Limited Capital Market Participation and Human Capital Risk*

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Abstract

Traditionally, the non-tradeability of human capital is often cited for the failure of traditional asset pricing theory to explain agents' portfolio holdings. In this paper we argue that the opposite might be true — traditional models might not be able to explain agent portfolio holdings because they do not explicitly account for the fact that human capital does trade (in the form of labor contracts). Our key point of departure from the existing literature is that we endow investors with the ability to generate wealth and derive wages endogenously as part of a dynamic equilibrium in a production economy. This flexibility allows agents to achieve a Pareto optimal allocation even when the span of asset markets is restricted to just stocks and bonds. Risk is shared in labor markets because firms have the flexibility to write bilateral labor contracts that insure workers. Capital markets facilitate this risk sharing because it is there that firms offload the labor market risk they assumed from workers. In effect, by investing in capital markets investors provide insurance to wage earners who then optimally choose not to participate in capital markets. The model can produce some of the most important stylized facts in asset pricing: (1) limited asset market participation, (2) the seemingly high equity risk premium and (3) the very large disparity in the volatility of consumption and the volatility of asset prices.

*Preliminary and incomplete. Comments are welcome.

A commonly held view amongst financial economists is that a significant fraction of wealth consists of non-tradeable assets, most notably human capital wealth. Indeed, this hypothesis is often used to explain why one of the key predictions of the CAPM does not hold, that all agents hold the same portfolio of risky assets. Because investors should use the capital markets to diversify as much risk as possible, and because non-tradeable human capital exposure varies across individuals, investors should optimally choose to hold different portfolios of risky assets. Although this explanation certainly has the potential to explain the cross sectional variation in portfolio holdings, it also necessarily implies wide stock market participation. However, the fact is that the majority of people do not participate in the capital markets. Not only do these individual appear to eschew the opportunity to partially hedge their human capital exposure, the hedging of human capital risk does not appear to be a primary motivator for the minority of people who do participate in capital markets. Instead, the anecdotal evidence suggests that rather than a desire to hedge, what motivates most investors is a willingness take on *additional* risk because they find the risk return tradeoff attractive.[1] The object of this paper is to put forward a plausible explanation for these two characteristics of investor behavior.

No convincing explanation for why most people do not participate in capital markets has been put forward. The most commonly cited explanation are barriers to entry, although in economies such as the United States it is difficult to accept that significant economic barriers to entry exist that prevent people from participating. Instead most researchers cite educational barriers to entry, and research has shown that education level is strongly correlated with participation.[2] But the problem with this explanation for limited stock market participation is that it does not address the question of *why* the educational barriers exist at all. After all, we see wide participation in arguably more complicated financial products such as mortgages, auto leases and insurance. In these cases the educational barriers to entry were removed by the motivation to make profits — firms invested considerable resources educating people so they could sell these products. Given the welfare gain to hedging non-tradeable human capital, why does a similar economic motivation to educated consumers to hold stocks apparently not exist?

One possible reason for non-participation might be the "narrowness" of the asset span — asset markets are so incomplete they offer little opportunity for Pareto improving trades. Although rarely cited explicitly, this explanation is implicit in the literature on non-traded wealth. But for this explanation to hold water, one must also then account for why the asset span does not endogenously expand. After all, if as much as 80% of wealth is non-tradeable,[3] one would expect there to be significant economic incentives to expand the asset span. In fact, the span of traded assets has changed only marginally (if at all) despite the fact that the number of

[1] For example, considerable resources are devoted to advising people on how to find high return investments whereas advice on investments with good hedging characteristics is largely non-existent.

[2] Mankiw and Zeldes (1991) document the relation between education and participation and Hong, Kubik, and Stein (2004) document that non-formal education, such as social interaction, is also correlated with participation.

[3] Need citation

new assets has exploded in recent years. Perhaps more importantly, one would not naturally expect incompleteness to result in non-participation, as Telmer (1993) demonstrates. Indeed the low correlation between human capital and stock market returns documented in Lustig and Van Nieuwerburgh (2008) would seem to imply that despite the incompleteness, there are still large benefits to asset diversification and so all agents should be participating.

If agents are not being prevented from participating, then then they must be choosing not to participate. On possibility is that agents' initial endowments are naturally so close to a Pareto optimal allocation there is little reason to engage in further trade. But considering the heterogeneity in actual endowments (some agents inherit money, others are naturally skilled, etc.), this explanation seems implausible. A more plausible possibility is that agents are able to achieve a Pareto optimal allocation through trade in other markets — that is, what traditional models of investment ignore are other markets on which people share risk. Building on this insight, we identify the labor market as one such market and posit that the unwillingness of some individuals to use capital markets is a consequence of the fact that they are able to share enough risk through their wage contracts so that the capital markets offer little incremental benefit.

We focus on labor markets because they are an ideal place to share risk. The structure of most firms has historically been built around long-term tailored labor contracts between agents. In such an environment we show that it is possible for agents to achieve a Pareto optimal allocation of risk even with limited capital market participation and a "narrow" asset span. This insight has important implications. First, it calls into question one of the basic assumptions in asset pricing — that because asset markets do not span labor risk, human capital is not traded and so most wealth is non-tradeable. Second, it can explain limited stock market participation as an efficient *equilibrium* outcome. Third, it has the potential to explain a number of important asset pricing puzzles, including the high equity risk premium and equity volatility observed in the data, as well as time varying correlation between consumption growth and equity returns and low unconditional correlation between human capital and equity returns.

Because firms insure workers through their labor contracts, shareholders are the ultimate insurers of human capital risk. Shareholders must be compensated for taking on this risk, so the risk premium for holding equity is considerably higher than the risk premium for holding pure consumption risk. Indeed, by parameterizing our model we are able to show that it is not difficult to reproduce the observed level of the equity risk premium with realistic values of risk aversion and consumption volatility.

There is perhaps no greater challenge to the neoclassical model of asset prices than the seeming disconnect between the volatility of consumption and asset prices. Not only is consumption volatility significantly lower than the volatility of asset prices, but the two series behave manifestly differently. For example, average quarterly volatility of the S&P 500 index is 68% higher during recessions (as identified by the NBER).[4] Yet, a concomitant increase in consumption volatility is much smaller if it exists at all — over the period 1947-2009, the point estimate of the volatility of (seasonally adjusted) quarterly GDP growth in NBER recessions is only 11% higher than in expansions.[5] By recognizing the important role of labor markets, we demonstrate why, in a otherwise standard neoclassical model of asset prices, the volatility of asset prices should bear little apparent relation to the volatility of consumption. Our model also readily explains the stylized fact that equity wealth is considerably more volatile than total wealth and why human capital wealth is relatively less volatile:[6] Human capital wealth is traditionally measured using *wage income*, that is, the income that results once risk sharing has already taken place in the labor market.

Finally, the model implies time varying (and low unconditional) correlation between consumption growth and equity returns, as well as between return on human capital and return on equity, in line with what has been observed empirically. For example, Duffee (2005) shows that the conditional correlation between consumption growth and equity returns is almost zero when the equity wealth-to-consumption ratio is low, whereas the correlation is high when the equity wealth-to-consumption ratio is high. This is exactly what our model predicts, and further, it suggests that the degree of heterogeneity in labor productivity should be a powerful instrument in a conditional asset pricing test.

The paper is organized as follows. In the next section we provide a brief literature review. In the next section we provide a literature review. In sections 2-4 we introduce the model and derive several theoretical implications. Section 5 provides a simple calibration to real parameters, and section 6 makes some concluding remarks. All proofs are left to an appendix.

1 Background

The idea that one role of the firm is to insure its workers' human capital risk dates at least as far back as Knight (1921). Knight takes as a primitive that the job of worker and manager entails taking on different risks and notes that entrepreneurs bear most of the risk. Using this idea Kihlstrom and Laffont (1979) endogenizes who, in a general equilibrium, becomes an entrepreneur and who becomes a worker. Less risk averse agents choose to be entrepreneurs who then optimally insure workers. However, the wage contract in that paper is exogenously imposed rather than an endogenous response to the desire to optimally share risk and so the resulting equilibrium is not Pareto efficient.

[4]It is 21.4% in recessions and 12.7% at other times. Quarterly volatility is defined to be the standard deviation of daily returns of the S&P 500 index over the quarter over the period 1962-2009. This difference is highly statistically significant.

[5]Using quarterly data published by the BEA, the volatility of GDP growth in recessions is 4.66% while is is 4.19% at other times over the 1947-2009 time period.

[6]See, for example, Lustig, Van Nieuwerburgh, and Verdelhan (2007).

The papers that first recognized the importance of endogenizing the wage contract, and therefore the ones most closely related to our paper, are Dreze (1989) and Danthine and Donaldson (2002). Like us, Dreze (1989) considers the interaction between a labor and capital market in general equilibrium and focuses on efficient risk sharing. Our point of departure is how we model production — Dreze does not consider the implication of productive heterogeneity. Consequently, there is no natural reason (beyond differences in risk aversion and wealth) for some workers to insure other workers in Dreze's model. Hence the model does not explain limited capital market participation or focus on the return to bear labor risk.

