private information of shareholders, then the issue becomes who should control the board or various decisions made by the board, not whether shareholders should directly control these decisions. This is the topic addressed in Harris and Raviv (2008a). Here, we make the opposite assumption that the board does not effectively represent the interests of shareholders. For evidence on the extent to which CEO involvement in the selection of new board members results in appointments of less independent directors, see Shivadasani and Yermack (1999).

[16] Aggregating shareholder information and preferences may not be as great an issue as commentators think. Holderness (2009) offers evidence that the vast majority of U.S. public firms have large blockholders, similar to firms in the rest of the world.

As far as contracts contingent on reports, decisions, or outcomes are concerned, one can argue, as in most agency models, that these variables are difficult to verify in real-world situations and therefore not contractible. Another possible motivation for not considering explicit contracts contingent on the decision-maker's decision or on the outcome is that, to the extent such contracts are possible, their effects are already included in the preference functions specified below. That is, one can think of the preferences as being a reduced form model that includes optimal contracts. Obviously, we are assuming that these schemes do not fully eliminate managerial agency problems.[17]

Finally, it should be pointed out that we do not consider multi-stage communication between management and shareholders or intermediaries in the communication process. These elements are discussed briefly in the Conclusions where their likely impact on our results is considered.

As mentioned in the Introduction, we assume that, not only does management have private information, but so do shareholders. Who we have in mind here are activist shareholders such as Kirk Kerkorian or various fund managers. As evidenced by their attempts to affect corporate decisions, these individuals have, or at least believe they have, information (perhaps gleaned from their experience with other firms) about firm strategy which the firm's management does not have. Note that we are not assuming shareholders have better information than management, only that they may have different relevant information. The optimal (value-maximizing) decision is assumed to depend on the private information of both management ("agents"), $\tilde{a}$, and shareholders ("principals"), $\tilde{p}$. To ensure that the private information of the two parties is complementary for the decision, we assume that the optimal decision is given by the sum of $\tilde{a}$ and $\tilde{p}$. To the extent that the actual decision, *s*, differs from this optimum, there is a loss in firm value given by the quadratic function

$$\left(s-\left(\tilde{a}+\tilde{p}\right)\right)^2. \tag{1}$$

[17]With respect to contracts contingent on reports, see Krishna and Morgan (2008). Note that the assumption that contracts contingent on the decision-maker's decision are not feasible rules out the possibility of constrained delegation, i.e., delegating the decision but constraining the decision-maker to a specific subset of the possible decisions. On the issue of constrained delegation, see Alonso and Matouschek (2007, 2008).

For most of the paper, we assume that shareholders seek to maximize expected firm value, or, equivalently, to minimize the expected loss derived from (1), given whatever information they have.[18] It follows that shareholders' optimal decision is given by $s = E(\tilde{a} + \tilde{p}) = p + E(\tilde{a})$, where the expectation is conditional on whatever information shareholders have about $\tilde{a}$.

Management, on the other hand, is biased relative to value maximization. In particular, we assume that management seeks to minimize, given whatever information they have, the expectation of the loss function

$$\left(s - \left(\tilde{a} + \tilde{p} + b\right)\right)^2, \tag{2}$$

where ***management's bias***, *b*, is a positive parameter that measures the extent of the agency problem between shareholders and management.[19] Management's optimal decision is given by $s = E(\tilde{a} + \tilde{p}) + b = a + E(\tilde{p}) + b$, where the expectation is conditional on whatever information management has about $\tilde{p}$.

Because of the quadratic loss functions in (1) and (2), the difference between the expected loss that results when management makes the decision and the expected-loss-minimizing decision, for any given information, is b^2. We therefore refer to b^2 (and sometimes *b*) as the ***agency cost***.

The shareholders' information cannot be obtained by management, but may be communicated to them by shareholders. Similarly, shareholders cannot obtain management's information directly, but management may communicate it to them. Because of the agency problem described above, communication is strategic and will be analyzed in the next section.[20]

---

[18] In section 5, we consider non-value-maximizing behavior on the part of shareholders.

[19] It is not essential that *b* be positive. This is assumed only for expositional convenience.

[20] We assume that the truthfulness of a party's report is not verifiable. Verifiable information is considered in Milgrom and Roberts (1986), among others. This paper shows that verifiability leads to an equilibrium in which the party not making the decision always reports the truth to the decision-maker. Their result, however, depends on the assumption that preferences of the reporting party over the decision-maker's decision are monotone. In our model, the reporting party's preferences over the decision are single peaked at a value that depends on the information of both parties. Consequently, the results of Milgrom-Roberts do not carry over to our model.

We make the following assumptions regarding the distributions of $\tilde{a}$ and $\tilde{p}$:

Assumption 1. The variables $\tilde{a}$ and $\tilde{p}$ are independent with $\tilde{a}$ uniformly distributed on $[0, A]$ and $\tilde{p}$ uniformly distributed on $[0, P]$.[21]

The specific assumptions that the optimal decision is the sum of $\tilde{a}$ and $\tilde{p}$ and that the loss is quadratic in equations (1) and (2) and Assumption 1 are used in Harris and Raviv (2005, 2008a). The quadratic loss function case together with Assumption 1 was originally solved by CS and used by Dessein (2002), assuming one-sided private information. These can be generalized somewhat for some of our results, but they greatly simplify the analysis. We believe that the insights derived from this model do not depend on these assumptions.

In some cases, it will be more convenient to work with the standard deviations of the random variables $\tilde{a}$ and $\tilde{p}$, as well as those of other uniformly distributed random variables, instead of the parameters *A* and *P*. Consequently, for any $x \geq 0$, denote by $\sigma(x)$ the standard deviation of a random variable uniformly distributed on an interval of width *x*, i.e., $\sigma(x) = x/\sqrt{12}$. We will use σ_a to denote $\sigma(A)$ and σ_p to denote $\sigma(P)$.

Because of the quadratic cost function in (1), it turns out that if an unbiased decision-maker chooses *s*, the difference in firm value between knowing $\tilde{p}$ (respectively, $\tilde{a}$) and having no information about $\tilde{p}$ (respectively, $\tilde{a}$) is exactly σ_p^2 (respectively, σ_a^2). We will therefore refer to σ_p^2 and σ_p (σ_a^2 and σ_a) as the ***importance of the shareholders' (management's) information***. We focus on the case in which the agency problem (as measured by *b*) is severe relative to the importance of shareholders' information. It will be shown below that this assumption implies that management will not want to delegate to shareholders, which we believe is realistic, and that shareholders will never directly reveal any

Although we haven't analyzed the model under the verifiability assumption, we suspect that revelation of information without distortion would not be possible.

[21] Our results depend only on the widths, *A* and *P*, of the supports of $\tilde{a}$ and $\tilde{p}$, not on their locations.

of their private information to management. These implications simplify the analysis relative to the more general case considered in Harris and Raviv (2005, 2008a). They also allow us to focus on what we think are the more realistic cases and to obtain a uniqueness result on the equilibrium when shareholders are in control, as well as additional comparative statics results. CS and Dessein (2002) have no corresponding assumption, since they consider only private information on the part of the agent. Formally, we assume

*Assumption 2***:** $\sigma_p < b$.

Recall that control of a decision empowers the controlling party to make the decision or to delegate it to the other party. If the controlling party does not delegate, it will make the decision based on its own information and any information communicated to it by the other party. The sequence of events is assumed to be the following. After observing its private information, the controlling party decides whether to delegate to the other party or not. The party not making the decision may communicate some or all of its private information to the decision maker. Finally, the decision maker chooses *s* and firm value is realized.

3 Base Case Analysis

In this section, we analyze the case in which shareholders are value-maximizers and understand perfectly the extent of their private information as well as all other parameters of the model. Since the current model is a special case of the model in Harris and Raviv (2008a) as explained earlier, we will borrow liberally from the results in that paper. We first analyze separately the two cases in which shareholders are assumed to be in control of the decision and management is assumed to be in control. We then determine optimal control by comparing the equilibrium firm values for the two cases.

3.1 Shareholder Control

First, assume that shareholders are in control but do not delegate the decision to management. In this case, shareholders will choose *s* based on their own information and any information communicated to them by management. Denote by $r(a)$ the report of management to shareholders if management observes $\tilde{a} = a$. Because of the agency problem, the report will not fully communicate the reporting party's information, as will be seen below.

As noted above, if shareholders observe $\tilde{p} = p$, their optimal decision is given by

$$s(p,r) = \bar{a}(r) + p, \tag{3}$$

where $\bar{a}(r) = E(\tilde{a}|r)$ is the mean of shareholders' posterior belief about $\tilde{a}$, given management's report, *r*.

Because of the agency problem, management will not fully reveal their information. Instead, management's report will allow the shareholders only to narrow the range of possible values of $\tilde{a}$ to an interval. The width of the interval is an inverse measure of the precision of the information communicated by management. For example, the greater is the agency cost, *b*, the less informative is management's report, i.e., the wider is the interval. More precisely, in the Pareto-best Bayes equilibrium of the game in which shareholders choose *s*, management will partition the support of $\tilde{a}$, $[0, A]$, into cells $[a_i, a_{i+1}]$ (of unequal widths) and report a value that is uniformly distributed on the cell in which the true realization of $\tilde{a}$ lies.[22] Thus shareholders learn only the cell in which the true value of management's information lies, and their posterior belief is that $\tilde{a}$ is uniformly distributed on that cell. It follows that if the report *r* is in $[a_i, a_{i+1}]$, $\bar{a}(r) = \frac{a_i + a_{i+1}}{2}$. The number of cells in the partition is denoted by $N(b, A)$ (see Harris and Raviv (2008b) for an explicit formula). Note that the number of cells is a measure of the extent to which management communicates its information to shareholders. For example, if there is only one cell, $N(b, A) = 1$, management communicates nothing shareholders do not already know. This will be the case, for example, if management's information is less important than agency cost, i.e., $\sigma_a \le b$. On the other hand as $N(b, A)$ gets very large (as will be the case if agency cost, *b*, approaches zero), the

[22] The game in which shareholders choose *s* is analyzed formally in Harris and Raviv (2005) by extending the results of CS to the case of bilateral asymmetric information. Here we simply summarize the results and provide intuition.

information communicated approaches perfect information about $\tilde{a}$. We state the following result about the function $N(b,x)$ for future reference (the proof can be found in Harris and Raviv (2005)).

Lemma 1. $N(b,x)=1$ if and only if $x \leq 4b$ or, equivalently, $\sigma(x) \leq 2b/\sqrt{3}$.

Since $2 \geq \sqrt{3}$, the lemma implies that $N(b,A)=1$ if $\sigma_a \leq b$, as mentioned above.

Since management does not fully communicate its private information, there is a consequent loss in firm value. Let $L(b,A)$ denote this information cost, i.e., the expected loss in firm value due to having only information about $\tilde{a}$ that is transmitted in equilibrium (as opposed to full information). That is,

$$L(b,A) = E\left[\bar{a}\left(r(\tilde{a})\right) + p - (\tilde{a} + p)\right]^2 = E\left[\bar{a}\left(r(\tilde{a})\right) - \tilde{a}\right]^2. \quad (4)$$

The expected loss $L(b,A)$ depends on how much information is transmitted, on average, in the report, as measured by $N(b,A)$.[23] In particular, if $N(b,A)=1$, i.e., no information is transmitted to shareholders, then $L(b,A)$ is the entire variance, σ_a^2, of $\tilde{a}$, as is obvious from (4). If some information is transmitted ($N(b,A)>1$), the expected cost is smaller than the variance.

Now suppose that shareholders do delegate the decision to management. Then management will choose *s* based on their own information and any information communicated to them by shareholders. Lemma 1, together with Assumption 2, implies that $N(b,P)=1$. Thus, the assumption that agency costs exceed the importance of shareholders' information (Assumption 2) implies that shareholders' report will convey no information about $\tilde{p}$ to management. Although shareholders do not directly share any information about $\tilde{p}$, they may reveal some information through their delegation decision, as will be seen presently. If management observes $\tilde{a} = a$, they choose

$$s(a) = a + \hat{p} + b, \quad (5)$$

---

[23] An explicit formula for *L* is given in Harris and Raviv (2008b) where it is shown that this expected loss depends only on the agency cost, *b*, and the width *A* of the support of $\tilde{a}$.

where $\hat{p} = E(\tilde{p} | delegation)$ is the mean of management's posterior belief about $\tilde{p}$ *given the fact that the decision has been delegated*. Therefore, to analyze the equilibrium in this case, one must first understand in which circumstances, i.e., for which values of $\tilde{p}$, shareholders will delegate.

To understand when shareholders will delegate, consider the loss in firm value from delegating and the loss from not delegating. There are two components of the loss from delegating, namely the direct agency cost, b^2, and the loss due to imperfect communication of shareholders' private information. The loss from not delegating is the loss due to imperfect communication of management's private information, i.e., $L(b, A)$.

If shareholders have no private information ($\sigma_p = 0$), there is no loss due to imperfect communication about shareholders' private information, so the relevant comparison is between $L(b, A)$ and the direct agency cost, b^2. Consequently, shareholders delegate in this case if and only if $L(b, A) \geq b^2$. It is shown in Lemma 1 of Harris and Raviv (2008b), however, that $L(b, A) \geq b^2$ if and only if $\sigma_a \geq b$. Therefore, when shareholders have no private information, they will delegate if and only if management's information is more important than agency costs. This result is quite intuitive, and the delegation policy maximizes firm value. This is not the case when shareholders have private information.

If shareholders have private information ($\sigma_p > 0$), there is a cost of delegating due to loss of information about $\tilde{p}$. The extent of this loss depends on what management infers about $\tilde{p}$ from the fact that shareholders have chosen to delegate. Moreover, the set of realizations of $\tilde{p}$ for which shareholders choose to delegate, i.e., the "delegation region," depends on what management infers from delegation. In equilibrium management inferences about $\tilde{p}$ based on shareholders' decision to delegate must be consistent with shareholders' optimal delegation region. Consequently, the delegation region must satisfy the following, equilibrium property. Suppose management believes that delegation occurs if and only if

$\tilde{p}$ is in some region $D \subset [0, P]$. Then it must be optimal for the shareholders to delegate if and only if $\tilde{p}$ is in the region *D*.

We argue that the equilibrium delegation region is of the form $[p^*, P]$ for some threshold $p^* \in [0, P]$, i.e., that shareholders will delegate if and only if $\tilde{p}$ exceeds some threshold p^*. To see this, suppose the interval $D = [d_1, d_2] \subset [0, P]$ is the equilibrium delegation region.[24] In this case, because $\tilde{p}$ is uniformly distributed, management's expectation of $\tilde{p}$ given delegation, $\hat{p}$, is the midpoint of *D*. Since management chooses $s = a + \hat{p} + b$, shareholders' loss from delegating if they observe $\tilde{p} = p$ is $[a + \hat{p} + b - (a + p)]^2 = (\hat{p} + b - p)^2$, i.e., the squared distance between $\hat{p} + b$ and *p*. But since $b > 0$ and $\hat{p}$ is the midpoint of *D*, $\hat{p} + b$ is to the right of the midpoint of *D*. If $d_2 < P$, then for some values of *p* strictly between d_2 and *P*, *p* is closer to $\hat{p} + b$ than some values of *p* in the delegation region *D*. But this means that for these values of $\tilde{p}$ shareholders would prefer to delegate even though $\tilde{p}$ is not in the delegation region. Hence, if $d_2 < P$, *D* cannot be an equilibrium delegation region, i.e., any equilibrium delegation region must have *P* as its right endpoint. This shows that delegation by shareholders, if it occurs, will occur for all values of $\tilde{p}$ above some threshold, p^*.

It is clear from this discussion that whenever the threshold p^* is strictly between 0 and *P*, p^* must be such that shareholders are indifferent between delegating and not delegating when $\tilde{p} = p^*$, given that management believes that shareholders delegate if and only if the realization of $\tilde{p}$ is above p^*. Therefore, in this case, p^* must satisfy

$$L(b, A) = (\hat{p} + b - p^*)^2 = \left(\frac{P + p^*}{2} + b - p^* \right)^2 . \quad (6)$$

[24] This is an intuitive argument that assumes the delegation region is an interval. In the proof of Proposition 1 (in the appendix), we show that, in the unique pure-strategy equilibrium, shareholders delegate if and only if $\tilde{p}$ exceeds a threshold.

In equation (6), the left hand side is the loss due to not delegating, i.e., the loss due to imperfect communication of $\tilde{a}$ by management. The middle expression is the loss due to delegating when $\tilde{p} = p*$, where $\hat{p}$ is the midpoint of the interval $[p*, P]$, since management's choice of *s* reflects their belief that delegation implies that $\tilde{p} \in [p*, P]$. Solving (6) for $p*$ results in the expression for $p*$ when $p*$ is strictly between 0 and *P* given in the next proposition.