Danthine and Donaldson (2002), like us, explicitly model both labor and financial markets with agent heterogeneity. Their model features investors and workers, but, importantly, Danthine and Donaldson (2002) do not allow workers to invest or investors to work. As such, although limited stock market participation is a feature of their equilibrium, prices do not adjust to induce this behavior. This is a key difference between the two models and is responsible for the stark differences in some of the models' implications. Because equity holders have to be induced, in our model, to hold equity, we get a large equity premium while the equity risk premium in Danthine and Donaldson (2002) is small.[7]

Our paper also contributes to the large literature, that started with Mayers (1972), studying the effect of non-tradeable wealth in financial markets. The main results in that literature are that investors should no longer hold the same portfolio of risky assets and the single factor pricing relation must be adjusted. Although Fama and Schwert (1977) finds little evidence supporting Mayer's model, both Campbell (1996) and Jagannathan and Wang (1997) find that adding a measure of human capital risk significantly increases the explanatory power of the CAPM. Santos and Veronesi (2006) find that the labor income to consumption ratio has predictive power for stock returns and can help explain risk premia in the cross section. The results in this paper demonstrate that the importance of non-traded wealth might be overstated. Furthermore, because wage contracts provide insurance for human capital risk, it is not surprising that wages (the typical measure of human capital risk used in the literature) have explanatory power for stock returns.

Because most of the theoretical predictions of the neo-classical asset pricing models rely on effectively complete markets, initially researchers were tempted to attribute the failure of those models on market incompleteness. However, Telmer (1993) and Heaton and Lucas (1996) convincingly argue that market incompleteness cannot account for the important puzzles such as the apparently high equity risk premium. As we show in this paper, quite the opposite intuition might be true. The failure of the models might stem from the fact that agents actually share risk more completely than is supposed in the literature. If labor markets effectively share risk, then because equity holders are the ultimate insurers of human capita risk, they will demand a

[7]Danthine and Donaldson (2002) can only get a large premium by introducing market frictions that restrict risk sharing.

high risk premium. Because most workers will demand rather than supply this insurance, they will have no reason to participate in equity markets. As we will demonstrate, our results are consistent with the findings in Mankiw and Zeldes (1991) and Brav, Constantinides, and Geczy (2002) in that those who choose not to participate are less wealthy, less educated and more reliant on wage income as their source of wealth. Furthermore, consistent with the anecdotal evidence, the primary motivation for investing in capital markets is the attractive risk return tradeoff offered, not a desire to hedge human capital risk.

2 Model

Like any source of risk, human capital risk has both an idiosyncratic component and a systematic component. Although the idiosyncratic component is likely to be large, especially early in a person's career, we will focus exclusively on the systematic component because we are interested in the implications of how agents share risk in the economy. Idiosyncratic risk, by its very nature, can be diversified away, so there is little reason for any agent to hold this risk in equilibrium. Consequently, the risk sharing implications of sharing idiosyncratic risk are well understood.[8]

Given our objective to study how systemic risk is shared in the economy, our model must include heterogeneous agents. An important source of individual heterogeneity in the economy is worker flexibility: Some workers have more productive options than others. Building on this insight we model productivity as follows. Our economy consists of a continuum of workers that produce a single, perishable, consumption good. Workers produce goods using a productive technology that has a common component and a idiosyncratic component we term a worker's *production technology*. Some workers only have access to a single production technology. Others can choose between production technologies.

Formally, there is a closed set of production technologies, $\mathcal{P} \subset [0,1] \times [0,\overline{K}]$, for some $\overline{K} > 0$. Each *inflexible* agent only has access to a single production technology in this set, (b, f), that has a constant component that does not depend on the state of the economy and a stochastic component that varies linearly in the state variable s. So a particular inflexible worker produces $A_t(b + fs)$ consumption good, where $A_t \equiv e^{rt}$ is the non-stochastic[9] part of individual production common to all agents, and s is a state variable that captures the current state of the economy. The dynamics of A_t are meant to capture overall economic growth and allows us to model recessions has a relative drop in productivity. The production technology set

[8]Although it's not the focus of the paper, Harris and Holmström (1982) makes it clear how agents share idiosyncratic labor risk. The paper shows that most, but not all, of this risk can be removed by the labor contract. Under the optimal labor contract firms insure all agents against negative realizations of idiosyncratic labor risk but agents remain exposed to some positive realizations. Of course, the owners of these firms do not have to expose themselves to this labor risk because by holding a large portfolio of firms, they can diversify the risk away.

[9]It is straightforward to extend our analysis to allow stochastic growth in A_t, as long as innovations in A_t are independent of innovations in s.

have the properties that $(0, \overline{K}) \in \mathcal{P}$, $(1, 0) \in \mathcal{P}$, $(b, \overline{K}) \in \mathcal{P} \Rightarrow b = 0$ and $(1, f) \in \mathcal{P} \Rightarrow f = 0$.

The stochastic process s is a diffusion process on $\mathbb{R}_+$ that summarizes the state of the world:

$$ds = \mu(s)\, dt + \sigma(s)\, d\omega,$$

We will model s as a mean-reverting square root process,

$$ds = \theta(\overline{s} - s)\, dt + \sigma\sqrt{s}\, d\omega, \tag{1}$$

where the condition $2\theta a > \sigma^2$, ensures positivity. The mean-reversion introduces a business cycle interpretation and is useful for calibrations, although, as we shall see, much of the theory goes through for general $\mu(s)$ and $\sigma(s)$ so long as μ and σ are smooth, that σ is strictly positive, and that they satisfy growth conditions $|\mu(s)| \le c_1(1 + s)$, $\sigma(s) \le c_2(1 + s)$ for finite constants, c_1 and c_2. It is natural to define a recession as states for which $s < \overline{s}$, whereas a boom occurs when $s > \overline{s}$.

Let the inflexible agents be indexed by $i \in \mathcal{I} = [0, \alpha]$, where $0 < \alpha < 1$, with agent i working in industry (b_i, f_i), and assume that b_i and f_i are measurable functions that are nondegenerate in the sense that it is neither the case that the full mass of agents work in industry $(0, \overline{K})$, nor in industry $(1, 0)$. Then the total productivity of all inflexible agents in the economy is

$$A_t K_I(s) = A_0 e^{rt} K_I(s)$$

where

$$K_I(s) \equiv \int_{i \in \mathcal{I}} (b_i + f_i s) di = \bar{b} + \bar{f} s. \tag{2}$$

Note that $0 < \bar{b} < 1$ and $0 < \bar{f} < \overline{K}$.

The rest of the agents in the economy are *flexible* agents, comprising mass $1 - \alpha$. These agents have access to any production technology in $\mathcal{P}$ and are free to move between production technologies (industries) at any point in time. So for a given s, it is optimal for them to work in the industry (b^*, f^*), that solves

$$(b^*, f^*) = \underset{(b,f) \in \mathcal{P}}{\arg\max}\ b + fs, \tag{3}$$

leading to the optimal productivity of flexible agents

$$A_t K_F(s) = A_0 e^{rt} K_F(s)$$

where

$$K_F(s) \equiv b^*(s) + f^*(s)s. \tag{4}$$

Notice that, at any point in time, all flexible agents choose to work in industries that generate the same output. We have that

Lemma 1 *The optimal production function of flexible agents satisfies:*

(a) $K_F(0) = 1$*,*

(b) $\lim_{s\to\infty} \frac{K_F(s)}{s} = \overline{K}$*,*

(c) $K_F(s)$ *is a convex function of* s*.*

We next assume that flexible agents can work part time in different industries, i.e., if $(b_1, f_1) \in \mathcal{P}$ and $(b_2, f_2) \in \mathcal{P}$, then $(\lambda b_1 + (1-\lambda)b_2, \lambda f_1 + (1-\lambda) f_2) \in \mathcal{P}$ for all $\lambda \in [0,1]$. This implies that for all $b \in [0,1]$ there is a $(b, f) \in \mathcal{P}$. Now, flexible agents will only consider production technologies on the efficient frontier, $(b, f(b))$, where $f(b) \stackrel{\text{def}}{=} \max_{(b,f)\in\mathcal{P}}\{f : (b, f) \in \mathcal{P}\}$, and it follows immediately that f is a strictly decreasing, concave function defined on $b \in [0,1]$, such that $f(0) = \overline{K}$ and $f(1) = 0$. Going forward, we make the additional technical assumptions that f is strictly concave, twice continuously differentiable, and that $f'(0) = 0$, and $f'(1) = -\infty$. Under these assumptions it is easy to show that

Lemma 2 $K_F(s)$ *is a twice continuously differentiable, strictly convex function, such that* $K_F'(0) = 0$ *and* $\lim_{s\to\infty} K_F'(s) = \overline{K}$*.*

Lemma 2 ensures that $K_F(s)$ is a diffusion process (which is, of course, also true of $K_I(s)$), and the following implications follow immediately:

Lemma 3 *The following results hold for the volatility of the agents' productivity:*

(a) For low s*, the volatility of the flexible agent's productivity is lower than that of the inflexible agent:*

$$Vol\left(\frac{dK_F}{K_F}\right) < Vol\left(\frac{dK_I}{K_I}\right).$$

(b) For high s*, the volatility of the flexible agent's productivity is higher than that of the inflexible agent,*

$$Vol\left(\frac{dK_F}{K_F}\right) > Vol\left(\frac{dK_I}{K_I}\right).$$

From the concavity of $f(\cdot)$, $\bar{f} \le f(\bar{b})$, so flexible agents can be no less productive than inflexible agents. Notice also that the average flexible agent will be relatively more wealthy than the average inflexible agent both for very low values of s and for very high values of s (see Figure 1).

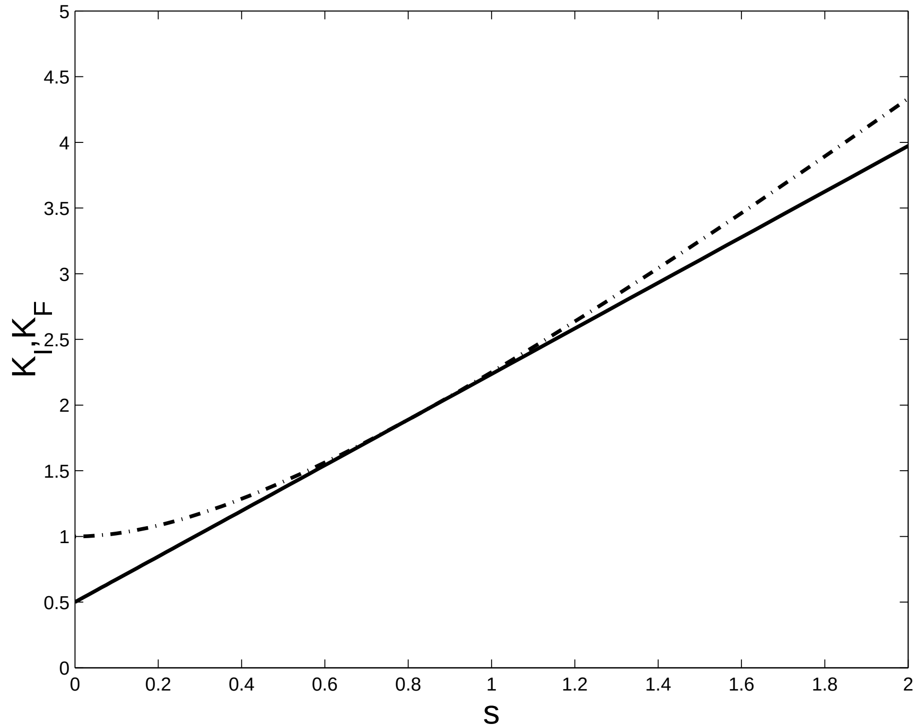

Figure 1: **Optimal production technology functions of flexible and inflexible agents,** $K_F(s)$ **and** $K_I(s)$: In this example, all inflexible agents work in the same industry, so K_I and K_F intersect at the point where flexible agents choose the same productivity as inflexible agents have. If there is dispersion among inflexible agents, K_I lies strictly below K_F, since f is concave.

Total output in the economy at time t is:

$$A_t K_{tot}(s_t) = A_0 K_{tot}(s_t) \tag{5}$$

where

$$K_{tot}(s) \equiv \alpha K_I(s) + (1-\alpha) K_F(s), \tag{6}$$

implying that $K_{tot}(s)$ is also a diffusion process.

Workers and firms are organized as follows. A worker can choose either to work for himself and produce the consumption good, or he can choose to "sell" his production to a firm and earn a wage instead. Firms are specialized — they can only employ workers with a single production technology. Consequently, if a flexible worker decides to switch production technologies, then if she is working for a firm she must quit her job and find a job in the industry she wishes to switch into (or simply work for herself).[10] Workers are also owners — they are free to invest in firms through the capital markets and consume any dividend payments. In equilibrium markets

[10]**?** document that 63% of workers who changed occupations also changed employers.

must clear — all firms must attract enough investment capital to fulfill their wage obligations.

All agents have constant relative risk-aversion (CRRA), with risk-aversion coefficient, $\gamma > 0$, and utility of consumption

$$U_a(t) = E_t \left[\int_t^\infty e^{-\rho(s-t)} u(c_s)\, ds \right]. \tag{7}$$

Here,

$$u(c) = \begin{cases} \log(c), & \gamma = 1, \\ \frac{c^{1-\gamma}}{1-\gamma}, & \gamma \neq 1. \end{cases} \tag{8}$$

3 Complete Markets Equilibrium

We begin by deriving the complete market Pareto optimal equilibrium and then consider how this equilibrium is implemented. First note that because $A_t K_{tot}$ maximizes total output, any Pareto optimal equilibrium must have this output. Under the complete markets assumption a representative investor exists with utility u_r such that the solution to this single representative agent problem is identical to the solution of the multi-agent problem. Moreover, all agents have CRRA utility functions with the same γ, so u_r is also of CRRA form, with the same γ. Thus, in equilibrium, the value of any asset generating instantaneous consumption flow $\delta(s_t, t)dt$, is

$$\begin{aligned} P(s_t) &= \frac{1}{u_r'(A_t K_{tot}(s_t))} E\left[\int_t^\infty e^{-\rho(\tau-t)} u_r'(A_\tau K_{tot}(s_\tau)) \delta(s_\tau, t) d\tau \right] \\ &= K_{tot}(s_t)^\gamma E\left[\int_t^\infty e^{-\hat{\rho}(\tau-t)} K_{tot}(s_\tau)^{-\gamma} \delta(s_\tau, t) d\tau \right], \end{aligned} \tag{9}$$

where $\hat{\rho} = \rho + \gamma r$. Hence, the total value of human capital of all agents of each type (their total wealth) at time $t = 0$ is:

$$W_I \equiv \alpha\, A_0 K_{tot}(s_0)^\gamma E\left[\int_0^\infty e^{-(\hat{\rho}-r)\tau} K_{tot}(s_\tau)^{-\gamma} K_I(s_\tau) d\tau \right], \tag{10}$$

$$W_F \equiv (1-\alpha)\, A_0 K_{tot}(s_0)^\gamma E\left[\int_0^\infty e^{-(\hat{\rho}-r)\tau} K_{tot}(s_\tau)^{-\gamma} K_F(s_\tau) d\tau \right]. \tag{11}$$

Any Pareto optimal equilibrium features perfect risk sharing — all agents' consumption across states have the same ordinal ranking. Moreover, because of the CRRA assumptions, it is well known that a stronger result applies in our equilibrium — all agents' ratio of consumption across any two states is the same. In other words, every agent consumes the same fraction of total output in every state:

$$
\begin{aligned}
c_F(s,t) &= (1-\eta)\ A_t K_{tot}(s) = (1-\eta)\ A_t(\alpha K_I(s) + (1-\alpha)K_F(s)) && (12)\\
c_I(s,t) &= \eta\ A_t K_{tot}(s) = \eta\ A_t(\alpha K_I(s) + (1-\alpha)K_F(s)) && (13)
\end{aligned}
$$

where c_F and c_I are the aggregate consumption of all the flexible and inflexible agents respectively and η is the fraction of the total output consumed by all the inflexible agents. W. From the budget constraint at time 0 it follows that

$$
\eta = \frac{W_I}{W_I + W_F}. \tag{14}
$$

We can also view η as a function of the initial condition, $\eta(s_0)$. The following properties of η follow immediately:

Lemma 4 *The function $\eta(s_0)$, defined by (14) satisfies the following conditions*

(i) *η is twice continuously differentiable.*

(ii) $\eta \in \left[\min\{\frac{1}{1+\frac{1-\alpha}{\alpha}\frac{1}{b}}, \frac{1}{1+\frac{1-\alpha}{\alpha}\frac{K}{f}}\}, \alpha\right]$.

(iii) *η is increasing for small s_0 and decreasing for large s_0.*

(iv) $\lim_{s_0\to\infty}\eta(s_0) = \frac{1}{1+\frac{1-\alpha}{\alpha}\frac{K}{f}}$.

From Lemma 4 it therefore follows that if we define $s^* \stackrel{\text{def}}{=} \min\{\arg\max_{s_0}\eta(s_0)\}$, then $0 < s^* < \infty$. The wealth share of the inflexible agent is thus maximized at a finite value, s^*, and we denote wealth share a this point by $\eta^* \equiv \eta(s^*)$.