It is convenient to denote by *d* the width of the interval of values of $\tilde{p}$ over which shareholders delegate, i.e., $d = P - p*$. We summarize the above discussion and characterize $p*$ and *d* more precisely in the following proposition.

Proposition 1: Suppose shareholders are in control of a decision. The unique pure-strategy Perfect Bayes' Equilibrium of the resulting game is as follows:[25]

- If shareholders have no private information ($\sigma_p = 0$) they will delegate the decision if and only if $\sigma_a \geq b$, i.e., if and only if management's information is more important than agency costs, regardless of the realization of $\tilde{p}$. In this case, management infers nothing from the delegation decision, and, if shareholders do delegate, management chooses $s = a + b$, where *a* is the realization of $\tilde{a}$. [This is shown in Dessein (2002).]
- If shareholders have private information ($\sigma_p > 0$), shareholders will delegate the decision if and only if the realized value of their private information exceeds a threshold $p* \in [0, P]$. This threshold and the equilibrium strategies and beliefs of management depend on the values of the parameters, σ_a, σ_p, and b. There are three cases to consider.
  - Case 1: $\sigma_a \leq b$. In this case, shareholders never delegate, unless $\tilde{p} = P$ and $\sigma_a = b$, i.e.,

$$p* = P \text{ and } d = 0. \tag{7}$$

[25] Harris and Raviv (2005) show that there is an equilibrium of the form described in this proposition. Here, by simplifying the problem somewhat, we show that this is the only pure-strategy equilibrium.

If shareholders do not delegate (this is their equilibrium move, unless $\sigma_a = b$ and $\tilde{p} = P$), management's inference and strategy are irrelevant, since it has no moves. If shareholders delegate (this is an off-equilibrium-path move, unless $\sigma_a = b$ and $\tilde{p} = P$), management infers that $\tilde{p} = P$ with probability one and chooses $s = a + P + b$, where a is the realization of $\tilde{a}$.

- Case 2: $P \le 2\left[\sqrt{L(b,A)} - b\right]$. In this case, shareholders always delegate, i.e.,

$$p^* = 0 \text{ and } d = P. \tag{8}$$

Management infers nothing if shareholders delegate (this is their equilibrium move) and chooses $s = a + \frac{P}{2} + b$, where a is the realization of $\tilde{a}$. If shareholders do not delegate (this is an off-equilibrium-path move), management's inference and strategy are irrelevant, since it has no moves.

- Case 3: $\sigma_a > b$ and $P > 2\left[\sqrt{L(b,A)} - b\right]$. In this case, $p^* \in (0,P)$, $d \in (0,P)$ and satisfy

$$P - p^* = d = 2\left[\sqrt{L(b,A)} - b\right]. \tag{9}$$

If shareholders delegate, management's posterior belief about $\tilde{p}$ is uniform on $[p^*, P]$, and it chooses $s = a + \frac{P + p^*}{2} + b$, where a is the realization of $\tilde{a}$. If shareholders do not delegate, management's inference and strategy are irrelevant, since it has no moves. There are no off-equilibrium-path moves in this case.

Note that for $d \in (0,P)$, d is independent of P.

It follows from Proposition 1 that, when shareholders are in control, the expected loss in firm value is given by

$$L_S = \frac{d}{P}\left(b^2 + \sigma\left(d\right)^2\right) + \left(1 - \frac{d}{P}\right)L\left(b, A\right). \tag{10}$$

To understand this expression, first recall that if shareholders delegate, they do not share any information about $\tilde{p}$ with management other than the fact that $\tilde{p} \geq p^*$, due to Assumption 2. Consequently, the expected loss in firm value if shareholders delegate is the agency cost, b^2, plus the loss from knowing only that $\tilde{p} \in \left[p^*, P\right]$, $\sigma\left(P - p^*\right)^2 = \sigma\left(d\right)^2$. Thus the first term on the right hand side of (10) is the probability that shareholders delegate, d/P, times the expected loss if they do, $b^2 + \sigma\left(d\right)^2$. The second term on the right hand side of (10) is the probability that shareholders do not delegate, $1 - d/P$, times the expected loss if shareholders make the decision, $L\left(b, A\right)$.[26]

We now consider the relationship between the *equilibrium* delegation threshold, p^*, and the threshold that would be optimal for shareholders if they could commit *ex ante* (before learning $\tilde{p}$) to a delegation threshold, say p^{**}. The fact that these two thresholds turn out to be different plays an important role in the analysis of optimal control in section 3.3.

Suppose shareholders could announce and commit to any delegation threshold, $P - x$ with $x \in \left[0, P\right]$, before observing their private information. Since shareholders are committed to delegate if and only if $\tilde{p} \geq P - x$, if shareholders delegate, management correctly infers that $\tilde{p} \geq P - x$. In this case, the *ex ante* expected loss from shareholder control is given by equation (10) with *d* replaced by *x*:

$$\frac{x}{P}\left(b^2 + \sigma\left(x\right)^2\right) + \left(1 - \frac{x}{P}\right)L\left(b, A\right). \tag{11}$$

[26] In the special case in which shareholders have no private information, $d = P = 0$. Equation (10) is still correct if we take $d/P = 1$ if $b \leq \sigma_a$ and $d/P = 0$ otherwise.

The *ex-ante*-optimal threshold is found by choosing $x \in [0, P]$ to minimize the above expression.[27] Let d^* solve this problem, and let $p^{**} = P - d^*$. Then as is shown in Lemma 2 in the appendix, p^{**} is an *ex-ante*-optimal delegation threshold, $d^* \geq d$ and $p^{**} \leq p^*$, with strict inequalities whenever $\sigma_a > b$, and the equilibrium delegation decision does not involve always delegating, i.e., $d < P$.

Thus, in general, the equilibrium delegation threshold p^* results in too little delegation compared to an *ex-ante*-optimal threshold with commitment. The reason for this is that the delegation decision is a signal that conveys information about $\tilde{p}$ to management in addition to determining who actually makes the decision. If the delegation threshold is chosen *ex ante*, shareholders take into account the effect of the threshold on the information content of delegating. In particular, they realize that a lower threshold causes management to reduce their beliefs about $\tilde{p}$ inferred from the act of delegating. This has the beneficial effect of inducing management to select a lower value of *s* than they otherwise would, thus helping to mitigate management's bias. The beneficial effect from changing the information content of the delegation decision results in shareholders being willing to delegate for some realizations of $\tilde{p}$ below p^*, even though, *ex post* the cost of delegating is higher than the cost of not delegating for those realizations. Consequently, commitment on the part of shareholders to the *ex-ante-optimal* delegation threshold is indeed required. In the *ex post* equilibrium without commitment, however, shareholders take management's belief about the threshold, and hence the information conveyed by delegating, as given. In this situation, reducing the threshold below p^* has only costs and no benefits. That is, reducing the threshold below p^* results in delegation for some realizations of $\tilde{p}$ for which the cost of delegating exceeds that of not delegating but has no beneficial effect of inducing management to infer that $\tilde{p}$ is smaller.

---

[27] Note that we are not claiming the solution to this problem is an optimal delegation *policy*. We claim only that, among policies that are characterized by a lower threshold for delegating, the solution is optimal.

This result implies that the equilibrium delegation decision of shareholders is suboptimal for shareholders from the *ex ante* point of view. The reason this result is so important is that it will help us understand why the intuition described in the introduction that, since they can delegate, it is always optimal for shareholders to be in control is not correct. This is explained in detail in section 3.3, after we discuss management control.

3.2 Management Control

From the point of view of management, shareholders are biased toward smaller choices of *s* by $-b$. Consequently, when management is in control, their delegation decision is the mirror image of shareholders' delegation decision. The analysis of the previous subsection applies with the obvious renaming of players. In particular, management never delegates if $\sigma_p < b$. Assumption 2 therefore implies that management will never delegate the decision to shareholders. As before, Assumption 2 also implies that shareholders will refuse to share any information about $\tilde{p}$ with management. Consequently, the expected loss in firm value under management control is the agency cost, b^2, plus the loss from knowing only that $\tilde{p} \in [0, P]$, σ_p^2, i.e., the expected loss in firm value under management control is

$$L_M = b^2 + \sigma_p^2. \qquad (12)$$

3.3 Optimal Control

Our goal in this subsection is to characterize the values of the parameters *b*, σ_p, and σ_a that lead to control by each of the two parties. We obtain the somewhat counterintuitive result that shareholders should control all decisions about which they are ignorant but not some decisions about which they have private information. Indeed, many commentators have argued just the opposite.[28]

Obviously, shareholder control of the decision is optimal if and only if the net gain to shareholder control, $\Delta \equiv L_M - L_S$, is non-negative, where L_M is given by (12) and L_S is given by (10). The main

[28] See, e.g., Bainbridge (2006).

result of this section is stated in the following proposition and depicted in Figure 1 (a more formal statement of this result and the proof are given in the appendix).

Proposition 2. For any given value of agency costs, *b*, the possible combinations of the importance of management's and shareholders' information, (σ_a, σ_p), can be divided into three regions as depicted in Figure 1.[29] In particular:

- It is strictly optimal for shareholders to control all decisions for which management's information is less important than agency cost ($\sigma_a < b$). For decisions for which management's information is more important than agency cost ($\sigma_a \geq b$), it is optimal for shareholders to control those decisions for which their information is more important than a threshold level, $\sigma_p^U(\sigma_a)$. This threshold is an increasing function of the importance of management's information. Moreover, it is below the importance of management's information, i.e., shareholders should control all decisions for which their information is at least as important as management's.
- It is strictly optimal for management to control decisions for which the importance of shareholders' information is below the threshold required for shareholder control to be optimal mentioned in the previous bullet ($\sigma_p^U(\sigma_a)$) but above a lower threshold, $\sigma_p^L(\sigma_a) \leq \sigma_p^U(\sigma_a)$, that is also an increasing function of the importance of management's information.
- Control is irrelevant for decisions for which the importance of shareholders' information is below the lower threshold mentioned in the previous bullet, $\sigma_p^L(\sigma_a)$.

---

[29] The function Δ is homogeneous of degree two in the parameters (b, A, P). Consequently, if it is optimal for a party to control a decision described by (b, A, P), it is also optimal for that party to control all decisions described by $(\alpha b, \alpha A, \alpha P)$ for $\alpha > 0$. Therefore, in the three-dimensional version of Figure 1 with *b* on the *z*-axis, the regions of optimal control are cones in the positive orthant. Thanks to Robert Novy-Marx for calling our attention to this fact.

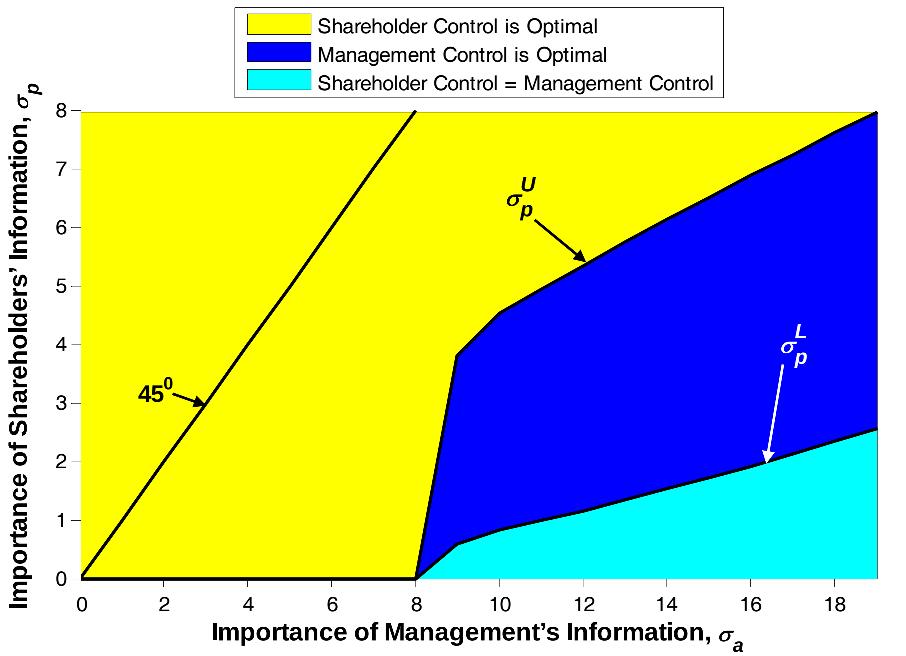

Figure 1

This graph shows how optimal control varies with the two information parameters, σ_a, the importance of management's information, and σ_p, the importance of shareholders' information. For this figure, $b = 8$.

The first bullet of Proposition 2 is quite intuitive. It states, essentially, that shareholders should control all decisions for which management's information is less important than agency cost, all decisions for which shareholders' information is at least as important as management's, and some decisions for which management has an informational advantage relative to its agency cost and relative to shareholders. For shareholder control to be optimal in these cases, shareholder information must be sufficiently important relative to management's information, i.e., the hurdle for the importance of shareholders' information increases as management's information becomes more important. The second bullet states that management should control decisions for which shareholders' information is less important than the hurdle mentioned above but not too much less important. The reason for this last caveat is that if shareholders' information is too much less important than management's, shareholders will always delegate to management, so control is irrelevant (this is the content of the third bullet).

Proposition 2 has several interesting and important implications. First, it is optimal for biased management to control some decisions. Given that shareholders' objective is to maximize firm value and given that, if shareholders believe that management's choice of s will result in higher value than their own, they may delegate the decision to management, one might ask why it would ever be optimal for management to control the decision. This follows from the fact that, when shareholders have private information, they do not delegate optimally as was discussed in the section 3.1 and shown in Lemma 2 in the appendix.

Indeed, from the point of view of assigning control, which is done *ex ante*, if shareholders were able, *ex ante*, to commit to a delegation policy, it would always be optimal to assign them control. To see this, note that a *feasible* solution to the problem of choosing a delegation threshold to minimize shareholders' expected loss, i.e., choosing $x \in [0, P]$ to minimize (11), is $x = P$ (always delegate), which results in an *ex ante* expected loss from shareholder control of $b^2 + \sigma_p^2$. But this is equal to the expected loss from management control, L_M (see equation (12)). Therefore, the *ex-ante*-optimal delegation threshold results in an expected loss from shareholder control that is no greater than the expected loss from management control. Consequently, if the threshold can be chosen *ex ante* to minimize the expected loss from shareholder control, and shareholders can commit to the chosen threshold, it is always at least weakly optimal for shareholders to control any decision. But Lemma 2 shows that the *ex-ante*-optimal delegation threshold is sometimes strictly smaller than the equilibrium delegation threshold, and, therefore, that commitment to the *ex-ante*-optimal delegation threshold is required for management to believe that it is the threshold. Since we assume no such commitment is possible, the equilibrium delegation threshold is sometimes suboptimal from the *ex ante* point of view. In such situations, it can be better for shareholders to have management in control. Indeed, putting management in control is like a commitment to delegate. This accounts for the non-empty region in Figure 1 in which it is optimal for management to control the decision. In this region, management's information is sufficiently more important than shareholders' information that firm value is higher if management always makes the decision but not sufficiently more important that shareholders will delegate to them for every realization of $\tilde{p}$.[30]

Given that suboptimal delegation causes shareholder control to be suboptimal for some decisions, and that delegation is suboptimal because the delegation decision conveys information, one might ask if the optimality of shareholder control can be restored, at least for some decisions, by requiring shareholders, when in control, to make the delegation decision *before* observing their private information. It turns out that, because all parameters of the decision are assumed known *ex ante*, this is equivalent to choosing between management control and shareholder control with delegation by shareholders prohibited.[31] In fact, it is easy to show that, for *any* decision, prohibiting shareholders from delegating when in control is worse for shareholders than allowing them to delegate contingent on their private information.[32] Consequently, requiring shareholders, when in control, to make the delegation decision before observing their private information will make shareholders worse off and result in more decisions for which management control is preferred.

A second important implication of Proposition 2 is that, when shareholders have no private information ($\sigma_p = 0$), it is strictly optimal for them to be in control when management's information is less important than agency cost ($\sigma_a < b$) and weakly optimal when management's information is more

---

[30] The fact that suboptimal delegation by outsiders leads to the conclusion that insider-control is sometimes optimal was present in Harris and Raviv (2008a) but was not fully explained there. In particular, Harris and Raviv (2008a) does not present a proof of the suboptimality of the delegation decision as we do here in Lemma 2.