We next move to general asset pricing formulas. We have:

Proposition 1 *The price, $P(s,t)$, of an asset that pays dividends $\delta(s,t)$ satisfies the PDE*

$$
\begin{aligned}
&P_t + \left(\mu(s) - \gamma R(s)\sigma(s)^2\right) sP_s + \frac{\sigma(s)^2}{2}P_{ss}\\
&\quad - \left(\hat{\rho} + \gamma\mu(s)R(s) - \frac{\sigma(s)^2}{2}\gamma(\gamma+1)R(s)^2 + \frac{\sigma(s)^2}{2}\gamma T(s)\right)P + \delta(s,t) = 0
\end{aligned}
\tag{15}
$$

where

$$
R(s) = \frac{K'_{tot}(s)}{K_{tot}(s)}, \qquad \text{and} \qquad T(s) = \frac{K''_{tot}(s)}{K_{tot}(s)}. \tag{16}
$$

It is well known that an immediate implication of this proposition is that the instantaneous risk free interest rate is[11]

$$r_s \equiv \hat{\rho} + \gamma\mu(s)R(s) - \frac{\sigma(s)^2}{2}\gamma(\gamma+1)R(s)^2 + \frac{\sigma(s)^2}{2}\gamma T(s). \tag{17}$$

In general, we will need to solve (15) numerically, which may be nontrivial, however, since it is defined over the whole of the positive real line, $s \in \mathbb{R}_+$, and it is not a priori clear what the boundary conditions are (neither at $s = 0$, nor at $s = \infty$, where P_t may become unbounded). We therefore make the transformation, $z \stackrel{\text{def}}{=} \frac{s}{s+1}$, used in **?** to get:

Proposition 2 *The price, $P(s,t)$, of an asset that pays dividends $\delta(s,t)$, where $\delta(s,t) \leq ce^{rt}K_{tot}(s)^\gamma$ for some positive constant c, is*

$$P(s,t) = K_{tot}(s)^\gamma Q\left(\frac{s}{s+1}, t\right),$$

where Q solves the PDE

$$Q_t + (1-z)^2\left(\mu\left(\frac{z}{1-z}\right) - \sigma\left(\frac{z}{1-z}\right)^2 z\right)Q_z + \frac{1}{2}(1-z)^4\sigma\left(\frac{z}{1-z}\right)^2 Q_{zz} - \hat{\rho}Q$$
$$+\delta\left(\frac{z}{1-z}, t\right)K_{tot}\left(\frac{z}{1-z}\right)^{-\gamma} = 0, \tag{18}$$

where $z \in (0,1)$.

The proof of Proposition 2 follows along identical lines as the analysis in **?**, where it is also shown that no boundary conditions are needed at $z = 0$ and $z = 1$, and is therefore omitted. Without loss of generality, we assume that $A_0 = 1$ going forward since all variables are homogeneous of degree zero or one in A_0. All the numerical solutions in this paper were derived by solving (18).

4 Implementation

A key insight in the previous section is that in the complete markets equilibrium, all agents consume a constant fraction of total production. Because inflexible agents' share of total production decreases in bad times, in this equilibrium wealth is transfered from flexible to inflexible agents in bad times, and the reverse happens in good times.[12] In other words, flexible agents provide insurance to inflexible agents. To gain insight into how actual markets, which are far

[11] see **?**, Section x.y

[12] Although we will not focus on this region there will always be a large enough s such that inflexible agents share of production becomes a decreasing function of s. Wealth will also be transfered from flexible to inflexible agents in these (very good) states as well.

from complete, share risk, it is important that we model these markets realistically. Hence we restrict agents' and firms' ability to write and trade contracts in the following ways:

Restriction 1 *(i) Binding contracts cannot be written directly between agents.*

(ii) Firms may enter into binding contracts with agents subject to the following restrictions: (1) Limited liability may not be violated. (2) Workers and equity holders cannot be required to make payments,

(iii) Banks may enter into short term debt contracts with agents and firms, paying an interest rate r_s.

These restrictions reflect the practical limitations of markets. Because individualized binding contracts cannot trade in anonymous markets a matching mechanism does not exist that would allow for widespread use of bilateral contracts as a risk sharing device. Perhaps because there are far fewer firms than agents in the economy, so it is easier to match firms and agents, we do observe binding bilateral labor contracts written between agents and firms. However, even these contracts are limited. Both equity and labor contracts are one sided in the sense that typically firms commit to make payments to agents. Agents very rarely commit to make payments to firms and courts rarely enforce such contracts. The only condition under which agents can enter a contract that commits them to make payments is if they take a loan from a bank. Both firms and agents can either borrow or lend from a bank subject to the condition that in equilibrium the supply of loans must equal the amount of deposits. Thus the span of traded assets consists of debt and equity. As we will see, there is no default in equilibrium so the interest rate banks pay is the risk free rate.

We also impose the following restriction on the industries in which a firm may operate

Restriction 2 *Firms are restricted to operate in only one industry. That is, all workers in a firm must have the same b and f.*

In reality, most firms operate in a single industry. Although conglomerates do exist, even these firms typically operate in only a few industries. Our results would not change if we allowed firms to operate in finitely many industries. What we cannot allow is a firm that operates in *every* industry.

We assume that there is a (very) small cost to dynamic trading in capital markets.

Restriction 3 *Dynamic trading in equity markets imposes a utility cost of $\epsilon = 0^+$ per unit time.*

This restriction captures transaction costs of active trading, as well as the utility cost of designating time and effort to active portfolio rebalancing strategies. Technically, the condition is needed to ensure uniqueness of the equilibrium outcome, since it implies that an equilibrium outcome that does not require active portfolio trading in asset markets dominates an equilibrium that is identical in real markets, but that *does* require active portfolio trading. We do not impose any transaction costs of switching jobs, although it can be argued that such costs are also present, and in fact may be higher than the costs a dynamic trading in asset markets. The reason we do not do this is that it would not change our results qualitatively, although the analysis would be less tractable[13]

We are now ready to describe how the complete markets equilibrium can be implemented under these restrictions. At first glance it might appear as if asset markets are unnecessary. After all, we allow firms to write bilateral contracts with agents, so by serving as an intermediary, firms can effectively allow agents to write bilateral contracts between themselves. For example, firms could hire both types of workers, pool their production and reallocate it by paying wages equal to a constant fraction of the total. However, such contracts alone cannot implement the Pareto optimal equilibrium. The reason is that in such an equilibrium, although risk is efficiently shared conditional on production, total production is not maximized — flexible workers must switch industries in order to maximize their production. But the only way for the firm to pool production and reallocate it would be to extract a commitment of lifetime employment from flexible workers. Such a commitment is suboptimal. Hence, both labor and asset markets are required to implement the complete markets equilibrium.

To achieve the complete market equilibrium all inflexible agents sign a binding employment contract with firms in the industry of their specialty that commits both parties to lifetime employment. Agents give up all their productivity and in return receive a wage equal to their Pareto optimal equilibrium allocation, $\eta(s_0)\ A_t K_{tot}(s)$, in every future state s. Flexible agents either choose to work for themselves, or work for firms and earn wages equal to their productivity. This means that in some states inflexible wages will exceed productivity. Because firms cannot force investors to make payments, firms require capital to credibly commit to the labor contract. They raise this capital by issuing limited liability equity. In states in which wages exceed productivity, the firm uses this capital to make up the shortfall and does not pay dividends. For the moment we will restrict attention to states in which the capital in the firm is positive.

Flexible agents purchase the equity by borrowing the required capital from the bank. Firms then redeposit the capital in the bank (ensuring that the supply of deposits equals the demand for loans) and pay instantaneous dividend flow equal to

$$A_t \max(\alpha K_I(s) + C_t r_s - \eta_0 K_{tot}(s), 0)$$

[13]The key point here is that, even with small friction costs of job-switching, it will still be optimal —for the flexible agent to switch industries (and for the inflexible agent to stay in the same industry) anyhow, since the productivity loss of not switching is a first order effect. The only difference is that he will switch industries less often when the friction is present.

where C_t is the amount of capital owned by the firm at time t and $\eta_0 \equiv \eta(s_0)$. Thus flexible agents consume

$$A_t \left[(1-\alpha)K_F(s) + \max(\alpha K_I(s) + C_t r_s - \eta_0 K_{tot}(s), 0) - C_t r_s\right] \tag{19}$$

where we assume (and later show) that flexible agents always choose to adjust their bank loans to match the amount firms deposit in the bank. Using (6), when dividends are positive, the term in square brackets in (19) becomes

$$(1-\alpha)K_F(s) + \alpha K_I(s) + C_t r_s - \eta_0 K_{tot}(s) - C_t r_s = (1-\eta_0)K_{tot}(s) \tag{20}$$

so flexible agents consume their complete market allocation. Similarly, when dividends are zero we get

$$(1-\alpha)K_F(s) - C_t r_s + \frac{dC_t}{dt}. \tag{21}$$

Now the change in firm capital equals the shortfall, that is,

$$dC_t = (\alpha K_I(s) + C_t r_s - \eta_0 K_{tot}(s))dt.$$

Substituting this expression into (21) gives

$$(1-\alpha)K_F(s) - C_t r_s + (\alpha K_I(s) + C_t r_s - \eta_0 K_{tot}(s)) = (1-\eta_0)K_{tot}(s). \tag{22}$$

So the flexible agent consumes his complete markets allocation in every state in which the firm's capital is positive.

Finally, consider the first time that either the value of the firm drops to zero or the firm's capital drops to zero. In such a state the firm can raise additional capital by issuing new equity (either by repurchasing existing equity for zero and issuing new equity to raise capital, or if the equity is not worth zero, issuing new equity at the market price). Hence by always issuing new capital in this state, the firm can ensure that neither its capital, nor its value, ever drops below zero and that it never pays negative dividends. Thus in this equilibrium both agents always consume their complete markets allocation which is Pareto optimal. This implies that flexible agents cannot be better off by following a different borrowing policy, justifying our assumption that they will always choose to borrow the amount firms deposit in the bank. Moreover, since this outcome implies a passive investment strategy for inflexible, as well as flexible, agents, this equilibrium implementation is the uniquely optimal one, since any other implementation leads to active rebalancing.

The following proposition summarizes these results:

Proposition 3 *The unique implementation that leads to the complete market Pareto efficient* *outcome is as follows:*

- *Flexible workers either work for themselves or a firm which pays the instantaneous wage* $w_F = K_F(s_t)$.
- *Inflexible workers work for publicly traded firms, which pay instantaneous wages equal to a constant multiple of aggregate production. In aggregate, firms pay the inflexible wage*

$$w_I = \eta_0 K_{tot}(s_t).$$

- *In states in which inflexible productivity plus interest on bank deposits exceeds wages, firms pay dividends equal to*

$$\alpha K_I(s) + C_t r_s - \eta_0 K_{tot}(s) \tag{23}$$

  and retain capital C_t *with* $dC_t = 0$

- *In states in which inflexible productivity plus interest on bank deposits does not exceed wages, firms pay no dividends and reduce capital to make wage payments*

$$dC_t = (\alpha K_I(s) + C_t r_s - \eta_0 K_{tot}(s))dt \tag{24}$$

- *The flexible workers own all the equity in the stock market. They pay for this equity by borrowing the capital from banks. Firms redeposit the capital in banks. Flexible workers optimally adjust their borrowing to ensure that at all times the supply of deposits equals the demand for loans.*
- *Whenever: (1) the price of the firm drops to zero the firm raises new capital by repurchasing old equity for nothing and issuing new equity or (2) the amount of capital drops to zero the firm raises new capital by issuing new equity at the market price.*

There are three important distinguishing characteristics of this solution that reflect reality. First, it features limited capital market participation — only flexible workers participate in capital markets. Without understanding the importance of the labor market, one might naively look at inflexible workers' wealth and conclude that because this wealth is non-tradeable, they would be better off using asset markets to hedge some of this exposure. While it is true that most wealth in our model is non-tradeable, what is not true is that workers are better off hedging this wealth in asset markets. In equilibrium inflexible workers choose not to further hedge their human capital risk exposure because it is too costly. Flexible workers choose to hold equity, not because of a desire to hedge — they choose to increase the riskiness of their position — but because of the compensation they receive in terms of a high equity risk premium.

The implication, that inflexible workers choose not to participate in markets is consistent with one of the most robust findings in the literature — that wealth and education are positively correlated with stock market participation (see Mankiw and Zeldes (1991)). Clearly, flexible workers are wealthier in our model, but more importantly, if productive flexibility derives from education, then they are likely to be better educated. In fact, Christiansen, Joensen, and Rangvid (2008) show that the degree of *economics* education is casually (positively) related to stock market participation. They interpret this result as evidence that non-participation derives from informational barriers to entry. But their results are also consistent with flexibility. Not all education provides productive flexibility so we would expect to see variation in the type of education and stock market participation. Their study clearly documents this variation. Finally, note that non-participation in capital markets implies that inflexible works also do not hold bonds, that is, they choose not to save. This result might help to explain the low savings rate observed in the U.S. — the reason workers choose not to save is that their labor contracts effectively do the saving for them.

A second distinguishing characteristic of our solution is that firm equity can be thought of as an option on total consumption. We therefore expect the volatility of equity returns to exceeds the volatility of total consumption. Because we do not have idiosyncratic risk in our setting, this volatility imparts risk — equity is considerably more risky than total consumption. Consequently, the equity risk premium exceeds the consumption risk premium — the risk premium of a security that is a claim on total consumption.

That equity can be viewed as an option is well known. However, normally this insight is derived using financial leverage. In our case the firm has no debt, indeed it actually holds cash. In a standard setting this would mean that equity would not have option characteristics, indeed, because of the cash, equity would be *less* risky than the firm's assets. In our setting, it is not financial leverage that gives equity option like characteristics, but the operating leverage resulting from wage commitments. Notice that this operating leverage is considerably more risky than the typical kind of operating leverage studied in the literature. Typically, firms have the option to shut down — if a firm is losing money on the margin then it can reduce its scale or shut down altogether. However, in our case firms optimally choose to give up this option — they commit to continue to pay wages even when, ex post, the value maximizing decision would be to shut down and pay out the remaining capital to equity holders. As we will demonstrate in the next section, by giving up this option, the firm substantially increases the risk of its equity.

Two important challenges to the neoclassical asset pricing model are its inability to produce a realistic equity risk premium and the seemingly large disparity between consumption and equity volatility. In our setting (in which the consumption CAPM holds) equity is more volatile than consumption and therefore commands a risk premium. Our model therefore has the potential to meet these challenges. Consequently, in the next section, we investigate whether, for realistic parameter values, our model can reproduce the magnitude of the observed empirical anomalies.

Finally, note that asset returns will vary with the business cycle in a highly nonlinear fashion, because of the role of firms as insurers. This means that the unconditional link can look quite weak even though the instantaneous correlations of real variables (consumption) and asset returns are perfect. In fact, as we shall see, our results are in line with the findings in Duffee (2005), that consumption and equity returns are only weakly related in bad times, but are highly correlated in good times.

5 Parameterization

Parameter			Moment Values at s_0		
Variable	Symbol	Value	Variable	Symbol	Value
Inflexible constant production	$\bar{b}$	0.5	Consumption Growth		1.2%
Flexible Limiting Production Sensitivity	$\overline{K}$	2.5	Risk Free Rate	r_s	4.2%
Impatience Parameter	ρ	0.5%	Firm Expected Return	r_e	8.3%
Risk Aversion	γ	8.5	Firm Volatility	σ_p	16.6%
Consumption Growth	r	1%	Consumption Volatility	σ_c	4.3%
% of population Inflexible	α	67%	Wealth fraction	η, η^*	65%
Inflexible variable production	$\overline{f}$	1.736	Initial Capital	C_0	0.015
State Variable Volatility	σ	6%	HJ bound		2.4%
Initial s	s_0, s^*	2	Probability $s < 2$		96%
Mean reversion speed	θ	0.0036	Unconditional correlation	ρ_{pc}	0.40
Long-term mean	$\bar{s}$	0.67	Equity premium	$r_e - r_s$	4.2%

Table 1: **Parameter Values**

The objective of this section is to show that the effects we have identified are first order, that is, large enough to potentially explain the empirical facts. We will demonstrate that for a set of realistic parameter values (summarized in Table 1), the model can match the magnitude of the important moments in the data. Although we view this as an achievement in itself — without the labor market, the model in this paper, essentially a Lucas tree economy, could not come close to matching the important moments in the data — deciding whether labor markers really do explain the empirical puzzles such as limited market participation will require further empirical investigation.

We assume that flexible workers make up 1/3 of the working population, implying that 2/3 of the population choose not to participate directly in capital markets, in line with the estimates reported in **?**, Guiso, Haliassos, Jappelli, and Claessens (2003), Hong, Kubik, and Stein (2004), and Christiansen, Joensen, and Rangvid (2008). Assume that f has the form

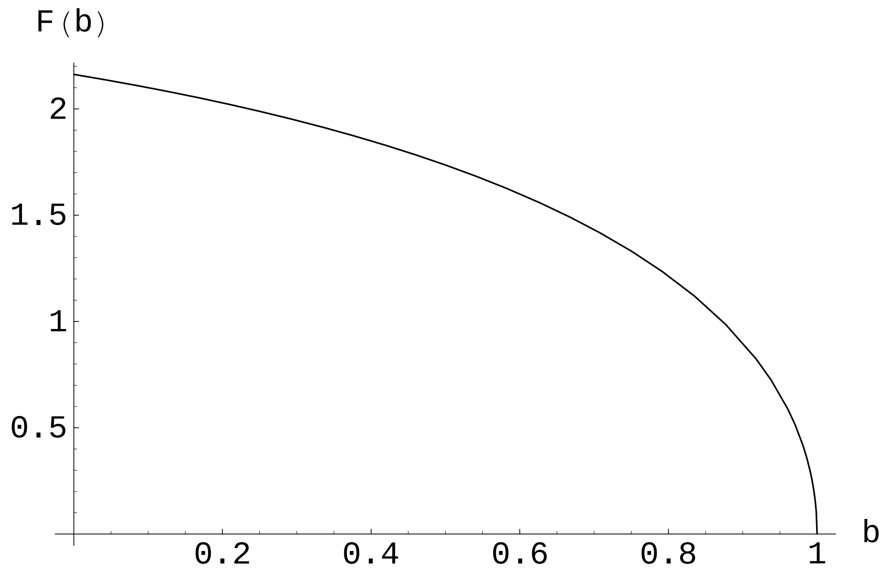

Figure 2: $f(b)$**:** Flexible Agents' optimal choice of f for a given choice of b

(shown in Figure 2):

$$f(b) = \frac{(1-b)\left(3\overline{K}\sqrt{\overline{K}-b\overline{K}}-(1-b)\sqrt{\overline{K}-b\overline{K}}+2\overline{K}^2\right)}{\left(\sqrt{\overline{K}-b\overline{K}}+(1-b)\right)\left(\sqrt{\overline{K}-b\overline{K}}+\overline{K}\right)}. \tag{25}$$

It is easy to see that f satisfies the required conditions, leading to the total production of flexible agents

$$K_F(s) = 1 + \overline{K}\frac{s^2}{s+1}. \tag{26}$$

We assume that flexible workers' limiting productive sensitivity to the state variable, $\overline{K}$, is 2.5 and that inflexible workers all work in an industry with $b = \bar{b} = 0.5$, and $f = \overline{f} = 1.736$.[14] The total production of inflexible agents is then

$$K_I(s) = 0.5 + 1.736s. \tag{27}$$

The two production functions are plotted in Figure 1.