[31] If shareholders are in control and must make the delegation decision before observing their private information, they will delegate if and only if $b^2 + \sigma_p^2 \le L(b, A)$, and expected losses will be given by $\min\{b^2 + \sigma_p^2, L(b, A)\}$. But, if management is in control, expected losses will be given by $b^2 + \sigma_p^2$, whereas, if shareholders are in control and cannot delegate, expected losses will be given by $L(b, A)$. An optimal (for shareholders) choice between these two regimes results in exactly the same expected losses as having shareholders in control and requiring them to make the delegation decision before observing their private information.

[32] The intuition for why it is never useful to prohibit shareholders from delegating is simple. The problem with shareholders' equilibrium delegation decision is that they delegate too little, not that they delegate too much.

important than agency cost ($\sigma_a \geq b$). This is because, when shareholders have no private information, they make an *ex-ante*-optimal delegation decision, namely to delegate if management's information is more important than agency cost and not otherwise. As noted in section 1, this result is in contrast to that of Aghion and Tirole (1997), where control is irrelevant in this case.

Third, the region of the $\left(\sigma_a, \sigma_p\right)$-plane in which it is strictly optimal for shareholders to be in control includes the diagonal. Thus, for decisions in which shareholders and management have information of approximately equal importance, shareholders should be in control, and management should control a decision only if their information is sufficiently more important than shareholders'. This result implies that the loss due to suboptimal delegation by shareholders is less than the loss due to management's bias, when the two parties have equally important information. Shareholders should also control all decisions for which management's information is less important than the agency cost, since for such decisions, shareholders, if in control, optimally do not delegate to management (see Proposition 1).

Fourth, in the region below the curve $\sigma_p = \sigma_p^L\left(\sigma_a\right)$, control doesn't matter, because for decisions in this region, management's information is of critical importance, so shareholders will always delegate and convey no information (see Proposition 1). Since management never delegates, in this region, management actually makes the decision with no information from shareholders, regardless of who controls the decision.

To gain additional insights into the determinants of optimal control of corporate decisions, in the next proposition, we examine the comparative statics results of optimal control with respect to the importance of the parties' information and the extent of the agency problem. The first part of this proposition is obvious from Figure 1 and Proposition 2. The second part is stated formally and proved in the appendix.

Proposition 3: Comparative Statics.

- An increase in the importance of shareholders' information can result in a shift from management control being strictly optimal to shareholder control being strictly optimal but not the reverse. Similarly, an increase in the importance of management's information can result in a shift from shareholder control being strictly optimal to management control being strictly optimal but not the reverse.

- Suppose agency costs increase from $b = b_L$ to $b = b_H$ with $b_H > b_L > 0$. This results in a switch of optimal control from management to shareholders if management's information is not too important. The increase in agency costs may also cause a switch in optimal control from shareholders to management if management's information is sufficiently important. Both cases are depicted in Figure 2.

Clearly, an increase in *b* shifts the intercepts of the two boundaries in Figure 1 to the right as shown in Figure 2. Since both boundaries are upward sloping, this always results in a region of values of (σ_a, σ_p) in which management control is optimal for the smaller value of *b*, but shareholder control is strictly optimal for the larger value of *b*. This region is shown in yellow in Figure 2. Figure 2 also illustrates the somewhat counterintuitive fact that an increase in *b* can result in a switch from shareholder control to management control for other values of (σ_a, σ_p). These values are shown as the blue region in Figure 2.

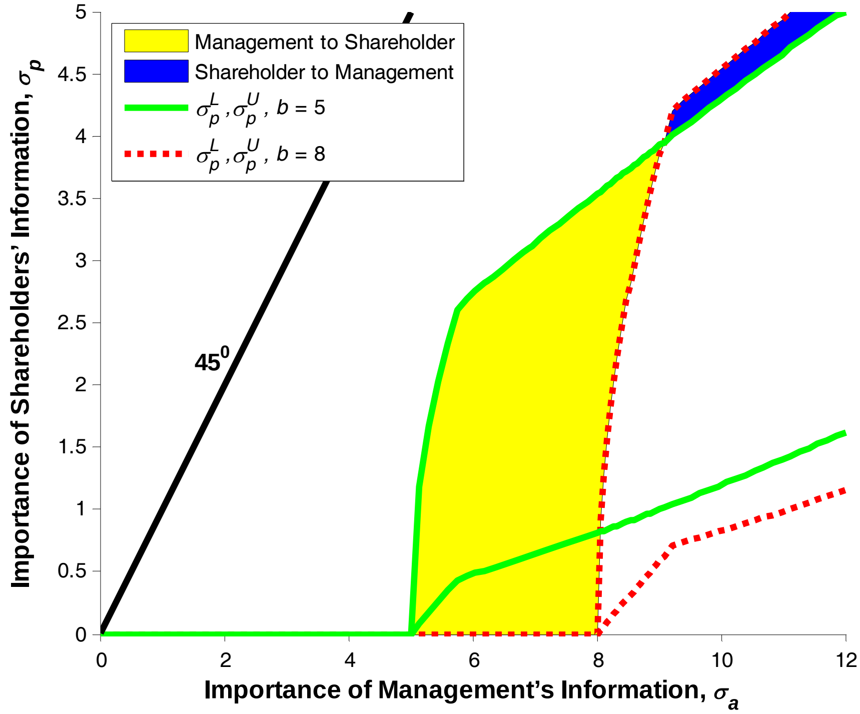

Figure 2

This figure shows the boundary curves σ_p^U and σ_p^L for two values of *b*, $b=5$ and $b=8$. For decisions for which (σ_a, σ_p) is in the yellow region, the increase in *b* from 5 to 8 results in a switch in control from management to shareholders. For decisions for which (σ_a, σ_p) is in the blue region, the increase in *b* from 5 to 8 results in a switch in control from shareholders to management. For this example, the value of σ_a at which the two upper boundary curves cross is approximately 9.0794.

Some intuition for these results can be obtained by examining the forces at work when the parameters are changed. First, consider an increase in the importance of shareholders' information, σ_p, holding agency costs and the importance of management information, b and σ_a, constant. Such an increase affects both the cost of management control, L_M, and the cost of shareholder control, L_S. The effect on the cost of management control is straightforward: since management, when in control, receives no information about shareholders' private information, any increase in the importance of this information increases the cost of management control (see equation (12)).

The effect of an increase in σ_p on the cost of shareholder control is a bit more complicated. Referring to equation (10), we see that the expected cost of shareholder control is a weighted average of the cost if shareholders delegate and the cost if they do not, weighted by the probabilities of delegating and not delegating, respectively. Neither of the costs is affected by an increase in σ_p, but the probabilities are. In particular, since an increase in σ_p (equivalently, *P*) does not affect the equilibrium width of the delegation region, *d* (see Proposition 1), it reduces the probability of delegation, d/P.[33] Consequently, an increase in σ_p shifts weight from the cost of delegating to the cost of not delegating. We argue that the *ex ante* expected cost of not delegating is greater than the *ex ante* expected cost of delegating. This is a consequence of the fact that the equilibrium delegation region is smaller than the *ex-ante*-optimal delegation region (Lemma 2). That is, since the *ex-ante*-optimal delegation region is larger than *d*, increasing *d* must decrease L_S. But increasing *d* makes delegating more likely and makes the cost of delegating higher, because management will have less information about $\tilde{p}$ with a larger delegation region. The increase in *d* has no effect on the cost of not delegating, $L(b,A)$, so, in order for the increase in *d* to reduce L_S, it must be the case that the cost of delegating at the initial value of *d* is smaller than the cost of not delegating. Consequently, by reducing the probability of the less-costly option (delegation), increasing σ_p also makes shareholder control more costly. Proposition 3 implies that the more direct effect on management control dominates the "delegation" effect on shareholder control.

[33] This is not true if $d = 0$ and may not be true if $d = P$. Since this discussion is only to explain the intuition for our results, and the intuition in these cases is similar, we don't make this distinction.

Second, consider an increase in σ_a, the importance of management's information, on the advantage of shareholder control relative to management control, Δ, holding agency costs and the importance of shareholder information, b and σ_p, constant. There are two opposing effects:

- An increase in the importance of management's information aggravates the loss due to imperfect communication of their information whenever shareholders do not delegate. Thus, for this effect, an increase in σ_a makes management control more attractive relative to shareholder control (reduces Δ).
- An increase in the importance of management's information also results in (weakly) more delegation by shareholders. Since shareholders, in general, delegate too little (Lemma 2), this effect makes management control less attractive relative to shareholder control.[34]

Proposition 3 implies that the first effect dominates.

Finally, consider a change in agency cost *b* on the advantage of shareholder control relative to management control, holding the importance of management and shareholder information, σ_a and σ_p constant. In this case, there are three effects:

- An increase in agency cost directly increases the cost of management control for sure, but directly increases the cost of shareholder control only if shareholders delegate. Since, in general, shareholders delegate with probability less than one, on balance, this effect generally makes management control less attractive relative to shareholder control.
- As was the case with an increase in the importance of management's information, an increase in agency cost aggravates the loss due to imperfect communication of management's information (management communicates less of their information) whenever shareholders do not delegate. Thus, for this effect, an increase in *b* makes management control more attractive relative to shareholder control (reduces Δ).

[34] It can be shown that, when the equilibrium delegation threshold is strictly between 0 and *P*, the magnitude of this effect is proportional to $d^{*2} - d^2$.

- Also as in the case of an increase in the importance of management's information, an increase in agency cost results in (weakly) more delegation by shareholders. Again, since shareholders, in general, delegate too little (Lemma 2), this effect makes management control less attractive relative to shareholder control.

As we see from Proposition 3, the first (direct) effect and the third (increase in delegation) effect outweigh the second (communication) effect when management's information is not too important, resulting in an increase in the advantage of shareholder control. The reverse may happen, however, if management's information is sufficiently important.

4 Shareholders Are Misinformed but Don't Realize It

As we have seen in the previous section, if shareholders fully understand their private information (or lack thereof), the case for shareholder control is actually stronger when shareholders are poorly informed because, in this case, they make better delegation decisions. Critics of shareholder control whose opposition is based on shareholder ignorance may argue, however, that not only are shareholders poorly informed but also overestimate the extent of their information. Consequently, in this section, we assume that shareholders misperceive their private information. Indeed, we rig the game against shareholder control by assuming that while shareholders believe they observe $\tilde{p}$, in fact they observe a random variable $\tilde{q}$ which is independent of $\tilde{p}$.

Note that, as mentioned in the Introduction, we take a normative point of view and ask what allocation of control maximizes firm value without addressing the issue of who allocates control. In the previous section, with shareholders who are aware of the extent of their information, if shareholders chose the control allocation before observing their private information, they would choose the allocation that maximizes value. In this section, however, we must take seriously our assumption that control is allocated to maximize value, e.g., by an unbiased regulator, since misinformed shareholders would not generally choose the value maximizing allocation of control. Indeed they would allocate control according to the solution in the base case of the previous section and would make mistakes in both directions. For some decisions they would allocate control to themselves when management control would maximize value, and, for other decisions they would make the opposite mistake.

Even though shareholders are misinformed, we find that it is optimal for them to control some decisions. In particular, as one would expect, we show that shareholder control is optimal when their misperception is not too extreme. More interestingly, however, we also show that, when shareholders are in control and do not delegate, shareholder misperception will be offset to some extent by management's strategically distorting their report relative to the base case of section 3. This "compensation effect" results in shareholder control being optimal in some cases when it otherwise would not be.

More specifically, in this section we assume both shareholders and management believe $\tilde{p}$ is uniform on $[0,P]$, but management realizes that shareholders actually observe $\tilde{q}$. The support of $\tilde{q}$ is assumed to be a subset of $[0,P]$. All other assumptions are the same as in section 3.

4.1 Shareholder Control

First, suppose shareholders are in control. Since shareholders believe they are in the situation modeled in section 3, they will delegate to management if and only if $q \geq p^*$, where q is the realization of $\tilde{q}$ and p^* is the equilibrium delegation threshold defined in section 3.1.

If, in fact, $q \geq p^*$, so shareholders do delegate to management, management, knowing that shareholders have no information about $\tilde{p}$, does not change its beliefs based on the fact that shareholders chose to delegate. Management chooses $s = \overline{p} + a + b$, where a is the realization of $\tilde{a}$. Shareholders' actual *ex ante* expected loss from being in control is

$$E\left(\overline{p} + \tilde{a} + b - \left(\tilde{p} + \tilde{a}\right)\right)^2 = \sigma_p^2 + b^2. \tag{13}$$

Shareholders' expected loss in this case consists of the loss due to having no information about $\tilde{p}$ plus the agency cost.

Now suppose that $q < p^*$, so that shareholders do not delegate, and management observes $\tilde{a} = a$. In the communication game when shareholders do not delegate, shareholders believe that the equilibrium partition, $\{a_0,\ldots,a_N\}$, is as given in section 3. Consequently, all reports in $[a_{i-1},a_i]$ lead shareholders to choose $s=\overline{a}_i+q$, where $\overline{a}_i=(a_{i-1}+a_i)/2$. As shown in the appendix, this results in management choosing a partition cell whose midpoint is closest to $a+b-\overline{e}$, where $\overline{e}=E(\tilde{q}-\tilde{p}|\tilde{q}<p^*)$. Thus if shareholders knew they do not observe $\tilde{p}$, management would choose a cell whose midpoint is closest to $a+b$. When shareholders believe they do observe $\tilde{p}$, management shifts its "ideal point" for reporting $\tilde{a}$ by $-\overline{e}$. This corrects for the expected error caused by shareholders' misperception. Since their misperception introduces a bias in their choice of *s* on average, we refer to $\overline{e}$ as shareholders' *average misperception bias*.

For the rest of this section we assume that $\overline{e}$ satisfies

$$-\frac{A}{N}+2b(N-1)=-a_1\leq\overline{e}\leq A-a_{N-1}=\frac{A}{N}+2b(N-1). \tag{14}$$

If $\overline{e}$ does not satisfy these bounds, some partition cells will be eliminated, reducing communication from management to shareholders.[35] It is obvious that shareholder-control is suboptimal for sufficiently large misperception biases. Our goal here is to see if shareholder-control is optimal for a range of misperception biases.

We now state the main result of this subsection.

Lemma 4. If shareholders control the decision, the *ex ante* expected loss is given by

$$L_S=\varphi\left(\sigma_p^2+b^2\right)+(1-\varphi)\left[\sigma_p^2+L(b,A)+\sigma_q^2+\overline{e}^2-\frac{N(b,A)-1}{N(b,A)}\overline{e}\left(\overline{e}+2b\right)\right], \tag{15}$$

where $\varphi=\Pr(\tilde{q}\geq p^*)$ is the probability of delegation and $\sigma_q^2=Var(\tilde{q}|\tilde{q}<p^*)$.

The right hand side of (15) has two main terms. The first is the probability that shareholders delegate times the loss if they do as as given in (13). The second main term is the probability that

---

[35] As an extreme example, suppose $\overline{e}>A-a_1$. Then, regardless of the realization of $\tilde{a}$, management's report will be in $[0,a_1]$.

shareholders do not delegate times the loss if they do not. This loss consists of five terms. The first, σ_p^2, is the loss due to knowing nothing about $\tilde{p}$ and will be present no matter who controls, since we assume, in this section, that no one observes $\tilde{p}$. The second term, $L(b, A)$, is the loss from knowing only the information about $\tilde{a}$ that would be communicated by management in equilibrium if shareholders were aware that they do not observe $\tilde{p}$. The third term, σ_q^2, is due to the uncertainty about the extent of shareholders' misinformation about $\tilde{p}$. The fourth term, $\overline{e}^2$, is the loss due to shareholder-misperception if management did not change its signal relative to the base case in response to this misperception. We refer to the sum of the third and fourth terms, $\sigma_q^2 + \overline{e}^2$, as the *direct cost* of shareholders' misperception. The fifth term is the extent to which the direct cost is offset by the fact that management's report compensates for shareholders' average misperception bias. We refer to this as the *compensation effect*.

If there is no information in management's report ($N = 1$), then the compensation effect is absent, since management's report is vacuous. If there is no misperception bias on average ($\overline{e} = 0$), again the compensation effect is missing. As long as $N > 1$ and $\overline{e} \neq 0$, the compensation effect is present and its impact on shareholders' choice of *s* is opposite to that of the misperception bias, as mentioned above. In some cases, this effect results in the optimality of shareholder-control when this would not otherwise be the case, as will be seen presently.

4.2 Management Control

Now suppose management is in control. Management will never delegate to shareholders, since shareholders have no information about $\tilde{p}$. Consequently, shareholders' expected loss if management controls the decision is the same as in section 3.2, namely $L_M = \sigma_p^2 + b^2$, and is the same as if shareholders were in control and delegated the decision to management.