We set the initial s equal to s^*, $s_0 = s^*$, ensuring that it is never optimal for the inflexible workers to quit the firm. s evolves according to (1), with parameter values $\theta = 0.0043$, $\sigma = 6\%$ and $\overline{s} = 0.67$. The economy is thus in a recession when $s < 0.67$ and when $s > 0.67$ it is in a boom. With these parameters, the unconditional probability that $0 < s_t < 2.5$ is 99%, and the

[14]Note that here we assume that $f(\bar{b}) = \overline{f}$ implying that a state exists in which all workers work in the same industry.

probability that $0 < s_t < 2$ is 97%, so we focus on this range. The growth rate of the economy is $r = 1.2\%$. This implies an unconditional consumption growth volatility of about 4%, which is in line with what was used in Mehra and Prescott (1985). We use standard values for the preference parameters. We pick a relative risk aversion coefficient of 8.5, within the range Mehra and Prescott (1985, p. 154) consider reasonable, and impatience parameter $\rho = 0.5\%$.

We solve for the complete markets equilibrium using (18) to derive the fraction of total consumption each agent type consumes in aggregate. To compute W_i, we set $\delta = K_i(s)$ for each agent type $i \in \{F, I\}$ in (15).[15] In Figure 3 we show the initial instantaneous relative productivity, $\frac{K_I}{K_{tot}}$, and the wealth share, η of the inflexible agents, as a function of s_0. We see that $\frac{K_I}{K_{tot}}$, reaches its maximum, α (i.e., 2/3), at $s_0 \approx 0.7$, whereas $\eta^* \approx 0.64$ occurs at $s_0 = s^* \approx 1.8$. The reason η is maximized to the right of the point where the inflexible agent's wealth share is maximized is that the value of insurance is greater in the bad states than the good states. Hence inflexible agents are willing to pay more for insurance at the point where their wealth share is maximized than point to the right, so *eta* continues to increase.

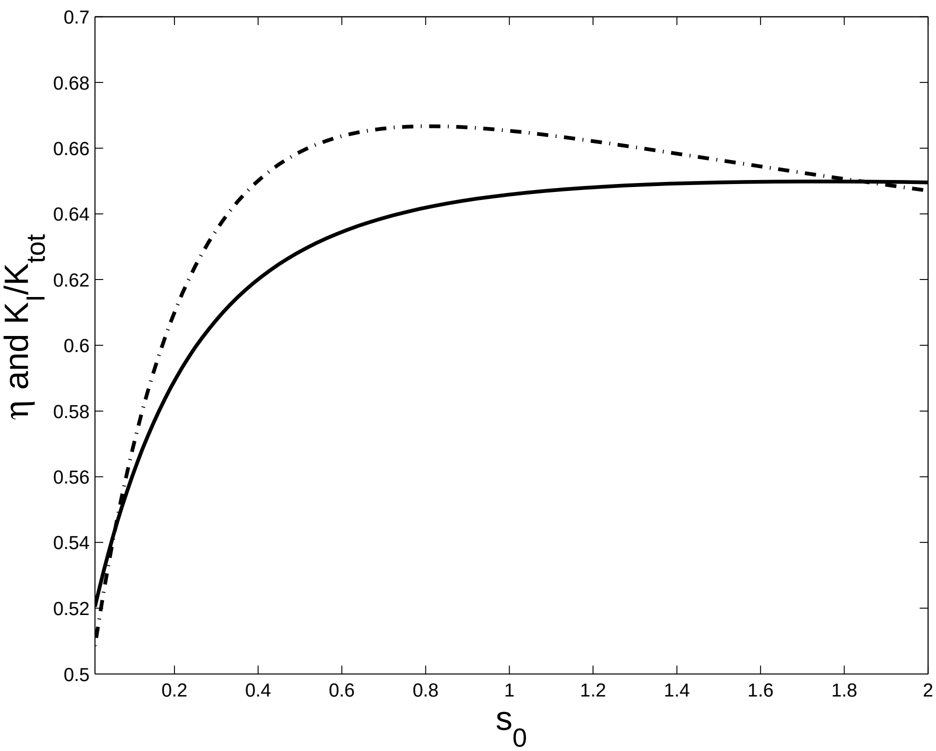

Figure 3: **The share of wealth of the inflexible worker at $t = 0$, η, as a function of s_0.**

[15]Notice that $K_i(s)$ is time independent, so (18) degenerates into an ODE, and the price function is also time independent. It is easy to show that the boundary condition at $z = 1$ for the ODE is $Q(1) = 0$. We know of no simple boundary specification at $z = 0$, however, since the boundary is not absorbing for the square root process. Therefore, we solve the PDE and let T become so large that the solution becomes effectively time independent.

The initial capital the firm raises is arbitrary in our model — because we have an effectively complete asset market the Modigliani-Miller proposition implies that the firm's capital structure is irrelevant. Of course, in a world with frictions the amount of capital raised will affected by a tradeoff between the benefits (e.g., more capital implies fewer visits to the capital markets and thus lower transaction costs) and costs (e.g. holding cash increases taxes and agency costs). We pick a level of initial capital to match the price volatility of the market, which we set to 16.6%. This leads to initial capital of $C_0 = 0.015$. The parameters and results are summarized in Table 1.

The dividends and price function are shown in Figure 5. We note that D and P are nonnegative within the range, and therefore there is no need for refinancing. For $s > 2$, there will be a point at which dividends turn negative, so that refinancing is needed at some point. This is a rare event, however. Note that the value of the firm, $V(s_t, C_t)$ is equal to the amount of cash plus the value of inflexible worker productivity minus the value of the wage commitment:

$$V(s_t, C_t) = \alpha W_I(s_t) + C_t - \eta W_{tot}(s_t) \tag{28}$$

where $W_{tot}(\cdot)$ is the total value of production. We note that the price function is highly nonlinear in s. It is quite flat — and even slightly decreasing — for very low s, it then increases rapidly in a convex fashion for low to high s, after which it reaches a maximum for very high s and becomes decreasing. The reason for this behavior is that the stock market's main role is *not* to distribute the value generation to its owners, but rather as an insurance vehicle. The price function is decreasing with D for very low s, since dividends are decreasing in s. Dividends, in turn, are decreasing, since an increase in s has a quite marginal effect on productivity (of which most is paid out through wages), whereas interest rates are actually decreasing in s for low values, so an increase in s decreases the size of of interest payments on C_0 capital. The key point though is that insurance motive makes the relation between prices and productivity (and thereby consumption) complicated.

Because of this nonlinearity of the price function, the firm's risk premium varies in the state variable the equity premium will be highly variable. To see this explicitly, note that instantaneous expected equity return is

$$r_e dt = \frac{E[dV]}{V} + \frac{D}{V}dt = \left(\frac{\mu s V' + \frac{\sigma^2}{2}s^2 V''}{V} + \frac{D}{V}\right)dt. \tag{29}$$

Using (28), Figure 5 plots this function, together with the risk-free rate. We see that the equity return is countercyclical, i.e., higher for lower s, except for very low s where it is nonmonotone. At the mean point, $s = 0.67$ expected stock returns are about 8%, which is close to the unconditional expected returns of 8.3%. The risk-free rate is increasing in s for high s — as we would expect — but is decreasing in s for low s. This is an artifact of the square root process: When s

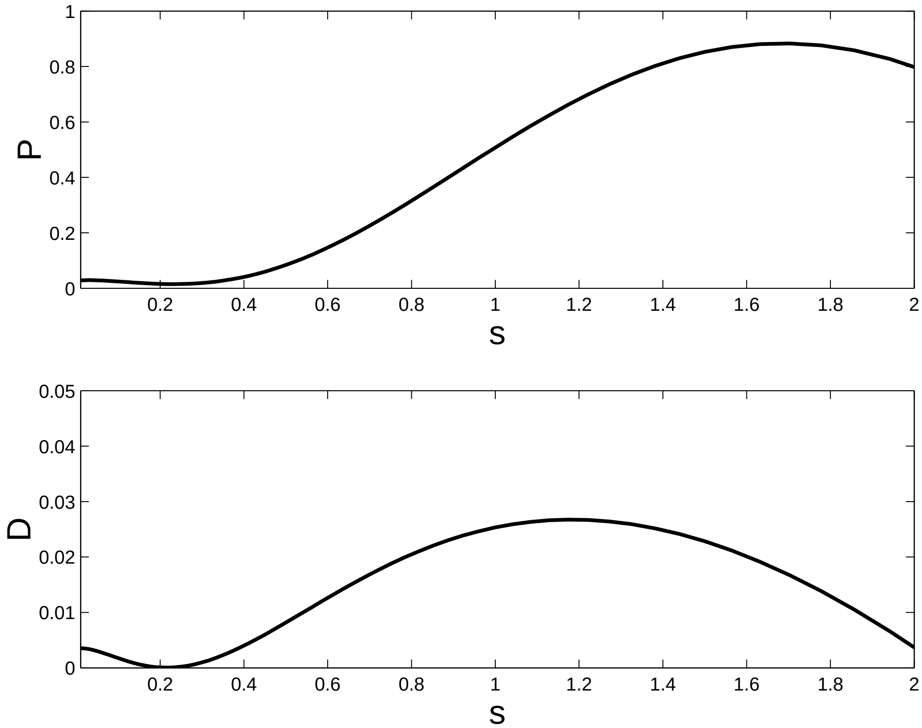

Figure 4: **Value of stock market (upper figure) and total dividends (lower figure) as a function of s.**

is very low, it is expected to grow very quickly which, due to consumption smoothing motives, makes investors want to consume immediately. Risk-free rates then adjust upwards to balance the demand and supply of instantaneous consumption, leading to high short-term rates. We conjecture that this artifact would disappear with a more slowly mean-reverting process in the bad states. The unconditional expected risk-free rate is 4.2%, implying an unconditional equity premium of $8.3\% - 4.2\% = 4.1\%$.

Notice that in good times the firm is not very risky. During these times, inflexible workers earn less than what they produce, and this difference, the insurance premium is paid out as a dividend to equity holders (flexible workers). In bad times, inflexible workers collect on their insurance, earn more than they produce and dividends cease. The firm finances the shortfall by using up its capital, imposing losses on its equity holders, and increasing the risk that its equity holders will be completely wiped out and the firm will need to refinance. The result is a very large risk premium.

The nonlinearity in the price function weakens the link between consumption and returns. For low s, since the price moves very little, and even decreases, with s empirical estimates of the correlation between returns and consumption growth may produce low numbers, even

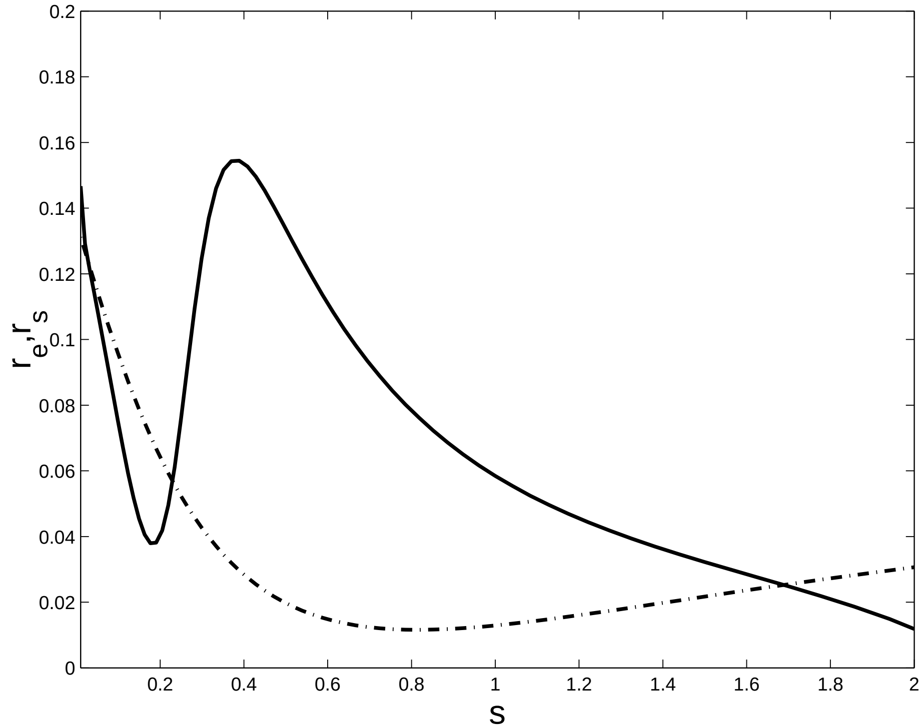

Figure 5: **Expected Return: Expected return on the firm's stock (solid blue line) and the (short) risk free rate (dotted red line).**

though the instantaneous correlation is perfect. This is in line with the results in Duffee (2005) that the correlation between stock returns and consumption growth is low in bad times (about 0), whereas it is high (about 0.6) in good times. The estimated correlation in our calibration for $s < 0.67$ is 0.23, whereas the estimated correlation for $s > 0.67$ is 0.81. The estimated unconditional correlation is 0.40. This also has implications for the Hansen Jagannathan (1989) bounds for the equity premium. Recall that the Hansen and Jagannathan bounds state that $r_e - r_s \leq \rho_{pc}\gamma\sigma_c\sigma_p$. Given our estimates, the right hand side is $0.40 \times 8.5 \times 4.3\% \times 16.6\% = 2.4\%$, which seems to be in violation with an equity premium of 4.1%. The violation occurs, since the correlation is perfect at each point in time, but the unconditional estimate implies only a weak link.

It is interesting to contrast our results with those of Danthine and Donaldson (2002). Although Danthine and Donaldson (2002) also model the effect of labor markets on asset prices, they are unable to match the equity risk premium in a model without frictions.[16] By introducing

[16] They introduce two frictions in their model: (1) Adjustment costs, which provides only a modest increase in the risk premium and (2) changes in the "bargaining power" of workers and investors that effectively prevents perfect risk sharing.

frictions they are able to match the risk premium, but at the cost of perfect risk sharing. In particular, to get a significant risk premium, their restriction limiting market participation becomes important. Without it, workers would invest in markets and thereby increase risk sharing. Not only would the equilibrium not feature limited stock market participation, but the ability to share additional risk would likely reduce the risk premium. Because we do not impose limited capital market participation, *prices* adjust to induce investment. In effect, flexible workers must be *induced* to take on additional risk in equilibrium. That implies that the return on equity (the means by which flexible workers take on this risk) has to be high enough to induce this behavior. This is the key insight in our model — rather than a place to hedge risk, asset markets are a place where investors are induced to take on extra risk.

Because we do not have idiosyncratic risk in our model, an increase in the risk premium must be associated with an increase in volatility. Figure 6 confirms this insight. The volatility of the firm explodes in bad states. More importantly, the volatility of consumption growth is virtually unaffected by the level of s. This figure, more than any other, makes clear how the interaction between labor and financial markets can dramatically affect the inferences of the neoclassical asset pricing model. In our model the consumption CAPM holds by construction. In line with the empirical evidence referenced in the introduction, consumption growth volatility is low and virtually unaffected by the business cycle. Asset volatility, on the other hand, is significantly higher and is very sensitive to the business cycle. So our model actually delivers an almost complete disconnect between consumption volatility and asset volatility, something that is traditionally regarded as perhaps the greatest challenge to the neoclassical asset pricing model.

6 Conclusions

Our objective in this paper is to demonstrate the potential importance of explicitly modeling labor markets within the neoclassical asset pricing model. We show that the inclusion of labor markets has the potential to explain some of the most important normative challenges faced by the model: (1) limited asset market participation, (2) the seemingly high equity risk premium, and (3) the very large disparity in the volatility of consumption and the volatility of asset prices.

Our model also sheds light on a related puzzle in asset pricing — the "narrowness" of the asset span. Traditionally economists have struggled to understand why the asset span of marketed assets does not endogenously expand given the historically high estimates of the fraction of wealth that is not traded.[17] In fact, the financial innovation that does take place almost exclusively consists of financial products that are already spanned by existing assets. Thus, it is hard to argue that the primary motivation for financial innovation is a desire to

[17] Lustig, Van Nieuwerburgh, and Verdelhan (2007) estimate that human capital comprises almost 90% of wealth.