4.3 Optimal Control

In this subsection we show that, even though shareholders are misinformed, there are nevertheless decisions for which shareholder control is optimal. When shareholders are in control, their misperception introduces three effects in addition to those considered in the base case. First, shareholders' delegation decision is affected. Second, if shareholders do not delegate, their misperception bias affects the information communicated by management to shareholders (the compensation effect described above). Third, if shareholders do not delegate, their decision is biased relative to the value-maximizing decision (the direct cost mentioned above). Opponents of shareholder control focus on this third effect which clearly weakens the case for shareholder control. Since shareholder control is strictly optimal for some decisions when shareholders are not misinformed, it will still be optimal in some cases if shareholders' misperception bias is small. Moreover, in some cases, the compensation effect results in the optimality of shareholder-control when the misperception bias would otherwise be too large.

Obviously, if shareholders always delegate ($\varphi = 1$), control is irrelevant, since management, if in control, never delegates, so management makes the decision with no information about $\tilde{p}$, regardless of who is in control. Formally, this is clear from a comparison of L_M and L_S with $\varphi = 1$. Consequently, in what follows, we assume $\varphi < 1$. In this case, a comparison of L_M with L_S reveals that it is optimal for shareholders to control the decision if and only if

$$b^2 > L(b,A) + \sigma_q^2 + \frac{\overline{e}}{N(b,A)}\left[\overline{e} - 2\left(N(b,A) - 1\right)b\right]. \tag{16}$$

Since delegating is the same as management control in this section, the comparison boils down to the loss due to management control *versus* the loss due to shareholder control when shareholders do not delegate. Notice that the cost of knowing nothing about $\tilde{p}$ cancels since this loss is borne regardless of control.

It can be shown that for $N > 2$, it is strictly optimal for managment to control the decision when shareholders delegate with probability less than one, regardless of $\overline{e}$.[36] It then follows from (16) that shareholder control is strictly optimal for

---

[36] The intuition for this result is not straightforward. In order for *N* to be "large," management's information must be much more important than agency costs, and, hence, the loss due to imperfect communication

$$\overline{e}^2 + \sigma_q^2 \le b^2 - \sigma_a^2 \text{ and } \sigma_a \in (0, b), \tag{17}$$

and for

$$(\overline{e} - b)^2 \le b^2 - \frac{\sigma_a^2}{2} - 2\sigma_q^2 \text{ and } \sigma_a \in \left(\frac{2b}{\sqrt{3}}, b\sqrt{2}\right).^{37} \tag{18}$$

These two regions are depicted in Figure 3. On the left side of Figure 3, where management's information is less important than agency costs, if shareholders are in control and do not delegate, management conveys no information to shareholders ($N = 1$) and, therefore, there is no compensation effect. Here, shareholder control is optimal only when the direct cost of shareholders' misperception, $\overline{e}^2 + \sigma_q^2$, is small. This is shown by the left yellow triangle. The upper bound on the direct cost of shareholders' misperception for shareholder control to be optimal decreases as the importance of management's information increases. In particular, when management has no private information, shareholder control is optimal whenever the direct cost of their misperception is less than the agency cost. This threshold decreases to zero as the importance of management's information increases from zero to *b*. The threshold increases with increases in management's bias.

---

of their information (the first term on the right hand side of (16)) must be large relative to agency costs, b^2. On the other hand, the net effect of shareholders' misperception bias (the second term on the right hand side of (16)) can be negative and more negative for large *N*. It turns out, however, that the right hand side of (16) is greater than $4b^2/3$ when $N > 2$.

[37] $N = 1$ for condition (17) and $N = 2$ for condition (18).

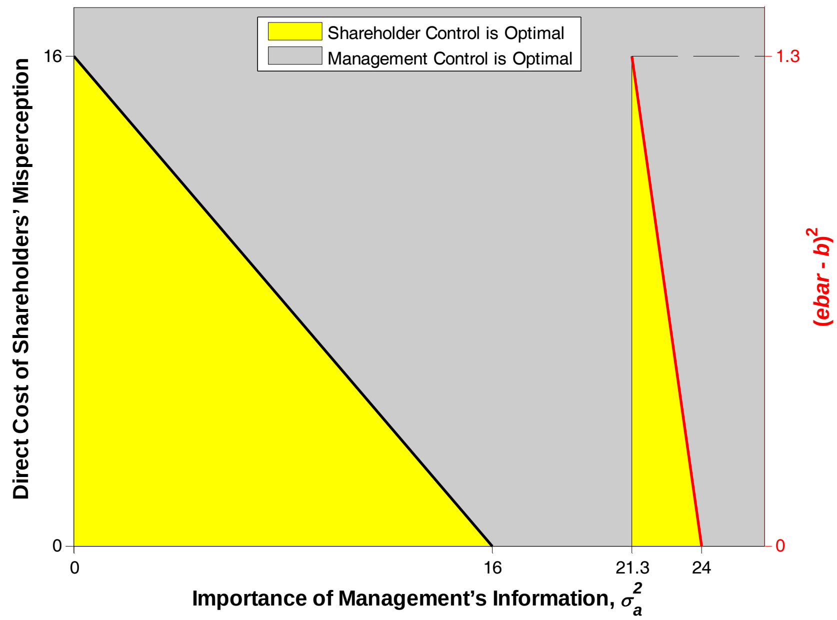

Figure 3

This figure shows the combinations of the importance of management's information, σ_a^2, and measures of the cost of shareholders' misperception such that shareholder-control is strictly optimal. The triangle on the left side corresponds to condition (17) and is plotted relative to the direct cost of shareholder misperception, $\sigma_q^2 + \bar{e}^2$, on the left vertical axis. The right triangle corresponds to condition (18) and is plotted relative to the squared distance between shareholders' average misperception bias and management's bias, $(\bar{e} - b)^2$, plotted on the right vertical axis. For this figure, $b^2 = 16$, $\sigma_q^2 = 2$, so

$4b^2/3 = 21\frac{1}{3}$, $2(b^2 - 2\sigma_q^2) = 24$, $2b^2 = 32$ and $\frac{b^2}{3} - 2\sigma_q^2 = 1\frac{1}{3}$.

On the right side of Figure 3, management's information is sufficiently important that, if shareholders are in control and do not delegate, management will tell shareholders whether $\tilde{a}$ is "low" or "high" ($N = 2$). In this case, shareholder control is optimal when shareholders' average misperception bias is close to management's bias. This is due to the compensation effect. If it were not for the compensation effect, management control would be optimal for all values of $\bar{e}$ when $N = 2$. Again the upper bound on the distance between shareholders' misperception bias and management's bias for shareholder control to be optimal decreases to zero as the importance of management's information increases over the range in which $N = 2$. Also, the maximum distance between shareholders' misperception bias and management's bias for shareholder control to be optimal increases with increases in management's bias and with decreases in the variance of shareholders' signal. An increase in management's bias makes management control less attractive while a reduction in uncertainty about shareholders' decision if they do not delegate makes shareholder control more attractive.

Recall that, when shareholders know they are uninformed, it is always optimal for them to control the decision, strictly if management's agency cost exceeds the value of their information and weakly otherwise. This results from the fact that uninformed shareholders make optimal delegation decisions. In contrast, misinformed shareholders do not make optimal delegation decisions. When shareholders are misinformed, it is strictly optimal for management to control decisions for which their agency cost exceeds the value of their information, provided that the direct cost of shareholders' misperception is sufficiently large relative to the importance of management's information, i.e., fails to satisfy (17). On the other hand, it is strictly optimal for shareholders to control decisions for which the value of management's information exceeds their agency cost, provided shareholders' average misperception bias is sufficiently close to management's bias as made precise in (18).

As the importance of management's information increases, optimal control can switch from shareholders to management and back again, as is clear from Figure 3. This contrasts with the base case in which increases in the importance of management's information cause optimal control to switch from shareholders to management but not the reverse. The reason for the reversal in this case is that increases in the importance of management information can trigger the compensation effect. This effect is not present in the base case.

Intuitively, it seems obvious that, as agency costs increase, for any given value of the importance of management's information, shareholder control should become more attractive. Indeed, this is generally the case, but if the increase in agency costs is not too large, it can result in the counter-intuitive result that optimal control switches from shareholders to management. This is due to the fact that a small increase in agency costs can reduce communication from management and eliminate the compensation effect. That is, an increase in agency costs may trigger a reduction in communication from management that increases the cost of shareholder misperception by more than it increases the cost of management control.[38] This is shown in Figure 4.

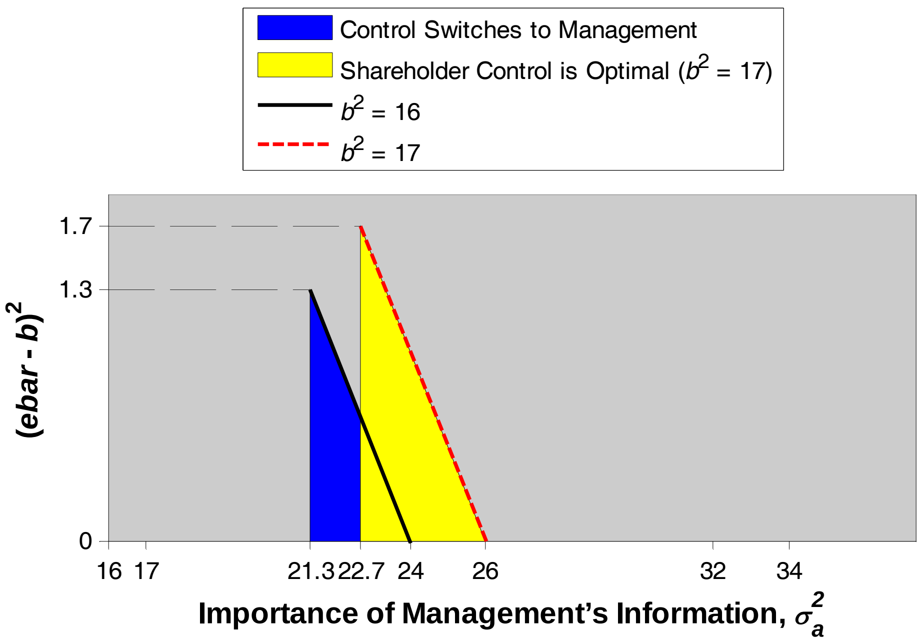

Figure 4

This figure shows how a small increase in agency costs can cause a switch in control from shareholders to management. With $b^2 = 16$, for parameters in the blue area,

[38] Oddly, when the base case equilibrium of the communication game is such that management would not communicate in that case, it cannot communicate in the present case of misinformed shareholders, even if it wanted to (which it might because of the compensation effect in offsetting shareholders' misperception bias). The reason is that shareholders, believing that they are in the base case, will ignore any report by management.

shareholder control is optimal, but with $b^2 = 17$, management control is optimal in this region. For this figure, $\sigma_q^2 = 2$.

We summarize the results of this section in the following proposition.

Proposition 4. When shareholders are misinformed, and their average misperception bias satisfies (14),

(i) **Shareholder-control is strictly optimal** in either of the following two cases: (a) management's information is less important than agency costs and the direct cost of shareholders' misperception, $\sigma_q^2 + \bar{e}^2$, is sufficiently small relative to the importance of management's information as specified in (17) or (b) management's information is sufficiently important that management will reveal whether $\tilde{a}$ is "low" or "high" and shareholders' average misperception bias is sufficiently close to management's bias, as specified by (18). **Management control is strictly optimal** in all other cases.

(ii) **Comparative statics for the importance of management's information:** Increases in the importance of management's information may cause optimal control to switch from shareholders to management and back again.

(iii) **Comparative statics for agency cost:** For case (a) in the above paragraph, any increase in agency costs increases the range of values of the direct cost of shareholders' misperception for which shareholder control is optimal. For case (b), suppose agency costs increase from b to b', where $b < b' < 2b/\sqrt{3}$. Then, for $\sigma_a^2 \in (4b/3, 4b'/3)$ and $(\bar{e} - b)^2 \leq b^2 - \frac{\sigma_a^2}{2} - 2\sigma_q^2$, optimal control switches from shareholders to management. This is due to the elimination of the compensation effect.[39]

[39] The proof of this part follows trivially from (17) and (18) and is, therefore, omitted.

The main contributions of this section are the two, counterintuitive comparative statics results in Proposition 4. Without the formal model and analysis, one's intuition is likely to lead to the conclusion that increases in the importance of management's information would always increase the advantage of management control, while increases in agency costs would have the opposite effect. This intuition is flawed because it ignores the compensation effect.

5 Non-value-maximizing Shareholders

In this section we consider whether shareholder control may still maximize firm value even when some shareholders have goals other than value maximization. In particular, we have in mind a situation in which some shareholders prefer that the firm sacrifice some profits to further another goal, e.g., preservation of the environment, support of a political agenda, etc. The presence of non-value-maximizing shareholders raises the issue of whether value maximization is still an appropriate goal. There are two obvious possibilities. One is to assume that the goal is to maximize the objective of the NVM shareholders. This simply repeats the base case analysis of section 3 if we reinterpret management's bias as being relative to the objective of the NVM shareholders. The results of that section carry over to this case. The other possibility, which we adopt in this section, is to assume the objective is value-maximization and ask whether giving control to shareholders, some of whom do not want to maximize value, can still be value maximizing.[40]

As in the previous section, we rig the game against shareholder control by assuming that the non-value-maximizing shareholders hijack shareholder decisions. The question then is under what circumstances, if any, the value-maximizing (VM) shareholders are better off with their non-value-maximizing (NVM) co-investors in control than with management in control.

We model NVM shareholders as being biased, like management but with a potentially different bias, β. Formally, NVM shareholders choose a decision *s* that minimizes the loss function

[40] Alternatives to share-value maximization in general have been discussed by some commentators [see, e.g., Tirole (2006, section 1.8)].

$E\left(s-\left(\tilde{p}+\tilde{a}+\beta\right)\right)^2$. NVM shareholders' optimal decision is the same as that of management's, except that *b* is replaced by β. That is, NVM shareholders choose $s = E\left(\tilde{a}+\tilde{p}\right)+\beta$, where the expectation is conditional on whatever information they have about $\tilde{a}$ and $\tilde{p}$. The parameter β measures the extent to which these shareholders will deviate from the optimal decision to further their social agenda. Note that β could be either positive or negative. For example, suppose the decision, *s*, is a minimum acceptable bid for selling the firm and that NVM shareholders anticipate losing control if the firm is sold. In this case, they may prefer a higher-than-optimal minimum acceptable bid, i.e., have a positive β. On the other hand, suppose the decision is the size of a new plant and that NVM shareholders are willing to sacrifice profits to reduce emissions from the plant. In this case, these shareholders may prefer a smaller-than-optimal plant if this will reduce emissions, i.e., have a negative β.

Management minimizes $E\left(s-\left(\tilde{p}+\tilde{a}+b\right)\right)^2$, given their information, as before.

The difference between management's bias, *b*, and shareholders' bias, β, plays an important role in the analysis of this section. We denote this difference by $B = b-\beta$ and refer to it as the ***net bias***.

If $B > 0$, all the results of section 3 apply, except that *B* replaces *b* in all calculations. In particular, p^* and $d = P - p^*$ are as described in Proposition 1 with *b* replaced by *B*. If $B < 0$, then NVM shareholders delegate when $\tilde{p} \in \left[0, p^*\right]$, and p^* and *d* are calculated as in section 3.1 except that *b* is replaced by $|B|$ and $p^* = d$.[41]

Also, for $B < 0$, management never delegates to shareholders if and only if $\sigma_p \le |B|$. Thus the counterpart of Assumption 2 in this case is $\sigma_p < |B|$, which also implies that NVM shareholders do not communicate any private information to management (other than what may be communicated by their

---

[41] The argument for delegation when $\tilde{p}$ is *below* a threshold if $B < 0$ is similar to the argument for delegation when $\tilde{p}$ is above a threshold in section 3.1 and is therefore omitted.

delegation decision). Consequently, we assume that $\sigma_p < |B|$. Since our results in this section characterize optimal control in terms of NVM shareholders' bias, it is convenient to restate the assumption in terms of their bias as

Assumption 3: $\beta \le b - \sigma_p$ or $\beta \ge b + \sigma_p$.

Since $\sigma_p < |B|$, $L(B,P) = \sigma_p^2$ and $L(B,d) = \sigma(d)^2$. Assuming that $\sigma_p > 0$, if NVM shareholders are in control, the expected loss in firm value is

$$\frac{d}{P}\left(b^2 + \sigma(d)^2\right) + \left(1 - \frac{d}{P}\right)\left(\beta^2 + L(B,A)\right). \tag{19}$$

The expression in (19) is the same as in the base case (equation (10)), except for three effects that are similar to those discussed in section 4.3. First, the size of the delegation region, *d*, is determined by the net bias *B* rather than management's bias *b*. Second, the loss when shareholders do not delegate is increased by the cost of the NVM shareholders' bias, β^2. This effect obviously reduces the attractiveness of shareholder control and is the effect on which opponents of shareholder control focus. Third, the loss due to imperfect communication from management is determined by the net bias instead of management's bias. Since the net bias can be smaller than management's bias, communication of management's information can be more precise than in the base case, resulting in smaller loss. As is shown in Proposition 5 below, this effect causes shareholder control to be optimal in some cases.

If management is in control, the expected loss in firm value is $b^2 + \sigma_p^2$, as before. Therefore, if the objective is to maximize firm value, shareholders should control if and only if

$$\frac{d}{P}\left(b^2 + \sigma(d)^2\right) + \left(1 - \frac{d}{P}\right)\left(\beta^2 + L(B,A)\right) \le b^2 + \sigma_p^2. \tag{20}$$

The main result of this section is to characterize optimal control of decisions for various values of the NVM shareholders' bias, β, and the importance of management's information, σ_a^2, for fixed values of management's bias, *b*, and the importance of shareholders' information, σ_p. A more formal statement of this result and the proof are given in the appendix.

Proposition 5. Assume $b-\overline{p}>\sigma_p$.[42] For any given values of management's bias, *b*, and the importance of shareholders' information, σ_p, the possible combinations of the NVM shareholders' bias, β, and the importance of management's information, σ_a^2, can be divided into three regions as depicted in Figure 5. In particular:

- It is strictly optimal for NVM shareholders to control decisions if NVM shareholders' bias is smaller than $b-\sigma_p$, and the importance of management's information is below a threshold. This threshold is given by a function $H(\beta)$ for $\beta \le \overline{p}$ and by a function $G(\beta)$ for $\overline{p} \le \beta < b-\sigma_p$. The function $H(\beta)$ is continuous for $\beta \le \overline{p}$, $H(\beta)\equiv 0$, for $\beta \le \beta_S$, where $\beta_S = -\sqrt{b^2+\sigma_p^2}$, H is increasing in β for $\beta_S < \beta \le b/2$ and satisfies $H(\overline{p}) = G(\overline{p})$. The function $G(\beta)$, defined for $\beta \le b-\sigma_p$ and $\beta \ge b+\sigma_p$, is continuous and symmetric with respect to $\beta = b$.
- It is strictly optimal for management to control decisions if the importance of management's information is below the threshold given by $G(\beta)$ and either
  - NVM shareholders' bias is greater than $b+\sigma_p$, or
  - NVM shareholders' bias is smaller than $\overline{p}$, and the importance of management's information is above the threshold given by $H(\beta)$.
- Control is irrelevant for any combination of NVM shareholders' bias, β, and the importance of management's information, σ_a^2, such that the importance of management's information

[42] This is the richest case. When this inequality fails, the result is qualitatively similar.

exceeds the threshold given by $G(\beta)$, since, in this case, shareholders always delegate if in control.

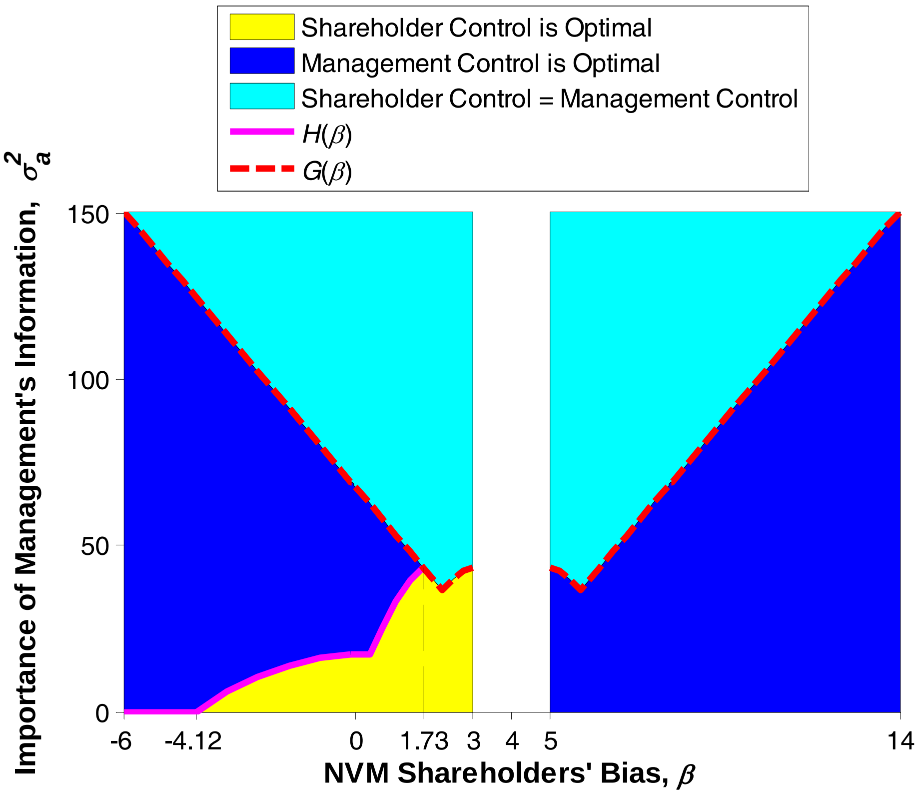

Figure 5

This figure shows, for various combinations of values of NVM shareholders' bias, β, and the importance of management's information, σ_a^2, which party optimally controls the decision. The values of the other parameters are $b = 4$ and $\sigma_p = 1$. These values imply that $\bar{p} = \sqrt{3} \approx 1.73$ and $\beta_S = -\sqrt{17} \approx -4.12$. Note that Assumption 3 implies that values of NVM shareholders' bias between $b - \sigma_p = 3$ and $b + \sigma_p = 5$ are not considered. This accounts for the white space in the middle of the figure.

The proposition is depicted in Figure 5 which shows that, indeed, shareholder control is optimal for some decisions. When NVM shareholders are either less biased in the same direction as management

($0<\beta<b-\sigma_p$) or are biased in the opposite direction ($\beta<0$), there is a tradeoff. When NVM shareholders' bias is similar to that of management but smaller ($\overline{p}<\beta<b-\sigma_p$), the net bias is small. In this case, management is willing to communicate much of their information to shareholders if shareholders are in control and decide not to delegate. Since NVM shareholders' bias is smaller than that of management, and little of management's information is lost if shareholders make the decision, the cost of letting shareholders decide is small, so it is optimal for them to control such decisions. Of course, when shareholders always delegate, i.e., $\sigma_a^2>G(\beta)$, shareholder control is weakly optimal. As NVM shareholders' bias decreases, holding management's bias fixed, the net bias increases. This reduces communication from management, but, if shareholders' bias is positive, like management's, the reduction in shareholders' bias also reduces the inefficiency of the shareholders' decision, *cet. par*. The net effect on control could go either way. If shareholders' bias is opposite that of management, i.e., $\beta<0$, however, further reductions in NVM shareholders' bias *increase* the inefficiency of the shareholders' decision. Now, both the communication effect and the direct effect on the decision of shareholders of the reduction in NVM shareholders' bias work against shareholder control of the decision. Thus when NVM shareholders' bias is negative (opposite that of management), as NVM shareholders' bias decreases (and the net bias increases), the importance of management's information must decrease in order for shareholder control to be optimal. That is, the threshold H in Figure 5 decreases as β becomes more negative, falling to zero when β reaches β_S.[43] That is, for sufficiently large (in absolute value) bias of NVM shareholders in the opposite direction from management's, i.e., larger in absolute value than $-\beta_S$, NVM shareholders' bias is so large (in absolute value) that it is optimal for management to control even if they have no private information.

[43] Actually, it is shown in Proposition 5 that H is upward sloping for $\beta\le b/2$. In Figure 5, $b/2=2$ and $\overline{p}=\sqrt{3}<2$, so H is upward sloping for all $\beta\le\overline{p}$.

Consider decisions for which NVM shareholders' bias is in the same direction as management's bias but larger, i.e., $\beta > b + \sigma_p$. In this case, if shareholders actually make the decision, it's even more biased than management's decision would be for the same information. On the other hand, shareholders have information about $\tilde{p}$ that management doesn't have and won't learn from shareholders, while shareholders may learn at least some information about $\tilde{a}$ from management. Moreover, shareholders, if in control, may delegate to management based on their (shareholders') private information. Consequently, it is not intuitively obvious who should control in this situation. It turns out that the fact that NVM shareholders' bias "outweighs" the combination of management's bias and the importance of their own information, i.e., $\beta > b + \sigma_p$, management control is always optimal in this region.

When management's information is sufficiently important ($\sigma_a^2 > G(\beta)$), shareholders, if in control, always delegate to management. As a result, the delegation decision conveys no information. Also, shareholders convey no information directly. Consequently control is irrelevant: regardless of who controls the decision, management will always actually make the decision with no information from shareholders.

This section shows that, even when controlling shareholders have biases that prevent them from choosing the value-maximizing decision, it may still be value-maximizing for them to control some decisions. In particular, they should control decisions for which the shareholders' bias is similar to that of management (i.e., the net bias is small), and the importance of management's information is also small. The farther shareholders' bias is from management's, the less important must be management's information for shareholder control to be optimal. If shareholders' bias is sufficiently far from management's bias, in either direction, management control is optimal even if management has no private information.

6 Conclusions

In this paper, we address the issue of which corporate decisions are best controlled directly by shareholders. Using a model that accounts for private information, delegation, communication and agency considerations, we show that popular arguments both for and against direct shareholder control are flawed. For example, a strong intuitive argument has been advanced by several commentators that shareholders should not control major corporate decisions because, unlike management, they do not possess the relevant information. We show, however, that shareholders should control decisions for which they have none of the information possessed by management and have no private information of their own, provided these shareholders are aware of their ignorance and the extent of management's private information. This result follows, in part, from the failure of the simple argument to take account of the fact that shareholders can delegate the decision to management. On the other hand, others have argued that, because shareholders can delegate and want to maximize value (i.e., have no agency problem), they should control every major decision. We show that this argument is incorrect, because, if shareholders have private information, they will fail to delegate optimally. The reason for this is that shareholders take account of the inference made by management about shareholders' private information based on shareholders' decision to delegate. Indeed, we show that management best controls decisions for which their private information is much more important than that of shareholders.

Others have argued against shareholder control on the grounds that either shareholders overestimate the extent of their information or that shareholders have agendas other than value maximization. We show that in both cases there are still some decisions for which shareholder control is optimal. This is due, in part, to the fact that shareholder biases, due either to misperception or non-value maximizing agendas, may improve communication from management to shareholders. This effect is ignored in popular arguments against shareholder control.

We view the main contribution of our analysis as improving our intuition about shareholder control of decisions by highlighting some less-than-obvious considerations involving strategic communication and delegation. Nevertheless, it is interesting to consider how the results of the basic model may be applied to some actual decisions. One example is how much cash to distribute to shareholders, a decision about which management is likely to have important information, while shareholders are likely to have little or no important private information. In this case one might be tempted to conclude that it is obvious that management should control the cash distribution decision. Our results imply just the opposite conclusion. This particular decision seems to be often contested by activist shareholders, generally without much success. Our results suggest that perhaps the governance rules should be changed to make it easier for such shareholders to exercise control over payout policy.

Another example is replacement of management. Both parties are likely to have private information about the distribution of talent in the population of potential replacements. It is reasonable to assume that management and shareholders have information of comparable importance in assessing the availability of replacements of various levels of ability. In this case, the model implies that shareholder control is optimal, regardless of the level of private benefits of control or how important the parties' private information is (as long as they are of similar importance).

As a final example, consider the optimal proportion of performance-based compensation. For this decision, management's information is likely to be more important than that of shareholders with regard to low level executives. On the other hand, for top executives, management's information about the optimal compensation scheme may be of roughly comparable importance to that of shareholders' information. The model then implies that, assuming similar agency costs for the two decisions, shareholder control is more likely to be optimal for top level compensation decisions than for lower level compensation. Moreover, it seems reasonable to suppose that agency costs for decisions involving one's own compensation are likely to be smaller than for decisions involving the compensation of others. This would reinforce the previous conclusion.

Obviously, we have neglected a number of important issues regarding the optimality of direct shareholder control of decisions. The most glaring of these omissions is our assumption that there are no differences of opinion, information, or preferences among shareholders (or at least among the controlling group of shareholders). When such differences exist, the issue arises as to how they are resolved in making decisions (both delegation decisions and "substantive" decisions). Obviously, this involves voting in some form or another. The same can be said about differences among managers. We have also assumed that all the parameters of the information structure and preferences are common knowledge. Relaxing these assumptions will, we believe, lead to interesting results. This, of course, is left for future work.

Another avenue for future work involves broadening the set of mechanisms for communicating information. Using an intermediary with preferences between those of management and those of shareholders, such as the board of directors or a group of shareholders sympathetic to management, could also improve outcomes and affect optimal control.

Another, counter-intuitive device that might improve communication is to introduce noise into the transmission of signals between shareholders and management. Blume, Board and Kawamura (2007) shows that in the quadratic-uniform version of the Crawford and Sobel (1982) model, introducing a small probability that the signal actually received is an independent random draw from a known, exogenous distribution on the message space will result in a Pareto improvement relative to the best Crawford-Sobel equilibrium. Therefore, improvements may be possible by adding this type of noise when shareholders are in control. If so, this would strengthen the case for shareholder control. One way that such noise could be introduced is by having management report to a disinterested third party who then randomly and secretly chooses whether to pass along management's report unchanged or instead pass along an independent random report.

A third possibility for improving communication involves multiple stages of communication as considered in Krishna and Morgan (2004).[44] Two of the results of Krishna and Morgan (2004) can be applied to the current model if delegation is ruled out. The first is that multi-stage communication cannot improve the outcome without exogenous randomization as part of the mechanism. The second result,

[44] See also Aumann and Hart (2003).

shown by example, is that with exogenous randomization, multi-stage communication can, indeed, improve the outcome. This is discussed in more detail in Harris and Raviv (2005). Since, in our model, management never delegates to shareholders, this result suggests that, when management is in control, multi-stage communication may improve the outcome. This would strengthen the case for management control.

If there are multiple decisions to be made at the same time, and shareholders and management each had private information about each decision, they could communicate a ranking of their private information across the various decisions, either in addition to or instead of information about the values of each decision's private information. For example, suppose the decisions were the size of two plants, management is in control, and both parties have private information about the optimal plant size of each plant. If we extend our assumptions about the size of management's bias relative to the importance of shareholder information to this setting, shareholders would communicate none of their information about the individual optimal plant sizes. Chakraborty and Harbaugh (2007) show, however, that under some symmetry assumptions in addition to the assumptions made here, it is an equilibrium for shareholders to reveal the ranking of the optimal plant sizes.

All of these communication-improving devices involve somewhat contrived examples of communication mechanisms. It is not clear to what extent these mechanisms have practical, real-world counterparts.

Finally, consider the question of who should control the decision of who controls substantive decisions. This is what Bebchuk (2005) refers to as controlling the "rules of the game." Suppose that such rules-of-the-game decisions can be made contingent on the parameters b, σ_a, and σ_p describing the substantive decisions, shareholders want to maximize firm value, and shareholders are not misinformed. In this case, shareholders should control rules-of-the-game decisions and, contingent on the parameters, allocate control as described in Proposition 2. Even if the rules-of-the-game decisions cannot be contingent on the relevant parameters, it seems clear that value maximizing shareholders should make them, provided there is no *private* information about their likely values. Matters become more complicated if there is private information about the likely values of parameters, shareholders are misinformed, or shareholders have other agendas. This topic is also left for future work.