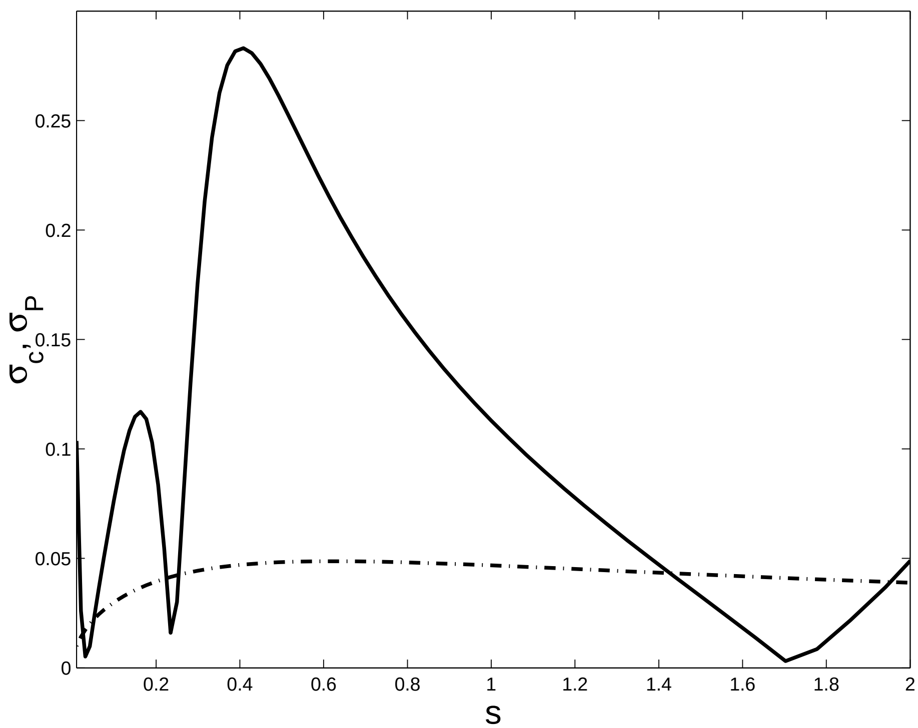

Figure 6: **Volatility: The blue solid curve is firm volatility ($\frac{V'}{V}\sigma(s)$), and the red dotted curve is consumption growth volatility ($\frac{K'_{tot}}{K_{tot}}\sigma(s)$).**

increase the span of traded assets. In this paper we demonstrate that by ignoring the role of labor contracts, financial economists overestimate the amount and importance of non-traded wealth. The reason for the absence of financial contracts that allow agents to trade part of or all of their non-tradeable wealth is not that these contracts are difficult to write or enforce because of frictions such as moral hazard concerns, but that they are not needed. What might appear initially to be non-traded wealth is in fact wealth that results from trading in labor markets.

Although our model is stylized, with one important exception, it captures most of the salient features of labor and asset markets. The one restriction that is common in labor markets that we do not impose in our model is that agents cannot commit to lifetime employment. In reality, although firms often commit to employ agents, rarely do agents *explicitly* commit to stay with firms.[18] But what firms do do is effectively allow agents to commit by offering them deferred compensation contracts. Examples include pension funds and stock vesting periods. In addition, workers themselves often negotiate contracts that increase the costs of switching jobs and thereby

[18]Notice that in our parameterization in Section 5, because $s_0 = s^*$, agents do not actually need to commit to lifetime employment, and so for this parameterization, we can impose the restriction without affecting the equilibrium.

allowing them to credibly commit to staying with their current employers. For example, many union contracts explicitly tie wages to seniority with the firm, making a job switch very costly. Finally, the costs to search for a new job also acts as a commitment device — if the gains to switching jobs is less the search costs, workers can effectively commit to lifetime employment.

Notice that while it is socially optimal for flexible workers to switch jobs, it is suboptimal for inflexible workers to do so (because by not be able to commit to lifetime employment risk sharing is reduced). Hence one might expect to see differences in the costs of switching jobs because incentives exist to lower the switching costs for flexible workers and increase these costs for inflexible workers. This observation might explain why, for example, head hunters tend to specialize in managerial jobs (where the switching costs are small and the skill set can be readily transfered across industries) while it is quite common for unions to negotiate contracts that explicitly reward tenure thereby increasing the costs of switching jobs.

Appendix

Proof of Lemma 1: (a) Follows since f is decreasing and $f(0) = 1$.

(b) Clearly, $Ks \leq C_F(s) \leq Ks + 1$ for all s, since the lower bound can be realized by choosing $b(s) = 1$, and the upper bound follows from the constraint that $b \leq 1$. (b) therefore immediately follows.

(c) Follows since $b + f(b)s$ is (weakly) convex as a function of s for each b and the maximum of a set of convex functions is convex. ∎

Proof of Lemma 2: The flexible worker solves $\max_{b\in[0,1]}\ b + f(b)s$. The first order condition is $f'(b) = -\frac{1}{s}$, and since f' is a continuously differentiable, strictly decreasing, mapping from $[0,1]$ onto $(-\infty, 0]$, the implicit function theorem implies that there is a unique, decreasing, continuously differentiable solution to the first order condition, $b^*(s)$, such that $b^*(0) = 1$ and $\lim_{s\to\infty} b^*(s) = 0$. Since the second order condition is $f''(b)s < 0$, this function indeed yields the maximal strategy, $K_F(s) = b^*(s) + f(b^*(s))s$.

Now, $K_F'(s) = b^{*\prime}(s) + b^{*\prime}(s)f(b^*(s))s + f(b^*(s)) = 0 + f(b^*(s))$, so $K_F'(0) = f(b^*(0)) = f(1) = 0$, and $K_F'(\infty) = f(b^*(\infty)) = f(0) = \overline{K}$. Moreover, $K_F''(s) = b^{*\prime}(s)f'(b^*(s)) = -\frac{b^{*\prime}(s)}{s}$, which is continuous and positive, so K_F is indeed strictly convex and twice continuously differentiable. ∎

Proof of Lemma 3: We have

$$Vol\left(\frac{dK_F}{K_F}\right)^2 = \left(\frac{K_F'}{K_F}\sigma s\right)^2 dt,$$

whereas

$$Vol\left(\frac{dK_I}{K_I}\right)^2 = \left(\frac{K_I'}{K_I}\sigma s\right)^2 dt = \left(\frac{\overline{f}}{\overline{f} + \overline{b}s}\sigma s\right)^2 dt.$$

From Lemmas 1 and 2, $\frac{K_F'}{K_F}$ converges to 0 for small s, whereas $\frac{\overline{f}}{\overline{f}+\overline{b}s}$ converges to 1, so the first inequality holds. ∎

For large s, we have

$$\frac{dK_I}{K_I} = \frac{\overline{f}}{\overline{f} + \overline{b}s} = \frac{1}{1 + \frac{\overline{b}}{\overline{f}}\frac{1}{s}}\sigma.$$

Moreover, from Lemma 2 it follows that

$$\frac{dK_F}{K_F} = \frac{f(b^*(s))}{b^*(s) + f(b^*(s))s}\sigma s = \frac{1}{1 + \frac{b^*(s)}{f(b^*(s)}\frac{1}{s}}\sigma.$$

Now, the inequality therefore follows if $\frac{b^*(s)}{f(b^*(s)} < \frac{\overline{b}}{\overline{f}}$, but since $b^(s) \to 0$, and therefore $f(b^*(s)) \to \overline{K}$, this inequality indeed is satisfied for large s. The lemma is proved. ∎

Proof of Proposition 1: From (9), the price of a general asset in this market, paying an instantaneous dividend stream, $g(s,t)dt$, at t, where g is a continuous function satisfying the growth conditions given in the proposition, is

$$P(s) = K_{tot}^\gamma\, E\left[\int_0^\infty e^{-\rho t}\frac{g(s,t)}{K_{tot}(t)^\gamma}dt\right] \stackrel{\text{def}}{=} K_{tot}^\gamma Q(s,0), \tag{30}$$

where

$$Q(s,t) \stackrel{\text{def}}{=} \int_t^\infty e^{-\rho(\tau - t)}\frac{g(s,\tau)}{K_{tot}(\tau)^\gamma}d\tau.$$

From Feynman-Kac's formula (see, e.g., Karatzas and Shreve (1991)) it follows that $Q \in C^{1,2}(\mathbb{R}_+ \times \mathbb{R}_+)$, and that Q satisfies the PDE

$$Q_t + \mu(s)Q_s + \frac{\sigma(s)^2}{2}Q_{ss} - \rho Q + \frac{g}{K_{tot}^{\gamma}} = 0. \tag{31}$$

Since K_{tot} is smooth, it follows that P is also smooth and since $Q' = \frac{P'}{K_{tot}^{\gamma}} - \gamma\frac{PK'_{tot}}{K_{tot}^{\gamma+1}}$ and $Q'' = \frac{P''}{K_{tot}^{\gamma}} - 2\gamma\frac{P'K'_{tot}}{K_{tot}^{\gamma+1}} - \gamma\frac{PK''_{tot}}{K_{tot}^{\gamma+1}} + \gamma(\gamma+1)\frac{P(K'_{tot})^2}{K_{tot}^{\gamma+2}}$. Plugging the expressions into (31), and defining $R(s) = \frac{K'_{tot}}{K_{tot}}$, and $T(s) = \frac{K''_{tot}}{K_{tot}}$, we arrive at (15).

■

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