7 Appendix

7.1 Proof of Proposition 1

Proposition 1: Suppose shareholders are in control of a decision. The unique pure-strategy Perfect Bayes' Equilibrium of the resulting game is as follows:

- If shareholders have no private information ($\sigma_p = 0$) they will delegate the decision if and only if $\sigma_a \geq b$, i.e., if and only if management's information is more important than agency costs, regardless of the realization of $\tilde{p}$. In this case, management infers nothing from the delegation decision, and, if shareholders do delegate, management chooses $s = a + b$, where a is the realization of $\tilde{a}$. [This is shown in Dessein (2002).]
- If shareholders have private information ($\sigma_p > 0$), shareholders will delegate the decision if and only if the realized value of their private information exceeds a threshold $p^* \in [0, P]$. This threshold and the equilibrium strategies and beliefs of management depend on the values of the parameters, σ_a, σ_p, and b. There are three cases to consider.
    - Case 1: $\sigma_a \leq b$. In this case, shareholders never delegate, unless $\tilde{p} = P$ and $\sigma_a = b$, i.e.,

      $$p^* = P \text{ and } d = 0. \quad (21)$$

      If shareholders do not delegate (this is their equilibrium move, unless $\sigma_a = b$ and $\tilde{p} = P$), management's inference and strategy are irrelevant, since it has no moves. If shareholders delegate (this is an off-equilibrium-path move, unless $\sigma_a = b$ and $\tilde{p} = P$), management infers that $\tilde{p} = P$ with probability one and chooses $s = a + P + b$, where a is the realization of $\tilde{a}$.
    - Case 2: $P \leq 2\left[\sqrt{L(b, A)} - b\right]$. In this case, shareholders always delegate, i.e.,

      $$p^* = 0 \text{ and } d = P. \quad (22)$$

Management infers nothing if shareholders delegate (this is their equilibrium move) and chooses $s = a + \frac{P}{2} + b$, where a is the realization of $\tilde{a}$. If shareholders do not delegate (this is an off-equilibrium-path move), management's inference and strategy are irrelevant, since it has no moves.

- Case 3: $\sigma_a > b$ and $P > 2\left[\sqrt{L(b,A)} - b\right]$. In this case, $p^* \in (0,P)$, $d \in (0,P)$ and satisfy

$$P - p^* = d = 2\left[\sqrt{L(b,A)} - b\right]. \tag{23}$$

If shareholders delegate, management's posterior belief about $\tilde{p}$ is uniform on $[p^*, P]$, and it chooses $s = a + \frac{P + p^*}{2} + b$, where a is the realization of $\tilde{a}$. If shareholders do not delegate, management's inference and strategy are irrelevant, since it has no moves. There are no off-equilibrium-path moves in this case.

Proof. We first show that the delegation region must be an upper interval, assuming that shareholders, when in control, always delegate when they are indifferent between delegating and not delegating. Let $D \subseteq [0,P]$ denote the delegation region. We want to show that, if $D \neq \emptyset$, then $D = [p^*, P]$, for some $p^* \in [0,P]$.

To show this, let $\lambda(p,D)$ denote the shareholders' expected loss from delegating when $\tilde{p} = p$, $D \neq \emptyset$ is the delegation region, and management chooses $s = \hat{p} + b + a$, where a is the realization of $\tilde{a}$ and $\hat{p} = E(\tilde{p} \mid \tilde{p} \in D)$. Then

$$\lambda(p,D) = E\left(\left(\hat{p} + b + \tilde{a} - (p + \tilde{a})\right)^2\right) = (\hat{p} + b - p)^2. \tag{24}$$

Recall $L(b,A)$ is the shareholders' expected loss from not delegating. Note that $L(b,A)$ is independent of D and the realization of $\tilde{p}$.

By definition of *D*,

$$D=\left\{p\in[0,P]\middle|\lambda(p,D)\le L(b,A)\right\}. \tag{25}$$

It follows from (24) and (25), that

$$D=[0,P]\cap\left[\hat{p}+b-\sqrt{L(b,A)},\hat{p}+b+\sqrt{L(b,A)}\right]. \tag{26}$$

Therefore *D* is a closed interval in $[0,P]$. Let $d_1,d_2\in[0,P]$ be such that $D=[d_1,d_2]$. Then, the definition of $\hat{p}$ and the fact that $\tilde{p}$ is uniformly distributed imply that

$$\hat{p}=\frac{d_1+d_2}{2}.$$

Suppose $d_2<P$. Then, it follows from (26) that

$$d_2=\hat{p}+b+\sqrt{L(b,A)}$$

and

$$d_1\ge\hat{p}+b-\sqrt{L(b,A)}.$$

Therefore,

$$\hat{p}=\frac{d_1+d_2}{2}\ge\frac{2(\hat{p}+b)-\sqrt{L(b,A)}+\sqrt{L(b,A)}}{2}=\hat{p}+b.$$

This contradicts $b>0$. Consequently, $d_2=P$, which completes the proof of our claim that the equilibrium delegation region is of the form $[p^*,P]$.

To characterize p^*, define a function *f* on $[0,P]$ as follows: for any $x\in[0,P]$, $f(b,P-x)$ is the loss to shareholders of delegating, given that management believes the threshold is *x*, and given that the actual realization of $\tilde{p}$ is exactly *x*. If management believes that shareholders delegate if and only if $\tilde{p}\in[x,P]$, $\hat{p}=\frac{P+x}{2}$, so management which observes $\tilde{a}=a$ chooses $s=a+\frac{P+x}{2}+b$ (recall that shareholders communicate no information about $\tilde{p}$ other than what can be inferred from the fact of delegation). Thus

$$f\left(b,P-x\right)=\left(a+\frac{P+x}{2}+b-\left(a+x\right)\right)^2=\left(\frac{P-x}{2}+b\right)^2. \tag{27}$$

Since *x* is the farthest point in the delegation region from management's choice of *s* (remember, $b>0$, so $\hat{p}$ is more than halfway between *x* and *P*), $f(b,P-x)$ represents the worst-case loss from delegating. In order for *x* to be an equilibrium threshold for delegating, this worst-case loss from delegating must be just equal to the loss from not delegating, $L\left(b,A\right)$, provided that $x\in\left(0,P\right)$. If $L\left(b,A\right)< f\left(b,P-x\right)$, the loss from delegating will be greater than the loss from not delegating for some values of $\tilde{p}>x$. If $L\left(b,A\right)> f\left(b,P-x\right)$, the loss from delegating will be less than the loss from not delegating for some values of $\tilde{p}<x$. Thus, if $p^*\in\left(0,P\right)$, p^* must satisfy

$$f\left(b,P-p^*\right)=\left(\frac{P-p^*}{2}+b\right)^2=\left(\frac{d}{2}+b\right)^2=L\left(b,A\right). \tag{28}$$

Solving (28) for *d* gives

$$d=2\left[\sqrt{L\left(b,A\right)}-b\right]. \tag{29}$$

Clearly, the formula for *d* in (29) is valid if and only if it results in a value between 0 and *P*. This is the case in Case 3 of the proposition. In Case 3, it is clear that the inference of management regarding $\tilde{p}$ claimed in the proposition satisfies Bayes' rule, given the delegation strategy of shareholders, and the strategy of management claimed in the proposition is optimal for management given their beliefs.

Now consider Case 1. If $L\left(b,A\right)<b^2$, then, by Lemma 1 of Harris and Raviv (2008b), $L\left(b,A\right)=\sigma_a^2<b^2$, and (29) results in $d<0$. In this case, if shareholders delegate, and management chooses $s=a+P+b$, shareholders lose $\left(a+P+b-a-p\right)^2\geq b^2$ for all $p\in\left[0,P\right]$. If shareholders do not delegate, they lose $L\left(b,A\right)=\sigma_a^2<b^2$. Consequently, it is optimal for shareholders not to delegate when $L\left(b,A\right)<b^2$, regardless of the realization of $\tilde{p}$. If $L\left(b,A\right)=b^2$, then $L\left(b,A\right)=\sigma_a^2$. The above argument for shareholders goes through for all $p\in\left[0,P\right)$. If $\tilde{p}=P$, then shareholders are indifferent

between delegating and not delegating, so it is optimal for them to delegate (and is consistent with our assumption above that shareholders always delegate when indifferent). As mentioned above, however, $L(b,A) \le b^2$ if and only if $\sigma_a \le b$, so we have that $d = 0$ and $p^* = P$ if and only if $\sigma_a \le b$. Since, when shareholders are in control, management moves only if shareholders delegate, management's beliefs and strategy are irrelevant if shareholders do not delegate. If $L(b,A) = b^2$, management's beliefs satisfy Bayes' rule when shareholders delegate. If $L(b,A) < b^2$, delegation is not on the equilibrium path, so management's beliefs need not be justified in this event. Given their assumed beliefs, if shareholders delegate in this case, the assumed strategy of management ($s = a + P + b$) is optimal.

If $L(b,A) \ge f(b,P)$, or, using (27), $P \le 2\left[\sqrt{L(b,A)} - b\right]$, the loss from not delegating exceeds the loss from delegating for all realizations of $\tilde{p} \in [0,P]$, and (29) results in $d \ge P$. In this case, management has sufficient information to warrant delegating to them regardless of the realization of $\tilde{p}$, so it is optimal for shareholders to delegate for all realizations of $\tilde{p}$, i.e., $d = P$ and $p^* = 0$. Clearly, the assumed beliefs of management follows Bayes' rule if shareholders delegate, and management's strategy is optimal given their beliefs. Of course, if shareholders do not delegate, management's beliefs and strategy are irrelevant.

With respect to uniqueness of the equilibrium, it is clear that there are no other pure-strategy equilibria when $\sigma_a > b$. When $\sigma_a < b$, one must consider the possibility that there are additional equilibria supported by different off-equilibrium-path beliefs. In particular, suppose that $\sigma_a < b$ and management's expectation of $\tilde{p}$ conditional on delegation is $\hat{p} < P$. In that case, the optimal delegation strategy for shareholders is to delegate whenever $p \in \left[\min\{\hat{p} + b - \sigma_a, P\}, \min\{\hat{p} + b + \sigma_a, P\}\right]$. If $\hat{p} + b - \sigma_a \ge P$, then $\hat{p} > P$, since $\sigma_a < b$. But this contradicts $\hat{p} < P$. If, on the other hand, $\hat{p} + b - \sigma_a < P$, then shareholders only delegate when $p > \hat{p}$, again since $\sigma_a < b$. But then $\hat{p}$ cannot be the expectation of $\tilde{p}$ conditional on delegation. Consequently, in any equilibrium when $\sigma_a < b$, management's expectation of $\tilde{p}$ conditional on delegation must be at least *P*. Since *P* is the upper bound of the support of $\tilde{p}$, this implies that management must have the claimed beliefs in equilibrium and, hence, this is the only equilibrium.

Q.E.D.

7.2 *Proof of Lemma 2*

Lemma 2. Let $d*$ minimize the expression in (11) with respect to *x*, subject to $x \in [0, P]$, and let $p** = P - d*$. Then $p**$ is an *ex-ante*-optimal delegation threshold, and $d*$ and $p**$ are given by

$$\text{If } \sigma_a \le b,\ d* = 0,\ p** = P; \tag{30}$$

$$\text{if } P \le 2\left[\sqrt{L(b,A) - b^2}\right],\ d* = P,\ p** = 0; \tag{31}$$

$$\text{otherwise, } d* = P - p** = 2\left[\sqrt{L(b,A) - b^2}\right] \in (0, P). \tag{32}$$

Moreover, $d* \ge d$ and $p** \le p*$, with strict inequalities whenever $\sigma_a > b$ and $d < P$.

Proof. The problem has a solution, since the objective function is continuous and the constraint set is compact. Obviously, the objective function in (11) is the *ex ante* expected loss if shareholders delegate if and only if $\tilde{p} \ge P - x$. Consequently, $p**$ is an *ex-ante*-optimal delegation threshold.

For $\sigma_a < b$, $L(b,A) < b^2$ (see Harris and Raviv (2008b), Lemma 1), so the objective function in (11) is strictly decreasing in *x*. Therefore, for $\sigma_a < b$, $d* = 0$ and $p** = P$.

Using $\sigma(x)^2 = x^2/12$, the first-order condition for minimizing (11) is

$$\frac{x^2}{4} + b^2 - L(b,A) = 0. \tag{33}$$

For $P < 2\left[\sqrt{L(b,A) - b^2}\right]$, the left hand side of (33) is strictly negative at $x = P$. Therefore, if $P < 2\left[\sqrt{L(b,A) - b^2}\right]$, $d* = P$ and $p** = 0$.

For $P \geq 2\left[\sqrt{L(b,A)-b^2}\right]$ and $\sigma_a \geq b$, solving (33) for x, yields $d^* = 2\left[\sqrt{L(b,A)-b^2}\right] \in [0,P]$.

Clearly, the second order condition is satisfied, since the left hand side of (33) is increasing in *x*.

Finally, if $\sigma_a > b$, and $d < P$, $d^* = \min\left\{2\left[\sqrt{L(b,A)-b^2}\right], P\right\}$ and $d = 2\left[\sqrt{L(b,A)}-b\right]$. It is easy to check that $\sqrt{L(b,A)}-b < \sqrt{L(b,A)-b^2}$ for $L(b,A) > b^2$, which is equivalent to $\sigma_a > b$ (see Harris and Raviv (2008b), Lemma 1). If $\sigma_a \leq b$, then $d^* = d = 0$, and if $d = P$, then $d^* = d = P$.

Q.E.D.

7.3 Proof of Proposition 2

Proposition 2.

(i) If the information of management is less important than agency costs ($\sigma_a < b$), then for any $\sigma_p \in [0,b]$, shareholder control is optimal ($\Delta > 0$).

(ii) For every $\sigma_a \geq b$, there are two boundaries for σ_p, $\sigma_p^L(\sigma_a)$ and $\sigma_p^U(\sigma_a)$, such that when the importance of the shareholders' information $\sigma_p \in \left(\sigma_p^L(\sigma_a), \sigma_p^U(\sigma_a)\right)$, management control is strictly optimal ($\Delta < 0$). When the importance of the shareholders' information $\sigma_p > \sigma_p^U(\sigma_a)$, shareholder control is strictly optimal ($\Delta > 0$). When $\sigma_p \leq \sigma_p^L(\sigma_a)$ or $\sigma_p = \sigma_p^U(\sigma_a)$, control is irrelevant ($\Delta = 0$).

(iii) The functions $\sigma_p^L(\sigma_a)$ and $\sigma_p^U(\sigma_a)$ satisfy the following properties: $\sigma_p^L(b) = \sigma_p^U(b) = 0$, for every $\sigma_a > b$, $\sigma_p^L(\sigma_a)$ and $\sigma_p^U(\sigma_a)$ are continuous and strictly increasing in σ_a, and $\sigma_a > \sigma_p^U(\sigma_a) > \sigma_p^L(\sigma_a) > 0$.

Proof. First suppose $\sigma_a < b$. In this case, as shown in Proposition 1, $d = 0$, so

$$\Delta = b^2 + \sigma_p^2 - L(b,A) > \sigma_p^2 > 0,$$

since, from Lemma 1 of Harris and Raviv (2008b), $\sigma_a < b$ implies that $L(b, A) < b^2$. This proves part (i). Henceforth, we assume $\sigma_a \geq b$.

Define σ_p^L by

$$f\left(b, \sigma_p^L \sqrt{12}\right) = \left(\frac{\sigma_p^L \sqrt{12}}{2} + b\right)^2 = L\left(b, \sigma_a \sqrt{12}\right), \tag{34}$$

or

$$\sigma_p^L(\sigma_a) = \frac{2}{\sqrt{12}}\left(\sqrt{L\left(b, \sigma_a \sqrt{12}\right)} - b\right). \tag{35}$$

From Lemma 1 of Harris and Raviv (2008b), $\sigma_a \geq b$ implies that σ_p^L as given in (35) is non-negative.

Since $\sigma_a \leq b$ implies that $L(b, A) = L\left(b, \sigma_a \sqrt{12}\right) = \sigma_a^2$, it is obvious from (35) that $\sigma_p^L(b) = 0$. Since *L* is strictly increasing and continuous in its second argument (Lemma 1 of Harris and Raviv (2008b)), it is also obvious from (35) that $\sigma_p^L(\sigma_a)$ is increasing in σ_a, $\forall \sigma_a > b$, as claimed in part (iii).

Since *f* is clearly increasing in its second argument, (34) implies that $f\left(b, \sigma_p \sqrt{12}\right) \leq L\left(b, \sigma_a \sqrt{12}\right)$ if and only if $\sigma_p \leq \sigma_p^L(\sigma_a)$. Hence, from (21)–(23) and (34), $d = \min\left\{\sigma_p^L(\sigma_a)\sqrt{12}, P\right\}$. It follows immediately that, for $\sigma_p \leq \sigma_p^L(\sigma_a)$, $d = P$ and $\Delta = 0$, as claimed in part (ii).

The next step is to develop a formula for $\sigma_p^U(\sigma_a)$. Assuming for the time being that $\sigma_p^U(\sigma_a) > \sigma_p^L(\sigma_a)$ and using $d = \min\left\{\sigma_p^L(\sigma_a)\sqrt{12}, P\right\}$, we can write the condition defining $\sigma_p^U(\sigma_a)$ as

$$\sigma_p^U\left(R - \left(\sigma_p^U\right)^2\right) = \sigma_p^L\left(R - \left(\sigma_p^L\right)^2\right), \tag{36}$$

where $R = L\left(b, \sigma_a \sqrt{12}\right) - b^2$. Rewrite (36) as

$$\left(\sigma_p^U\right)^3 - \left(\sigma_p^L\right)^3 - R\left(\sigma_p^U - \sigma_p^L\right) = 0. \tag{37}$$

Since we are assuming that $\sigma_p^U > \sigma_p^L$, we can divide (37) by $\sigma_p^U - \sigma_p^L$ to obtain

$$\left(\sigma_p^U\right)^2 + \sigma_p^U \sigma_p^L + \left(\sigma_p^L\right)^2 - R = 0 .$$

The solution of this equation of interest to us is given by

$$\sigma_p^U = \frac{-\sigma_p^L + \sqrt{4R - 3\left(\sigma_p^L\right)^2}}{2} . \tag{38}$$

For σ_p^U to be given by equation (38), we need only show that this value exceeds σ_p^L. For this, it suffices to show that $R > 3\left(\sigma_p^L\right)^2$. But from (35) and the definition of *R*, we have that $R > 3\left(\sigma_p^L\right)^2$ if and only if $\sqrt{L\left(b, \sigma_a\sqrt{12}\right)} + b > \sqrt{L\left(b, \sigma_a\sqrt{12}\right)} - b$, which is clearly true, since $b > 0$. Consequently, we have shown that σ_p^U is indeed given by equation (38) and that $\sigma_p^U\left(\sigma_a\right) > \sigma_p^L\left(\sigma_a\right)$, as claimed in (iii). For $\sigma_a = b$, $R = \sigma_a^2 - b^2 = 0$, and, as shown previously, $\sigma_p^L = 0$. Consequently, $\sigma_p^U\left(b\right) = 0$, as claimed in (iii).

To show that $\sigma_p^U\left(\sigma_a\right) < \sigma_a$, from (38), it suffices to show that

$$R = L\left(b, \sigma_a\sqrt{12}\right) - b^2 < \sigma_a^2 + \sigma_a \sigma_p^L + \left(\sigma_p^L\right)^2 . \tag{39}$$

It is easy to check that $L\left(b, \sigma_a\sqrt{12}\right) \le \sigma_a^2$.[45] Therefore (39) is clearly satisfied since *b*, σ_a, and σ_p^L are all positive.

To complete the proof of part (iii), it remains to show that $\sigma_p^U\left(\sigma_a\right)$ is continuous and strictly increasing in σ_a. For this, it suffices to show that $\sigma_p^U\left(\sigma_a\right)$ is continuous and strictly increasing in

---

[45] If $N\left(b, \sigma_a\sqrt{12}\right) = 1$, then $L\left(b, \sigma_a\sqrt{12}\right) = \sigma_a^2$, and we are done. Suppose $N\left(b, \sigma_a\sqrt{12}\right) = n \ge 2$. Then $L\left(b, \sigma_a\sqrt{12}\right) = \frac{\sigma_a^2}{n^2} + \frac{b^2\left(n^2 - 1\right)}{3} < \sigma_a^2$ if and only if $\sigma_a^2 > \frac{\left(bn\right)^2}{3}$. But, as shown in Lemma 1 of Harris and Raviv (2008b), $N\left(b, \sigma_a\sqrt{12}\right) = n$ implies that $A > 2bn\left(n-1\right)$ or $\sigma_a^2 > \frac{\left(bn\left(n-1\right)\right)^2}{3} \ge \frac{\left(bn\right)^2}{3}$, for $n \ge 2$.

$\sqrt{L\left(b,\sigma_a\sqrt{12}\right)}$. To make the formulas easier to read, let $z=\sqrt{L\left(b,\sigma_a\sqrt{12}\right)}$. Then, substituting for *R* and σ_p^L in (38), we have

$$\sigma_p^U=\frac{1}{2}\left[-\frac{2}{\sqrt{12}}\left(z-b\right)+\sqrt{3\left(z^2-b^2\right)+2b\left(z-b\right)}\right]. \qquad (40)$$

It is clear from (40) that σ_p^U is continuous in *z*. It is easy to check that the derivative of the right hand side of (40) with respect to *z* is positive if and only if $3z^2+2bz+b^2>0$, which is clearly true. This completes the proof of part (iii).

To complete the proof of part (ii), we must show that $\Delta<0$ for $\sigma_p^L\left(\sigma_a\right)<\sigma_p<\sigma_p^U\left(\sigma_a\right)$, and $\Delta>0$ for $\sigma_p>\sigma_p^U\left(\sigma_a\right)$. It is easy to check that Δ is convex in σ_p for $\sigma_p\geq\sigma_p^L$. Since $\Delta\equiv 0$ for $\sigma_p\leq\sigma_p^L$, Δ can cross zero at most once at some $\sigma_p>\sigma_p^L$ and only from below (see Figure 6). Consequently, this must occur at σ_p^U, and $\Delta<0$ for $\sigma_p^L\left(\sigma_a\right)<\sigma_p<\sigma_p^U\left(\sigma_a\right)$, $\Delta>0$ for $\sigma_p>\sigma_p^U\left(\sigma_a\right)$, and $\Delta=0$ for $\sigma_p=\sigma_p^U\left(\sigma_a\right)$ as claimed. Q.E.D.

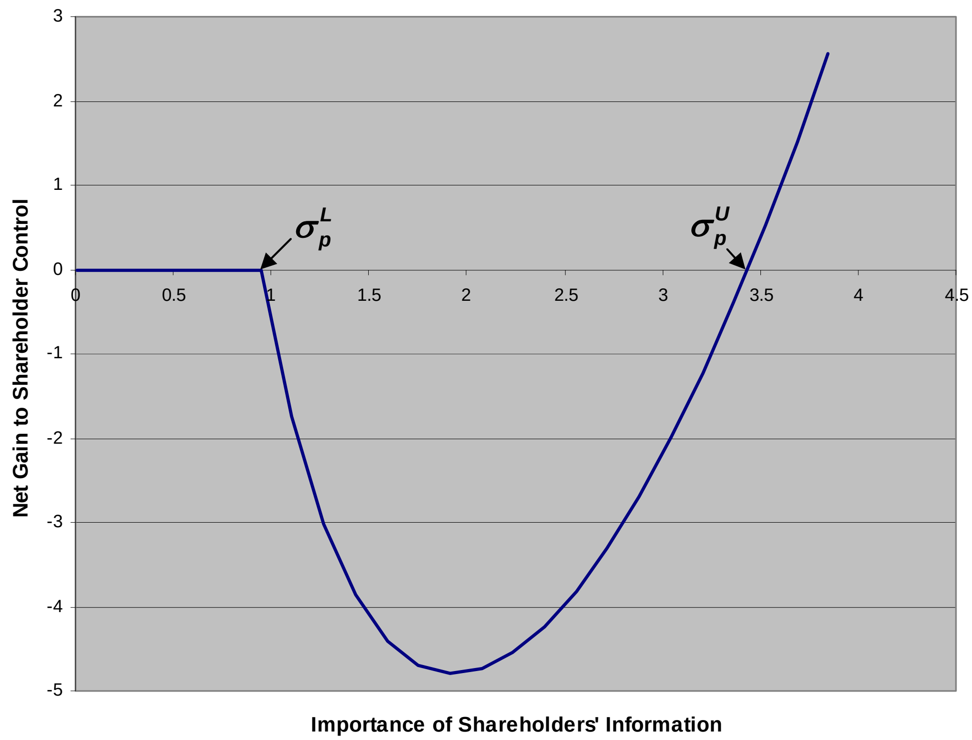

Figure 6

This graph shows the net gain to shareholder control, Δ, as a function of the importance of shareholders' information. Shareholder control is optimal whenever the importance of shareholders' information, σ_p, exceeds $\sigma_p^U \approx 3.45$. For this figure, $b = 4$, $\sigma_a = 8$, and $\sigma_p^L = 0.95658525 < b$.

7.4 Proof of Proposition 3

Proposition 3.

- An increase in the importance of shareholders' information can result in a shift from management control being strictly optimal to shareholder control being strictly optimal but not the reverse. Similarly, an increase in the importance of management's information can result in a shift from shareholder control being strictly optimal to management control being strictly optimal but not the reverse.

- Suppose agency costs increase from $b = b_L$ to $b = b_H$ with $b_H > b_L > 0$. There exists a largest value of σ_a, denoted $\bar{b}$, with $\bar{b} > b_H$ ($\bar{b}$ may be infinite), such that whenever $(\sigma_a, \sigma_p) \in [b_L, \bar{b}] \times [\sigma_p^U(\sigma_a, b_H), \sigma_p^U(\sigma_a, b_L)]$, the increase in agency costs results in a shift from management control being optimal to shareholder control being strictly optimal. If $\bar{b}$ is finite, then there is also a region of values of (σ_a, σ_p) in which the increase in agency costs results in a shift from shareholder control being strictly optimal to management control being strictly optimal. This case does occur for some values of b_L and b_H.

Proof. The first part of the proposition follows immediately from Proposition 2.

For the second part, we define σ_p^U to be zero for $\sigma_a \le b$. Since σ_p^U is strictly increasing in σ_a for $\sigma_a \ge b$ (see Proposition 2), $\sigma_p^U(\sigma_a, b_L) > \sigma_p^U(\sigma_a, b_H)$ for $\sigma_a \in (b_L, b_H]$. Consequently, since σ_p^U is continuous in σ_a (see Proposition 2), $\sigma_p^U(\sigma_a, b_L) > \sigma_p^U(\sigma_a, b_H)$ for some interval of values of $\sigma_a \ge b_H$. If, for some values of $\sigma_a > b_H$, $\sigma_p^U(\sigma_a, b_L) \le \sigma_p^U(\sigma_a, b_H)$, define $\bar{b}$ to be the minimum such value (the minimum exists, since, by continuity of σ_p^U in σ_a, the set of values of $\sigma_a > b_H$ such that $\sigma_p^U(\sigma_a, b_L) \le \sigma_p^U(\sigma_a, b_H)$ is closed and bounded below). If $\sigma_p^U(\sigma_a, b_L) > \sigma_p^U(\sigma_a, b_H)$ for all $\sigma_a > b_L$, define $\bar{b} = \infty$. Consequently, $[b_L, \bar{b}] \times [\sigma_p^U(\sigma_a, b_H), \sigma_p^U(\sigma_a, b_L)] \ne \emptyset$. The example in Figure 2 shows that there are values of b_L and b_H for which $\bar{b}$ is finite. The result now follows from Proposition 2.

Q.E.D.

7.5 Proof of Lemma 4

Lemma 4. If shareholders control the decision, the *ex ante* expected loss is given by

$$L_S = \varphi\left(\sigma_p^2 + b^2\right) + (1-\varphi)\left[\sigma_p^2 + L(b,A) + \sigma_q^2 + \bar{e}^2 - \frac{N(b,A)-1}{N(b,A)}\bar{e}(\bar{e}+2b)\right], \quad (41)$$

where $\varphi = \Pr(\tilde{q} \ge p^*)$ is the probability of delegation.

Proof. Management is indifferent among all reports in a given partition cell, as in the base case of section 3. Thus, given a realization a of $\tilde{a}$, management chooses a report in $\left[a_{i-1}, a_i\right]$ if and only if i solves

$$\min_{j\in\{1,\ldots,N\}} E\left\{\left[\overline{a}_j + \tilde{q} - \left(a + b + \tilde{p}\right)\right]^2 \middle| \tilde{q} \le p^*\right\},$$

where $N = N\left(b, A\right)$. It is easy to check that solving this problem is equivalent to solving

$$\min_{j\in\{1,\ldots,N\}} \left[\overline{a}_j - \left(a + b - \overline{e}\right)\right]^2, \tag{42}$$

Suppose $a + b - \overline{e}$ is equidistant from $\overline{a}_i$ and $\overline{a}_{i+1}$. By construction of $\left\{a_i\right\}$, however, $a_i + b$ is equidistant from $\overline{a}_i$ and $\overline{a}_{i+1}$. Therefore, we must have $a + b - \overline{e} = a_i + b$ or $a = a_i + \overline{e}$. It follows that the solution of (42) is the value of i such that $a \in \left[\alpha_{i-1}, \alpha_i\right]$, where, if $\overline{e} \ge 0$,

$$\begin{aligned} &\alpha_0 = a_0 = 0, \\ &\alpha_i = \min\left\{a_i + \overline{e}, A\right\}, i \in \left\{1,\ldots,N\right\}, \end{aligned} \tag{43}$$

and if $\overline{e} < 0$,

$$\begin{aligned} &\alpha_N = a_N = A, \\ &\alpha_i = \max\left\{a_i + \overline{e}, 0\right\}, i \in \left\{0,\ldots,N-1\right\}. \end{aligned} \tag{44}$$

The expected loss in value if shareholders are in control, given $\tilde{q} = q < p^*$ (so they do not delegate), is then

$$\begin{aligned} &\frac{1}{AP}\int_0^P \left[\sum_{i=1}^N \int_{\alpha_{i-1}}^{\alpha_i} \left[\overline{a}_i + q - \left(a + p\right)\right]^2 da\right] dp \\ &= \sigma_p^2 + \left(q - \overline{p}\right)^2 + \frac{1}{A}\sum_{i=1}^N \left(\alpha_i - \alpha_{i-1}\right)\left[2\left(q - \overline{p}\right)\left(\overline{a}_i - \overline{\alpha}_i\right) + \left(\overline{a}_i - \overline{\alpha}_i\right)^2 + \sigma\left(\alpha_i - \alpha_{i-1}\right)^2\right], \end{aligned} \tag{45}$$

where $\overline{\alpha}_i = \left(\alpha_{i-1} + \alpha_i\right)/2$. Taking the expectation of the right-hand side of (45) with respect to q, given that $q < p^*$, we see that the expected loss in value if shareholders are in control and do not delegate is

$$\sigma_p^2 + \sigma_q^2 + \overline{e}^2 + \frac{1}{A}\sum_{i=1}^N \left(\alpha_i - \alpha_{i-1}\right)\left[2\overline{e}\left(\overline{a}_i - \overline{\alpha}_i\right) + \left(\overline{a}_i - \overline{\alpha}_i\right)^2 + \sigma\left(\alpha_i - \alpha_{i-1}\right)^2\right]. \tag{46}$$

Condition (14) implies that $\alpha_i = a_i + \overline{e}$ for $i \in \{1, \dots, N-1\}$. It then follows from (46) that shareholders' expected loss in value if they are in control and do not delegate is

$$\sigma_p^2 + L(b,A) + \sigma_q^2 + \overline{e}^2 - \frac{N(b,A)-1}{N(b,A)}\overline{e}(\overline{e}+2b). \tag{47}$$

This results in the expression for the *ex ante* loss to shareholders as stated in the lemma.

Q.E.D.

7.6 Proof of Proposition 5

Lemma 3. Define $\beta_0 = \frac{1}{2}\left(b - \sqrt{b^2 + 2\sigma_p^2}\right)$. Then $\beta_0 < 0$ and if $b > \sigma_p/2$, then $b - \beta_0 < 2b$.

Proof. It is obvious that $\beta_0 < 0$ and that $-b < \beta_0$ if and only if $b > \sigma_p/2$. Clearly, $b - \beta_0 < 2b$ if and only if $-b < \beta_0$. Q.E.D.

Proposition 5 (see Figure 5)**.** Assume $b - \overline{p} > \sigma_p$. Then there exist continuous functions, $G(\beta)$, with *G* symmetric with respect to $\beta = b$, and $H(\beta)$ such that $H(\overline{p}) = G(\overline{p})$, $H(\beta) \equiv 0$ for $\beta \le \beta_S$, $H(\beta)$ is increasing in β for $\beta \le b/2$, and

- shareholders, when in control, always delegate to management and control is irrelevant whenever $\sigma_a^2 \ge G(\beta)$;
- for decisions for which $\beta > b + \sigma_p$, management control is strictly optimal if $\sigma_a^2 < G(\beta)$;
- for decisions for which $b - \sigma_p > \beta > \overline{p}$, shareholder control is strictly optimal if $\sigma_a^2 < G(\beta)$;
- for decisions for which $\overline{p} > \beta$, shareholder-control is strictly optimal if $\sigma_a^2 < H(\beta)$ and management control is strictly optimal if $H(\beta) < \sigma_a^2 < G(\beta)$; management control is also strictly optimal for $\sigma_a^2 = H(\beta) = 0$ when $\beta < \beta_S$.

Proof. Define $g(B)$ as the value of σ_a^2 such that $L\left(B,\sigma_a\sqrt{12}\right)=\left(\frac{P}{2}+|B|\right)^2$. It is easy to check, using the facts that $L\left(B,\sigma_a\sqrt{12}\right)=\sigma_a^2$ for $\sigma_a^2 \le B^2$ and $L\left(B,\sigma_a\sqrt{12}\right)\to\infty$ as $\sigma_a\to\infty$, and L is continuous in its second argument (see Lemma 1 in Harris and Raviv (2008b)), that such a value of σ_a^2 exists and is larger than B^2. Thus $g(B)>B^2$ for all B. Since L depends on B only through B^2, and $\left(\frac{P}{2}+|B|\right)^2$ depends on B only through $|B|$, g is symmetric with respect to $B=0$. Finally, since L and $\left(\frac{P}{2}+|B|\right)^2$ are continuous in B (for $B\neq 0$), so is g. From the definition of g, the fact that L is increasing in its second argument (see Lemma 1 in Harris and Raviv (2008b)), and Proposition 1, for $\sigma_a^2 \ge g(B)$, $d=P$, i.e., shareholders, if in control, always delegate to management. In this case, control is irrelevant, since management always makes the decision with no information from shareholders, regardless of control, i.e., (20) is satisfied as an equality. Define $G(\beta)=g(b-\beta)$. Therefore, for $\sigma_a^2 \ge G(\beta)$, shareholders, if in control, always delegate to management and, control is irrelevant, as claimed in the first bullet. Moreover, G is continuous and symmetric with respect to $\beta=b$. For the remainder of the proof, we consider only the case in which $\sigma_a^2 < G(\beta)$ or, equivalently, $\sigma_a^2 < g(B)$.

We split the remainder of the proof into two cases, $\sigma_a^2 \le B^2$ and $\sigma_a^2 > B^2$.

Case 1: $\boldsymbol{\sigma_a^2 \le B^2}$. In this case, neither party will delegate to the other nor will they communicate any of their private information. Therefore, condition (20) becomes $\beta^2+\sigma_a^2 \le b^2+\sigma_p^2$, which can be rewritten in terms of the net bias, B as

$$\sigma_a^2 \le \sigma_p^2-\left(B^2-2bB\right). \tag{48}$$

Define $h_1(B) = \sigma_p^2 - (B^2 - 2bB)$. Clearly, (48) is satisfied if and only if $\sigma_a^2 \le h_1(B)$. If $B < -\sigma_p$, or, equivalently, $\beta > b + \sigma_p$, then $B^2 - 2bB > B^2 \ge \sigma_p^2$, since $B < 0$, so $h_1(B) < 0$ for *B* in this range. Consequently, for $\beta > b + \sigma_p$, (48) cannot be satisfied, and management control is optimal.

At this point, it is convenient to redefine $h_1(B) = \max\{\sigma_p^2 - (B^2 - 2bB), 0\}$ If $B > \sigma_p$, it is easy to check that $h_1(B) \le B^2$ if and only if $B \ge b - \beta_0$ with equality if and only if $B = b - \beta_0$. Note that the assumption that $b - \overline{p} > \sigma_p$ implies that $b > \sigma_p/2$ which implies that $0 < b - \beta_0 < 2b$ by Lemma 3. Moreover, h_1 is strictly decreasing in *B* for $B_0 > B \ge b - \beta_0$, where $B_0 = b + \sqrt{b^2 + \sigma_p^2}$, and $h_1(B) \equiv 0$, for all $B \ge B_0$. Thus, for $\sigma_a^2 \le B^2$, management control is strictly optimal for $B < -\sigma_p$ (as shown above) and for $B > b - \beta_0$ if and only if $\sigma_a^2 > h_1(B)$. For $\sigma_p \le B < b - \beta_0$, shareholder control is optimal for all $\sigma_a^2 \le B^2 \le h_1(B)$. This completes the characterization of optimal control for $\sigma_a^2 \le B^2$. Define $H_1(\beta) = h_1(b - \beta)$. Thus H_1 is strictly increasing in β for $\beta_S < \beta \le \beta_0$ and is identically zero for $\beta \le \beta_S = b - B_0 = -\sqrt{b^2 + \sigma_p^2}$.

Case 2: $\sigma_a^2 > B^2$. In this case $d \in (0, P)$ (recall we assume $\sigma_a^2 < g(B)$), so we have $L(B, A) = f(B, d) = \left(\frac{d}{2} + |B|\right)^2$. Substituting $\left(\frac{d}{2} + |B|\right)^2$ for *L*, we can write the left hand side of (20) as

$$F(d) \equiv \frac{d}{P} b^2 + \left(1 - \frac{d}{P}\right)\beta^2 + \frac{d}{P}\sigma(d)^2 + \left(1 - \frac{d}{P}\right)\left(\frac{d}{2} + |B|\right)^2.$$

We claim that the function *F* is strictly concave in *d* on $(0, P)$. To see this, note that

$$F''(d) = \frac{1}{2} - \frac{2}{P}\left(\frac{d}{2} + |B|\right) < \frac{1}{2} - \frac{2|B|}{P}.$$

But $P = \sqrt{12}\sigma_p \le \sqrt{12}|B| < 4|B|$. Consequently, $F''(d) < 0$ and *F* is strictly concave as claimed. Also $F(0) = B^2 + \beta^2 = b^2 + 2B(B - b)$, and $F(P) = \sigma_p^2 + b^2$ = right hand side of (20).

Since F is strictly concave and $F(P)=\sigma_p^2+b^2$, if $F(0)\geq\sigma_p^2+b^2$, then $F(d)>\sigma_p^2+b^2$ for all $d\in(0,P)$. Therefore, in this case (20) is false, i.e., management control is strictly optimal, for all $\sigma_a^2<g(B)$, or, equivalently, for all $\sigma_a^2<G(\beta)$.

Now suppose $\beta>b+\sigma_p$, or, equivalently, $B<-\sigma_p$. Then, $2B(B-b)>B^2\geq\sigma_p^2$, so $F(0)>\sigma_p^2+b^2$. Consequently, management control is strictly optimal for all $B^2<\sigma_a^2<g(B)=G(\beta)$. Together with the previous result that management control is strictly optimal when $\beta>b+\sigma_p$ for all $\sigma_a^2\leq B^2$, we have completed the proof of the second bullet.

Next, suppose $B>\sigma_p$, or, equivalently, $\beta<b-\sigma_p$. Then it is easy to check that $F(0)<\sigma_p^2+b^2$ if and only if $B<b-\beta_0$. Consequently, if $B\geq b-\beta_0$, or, equivalently, $\beta\leq\beta_0$, $F(0)\geq\sigma_p^2+b^2$, so management control is strictly optimal. Now consider $\sigma_p<B<b-\beta_0$ (equivalently $b-\sigma_p>\beta>\beta_0$), so that $F(0)<\sigma_p^2+b^2$. There are two possible cases. Since F is strictly concave, if $F'(P)\geq 0$, then $F(d)<\sigma_p^2+b^2$ for all $d\in(0,P)$ which implies that it is strictly optimal for shareholders to control regardless of the value of $B^2<\sigma_a^2<g(B)$. If $F'(P)<0$, then there exists a unique $d_0\in(0,P)$ such that $F(d)\leq\sigma_p^2+b^2$ for all $d\leq d_0$, $F(d)>\sigma_p^2+b^2$ for all $d>d_0$, and $F'(d_0)>0$. That is, it is optimal for shareholders to control if and only if $d\leq d_0$. But d is a continuous, increasing function of σ_a^2, $d=0$ for $\sigma_a^2\leq B^2$, and $d=P$ for $\sigma_a^2\geq g(B)$, so, for each B such that $F'(P)<0$, there is a unique value of $\sigma_a^2\in(B^2,g(B))$ such that $d=d_0$ for that value of σ_a^2. Define $h_0(B)$ to be the value of σ_a^2 for which $d=d_0$ for B such that $F'(P)<0$. For such B, shareholder control is strictly optimal for $\sigma_a^2<h_0(B)$, and management control is optimal for $\sigma_a^2>h_0(B)$. It is clear from the construction that $g(B)>h_0(B)>B^2$. Since F, L, and f are continuous in B, so is h_0.

It is easy to check that

$$F'(P)=\frac{1}{\overline{p}}(b-\overline{p}-B)B.$$

Consequently, $F'(P)<0$ for $b-\overline{p}<B<b-\beta_0$, and $F'(P)\geq 0$ for $\sigma_p \leq B \leq b-\overline{p}$. It follows that shareholder control is strictly optimal for $\sigma_p \leq B \leq b-\overline{p}$ and $B^2<\sigma_a^2<g(B)$ and for $b-\overline{p}<B<b-\beta_0$ with $B^2<\sigma_a^2<h_0(B)$, while management control is strictly optimal for $b-\overline{p}<B<b-\beta_0$ for $h_0(B)<\sigma_a^2<g(B)$. Define $H_0(\beta)=h_0(b-\beta)$. Restated in terms of β, we have shown that shareholder control is strictly optimal for $b-\sigma_p \geq \beta \geq \overline{p}$ and $B^2<\sigma_a^2<G(\beta)$ and for $\overline{p}>\beta>\beta_0$ with $B^2<\sigma_a^2<H_0(\beta)$, while management control is strictly optimal for $\overline{p}>\beta>\beta_0$ for $H_0(\beta)<\sigma_a^2<G(\beta)$.

Now, since F' is continuous and $F'(P)=0$ for $B=b-\overline{p}$, $d_0 \uparrow P$ as $B \downarrow b-\overline{p}$. But, for any *B*, $g(B)$ is the smallest value of σ_a^2 such that $d=P$. Consequently, $h_0(B)\rightarrow g(b-\overline{p})$ as $B\downarrow b-\overline{p}$. Define $h_0(b-\overline{p})=g(b-\overline{p})$, so h_0 is continuous at $b-\overline{p}$. Moreover, as $B\uparrow b-\beta_0$, $d_0\downarrow 0$, so $h_0(B)\rightarrow B^2=h_1(b-\beta_0)$. Therefore, define

$$h(B)=\begin{cases} h_0(B) \text{ for } B\in[b-\overline{p},b-\beta_0], \\ h_1(B) \text{ for } B\geq b-\beta_0. \end{cases}$$

Then *h* is continuous in *B*, $h(b-\overline{p})=g(b-\overline{p})$, and $h(B)\equiv 0$ for all $B\geq B_0$.

Finally, note that, for $B>0$,

$$\frac{\partial F(d)}{\partial B}=2\left(1-\frac{d}{P}\right)\left(2B-b+\frac{d}{2}\right).$$

Consequently, $\frac{\partial F(d)}{\partial B}>0$ for all $d\in(0,P)$ if $B\geq b/2$. Since $F'(d_0)>0$, it follows that, for $B\geq b/2$, d_0 is strictly decreasing in *B*. Therefore, so is $h_0(B)$. Since we have already shown that $h_1(B)$ is strictly decreasing in *B* for $B_0>B\geq b-\beta_0$, $h(B)$ is strictly decreasing in *B* for $B_0>B\geq b/2$.

Finally, define $H(\beta)=h(b-\beta)$. Then H is continuous in β, $H(\overline{p})=G(\overline{p})$, $H(\beta)\equiv 0$ for all $\beta \le \beta_S$, and $H(\beta)$ is strictly increasing in β for $\beta_S < \beta \le b/2$. Q.E.D.

References

Adams, R. B. and D. Ferreira (2007), "A Theory of Friendly Boards," *The Journal of Finance 62*, 1, January, 217-250.

Adams, Renée, Benjamin E. Hermalin, and Michael S. Weisbach (2008), "The Role of Boards of Directors in Corporate Governance: A Conceptual Framework and Survey," Available at SSRN: http://ssrn.com/abstract=1299212.

Agarwal, Ashwini (2007), "Corporate Governance Objectives of Labor Union Shareholders," working paper, University of Chicago Graduate School of Business. Available at http://home.uchicago.edu/~aagrawa1/.

Aghion, Phillipe and Jean Tirole (1997), "Formal and Real Authority in Organizations," *Journal of Political Economy 105*, 1, February, 1997, 1-29.

Almazan, Andres and Javier Suarez (2003), "Entrenchment and Severance Pay in Optimal Governance Structures," *The Journal of Finance. 58*, 2, April, pp. 519-547.

Alonso, Ricardo and Niko Matouschek (2007), "Relational Delegation," *Rand Journal of Economics 38*, 4, 1070-1089.

Alonso, Ricardo and Niko Matouschek (2008), "Optimal Delegation," *Review of Economic Studies 75*, 1, 259-293.

Aumann, Robert J. and Sergiu Hart (2003), "Long Cheap Talk," *Econometrica 71*, 1619–1660.

Bainbridge, Stephen M. (2006), "Director Primacy and Shareholder Disempowerment," UCLA Law School Research Paper #05-25, forthcoming *Harvard Law Review*. Available at SSRN: http://ssrn.com/abstract=808584.

Bebchuk, Lucian Arye (2005), "The Case for Increasing Shareholder Power," *Harvard Law Review 118*, 835-914.

Becht, Marco, Patrick Bolton, and Ailsa Röell (2003), "Corporate Governance and Control," in George Constantinides, Milton Harris, and René Stulz, eds., *The Handbook of the Economics of Finance* (Amsterdam: North Holland).

Blume, Andreas, Oliver J. Board, and Kohei Kawamura (2007), "Noisy talk," *Theoretical Economics 2*, 395-440.

Burkhart, Mike, Denis Gromb and Fausto Panunzi (1997), "Large Shareholders, Monitoring, and the Value of the Firm," *Quarterly Journal of Economics 112*, 3, August, 693-728.

Chakraborty, Archisman and Rick Harbaugh (2007), "Comparative Cheap Talk," *Journal of Economic Theory 132*, 70-94.

Crawford, Vincent P. and Joel Sobel (1982), "Strategic Information Transmission," *Econometrica*, 50, 1431-1451.

Crémer, Jacques (1995), "Arm's Length Relationships," *Quarterly Journal of Economics CX*, 2, May, 275-295.

Dessein, Wouter (2002), "Authority and Communication in Organizations," *Review of Economic Studies*, 69, 811-838.

Harris, Milton and Artur Raviv (2005), "Allocation of Decision-making Authority," *Review of Finance 9*, 353-383.

Harris, Milton and Artur Raviv (2008a), "A Theory of Board Control and Size," *Review of Financial Studies 21*, 4, July, 1797-1832.

Harris, Milton and Artur Raviv (2008b), "Appendix to 'A Theory of Board Control and Size'," *Review of Financial Studies* web site, http://www.sfsrfs.org.

Hermalin, Benjamin E., and Michael S. Weisbach (1988), "The Determinants of Board Composition." *RAND Journal of Economics 19*, 589-606.

Hermalin, Benjamin E. and Michael S. Weisbach (2003), "Boards of Directors as an Endogenously Determined Institution: A Survey of the Economic Literature," *FRBNY Economic Policy Review 9*, 7-26.

Holderness, Clifford G. (2009), "The Myth of Diffuse Ownership in the United States," *Review of Financial Studies 22*, 4, April, 1377-1408.

Krishna, Vijay and John Morgan (2004), "The art of conversation: eliciting information from experts through multi-stage communication," *Journal of Economic Theory 117*, 147-179.

Krishna, Vijay and John Morgan (2008), "Contracting for Information Under Imperfect Commitment," *Rand Journal of Economics 39*, 4, 905-925.

Lipton, Martin (2002), "Pills, Polls, and Professors Redux," *University of Chicago Law Review 69*, 3, 1037-1065.

Milgrom, Paul and John Roberts (1986), "Relying on the information of interested parties," *Rand Journal of Economics 17*, 1, Spring, 18-32.

Shivdasani, A. and D. Yermack (1999), "CEO Involvement in the Selection of New Board Members: An Empirical Analysis," *The Journal of Finance 54*, 1829-1853.

Shleifer, Andrei and Lawrence H. Summers (1988), "Breach of Trust in Hostile Takeovers" in *Corporate takeovers: Causes and consequences*, Alan J. Auerbach, ed., Chicago: University of Chicago Press, 33-56.

Shleifer, Andrei and Robert W. Vishny (1997), "A Survey of Corporate Governance," *The Journal of Finance 52*, 2, June, 737-783.

Stout, Lynn A. (2006), "The Mythical Benefits of Shareholder Control," *Virginia Law Review*, forthcoming. Available at SSRN: http://ssrn.com/abstract=929530.

Strine, Leo E., Jr. (2006), "Towards A True Corporate Republic: A Traditionalist Response To Lucian's Solution For Improving Corporate America," Harvard Law School Discussion Paper #537. Available at SSRN: http://ssrn.com/abstract=883720.

Tirole, Jean (2006), *The Theory of Corporate Finance* (Princeton, NJ: Princeton University Press).