Carry Trades and Speculative Dynamics[*]

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Abstract

We develop a model of foreign exchange trading with imperfect liquidity. Speculators have a collective impact on prices through trades and their margin requirements fluctuate with market conditions. Such circumstances can turn carry trades into self-enforcing arbitrage opportunities: Carry trades generate all the more value because many speculators enter them. As a result, rational speculation destabilizes the exchange rate. Applying techniques from dynamic coordination games, we obtain a unique equilibrium exchange rate with high conditional skewness. Namely, extended periods of slow depreciation of the low rate currency are followed by abrupt reversals. Reversals are stochastic, but their distribution is uniquely determined by the distribution of the fundamentals.

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Currency carry trades consist in selling a low interest rate currency to fund the purchase of a high interest rate currency - that is, in selling currencies forward that are at a significant forward premium. The “yen carry trade” in particular has been a topical subject of debate over the last decade or more given the extended period of low interest rates in Japan. Carry trades aim at exploiting the failure of uncovered interest parity - the fact that low interest rate currencies fail to appreciate over time relative to high interest rate currencies. Profits from carry trades stem partly from the interest rate differential, but also from the subsequent depreciation of the low interest rate currency. In a recent study, Burnside et al. (2006) find that currency carry trades generate high Sharpe ratios that do not seem to correspond to a compensation for a variety of risk factors. More generally, from the perspective of asset pricing theory, the forward discount bias is by and large an anomaly. Deriving it from pure risk to consumption arguments has proven difficult for a whole range of ”plausible” preferences. Lustig and Verdelhan (2007) find that consumption growth risk can explain the cross section of returns on portfolios of currencies sorted by interest rates only if the representative agent has a level of risk-aversion that also explains the return on U.S. equity as a consumption risk premium.

A popular view is that the forward discount bias is not only the *precondition* for carry trades, but is also a *consequence* of carry trades.[1] The rationale behind this view starts with the observation that most central banks set official overnight interest rates mainly with domestic monetary policy considerations in mind, rather than the external exchange rate environment. Inflation-targeting central banks, for instance, set their policy interest rate mainly in response to the prospects for domestic inflation. When official

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[1] see, e.g., *Carry on Speculating*, The Economist, February 22nd 2007: ”One obvious possibility is that the actions of carry traders are self-fulfilling; when they borrow the yen and buy the dollar, they drive the former down and the latter up.”

interest rates are held fixed by central banks in this way, carry trades can become self-reinforcing. As more and more speculators pile into the carry trade, they sustain the appreciation of the high interest rate currency relative to the low interest rate currency. This notion that an arbitrage opportunity can be magnified by rational speculation is at odds with the usual view that arbitrage opportunities should become less, not more profitable as more speculators exploit them.

This paper develops a theoretical model that identifies plausible conditions under which carry trades lead to such *self-enforcing arbitrages*[2]. We also explore the impact of carry trades on speculative dynamics in the foreign exchange market. We consider speculators who have a collective ability to move an exchange rate because they face dealers who do not provide a perfectly elastic demand/supply curve for the currency.[3] By contrast, short-term funds are in perfectly elastic domestic supply and demand at the interest rates prevailing in each currency, as set by central banks.

With these two ingredients only, we obtain the unsurprising conclusion that speculation is stabilizing. Anticipating future corrections, speculators bring the market exchange rate in line with their view of the fundamental parity between the two currencies. This result has a particularly strong form in our setup. Stabilizing speculation is not only the unique Nash equilibrium of the trading game, it is the only possible outcome that is consistent with common knowledge that speculators are rational.

Imposing additional - possibly small - funding constraints on speculators can change everything in this framework. Following Brunnermeier and Pedersen (2007), we introduce the feature that speculators can structure a carry trade at a lower cost whenever there is a lot of liquidity in the market. The

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[2] We are grateful to the editor for suggesting this term.

[3] In line with the important role played by illiquidity in our model, Burnside et al. (2006) argue that the main limitation of the arbitrage profits generated by carry trades are liquidity issues in FX markets.

speculative dynamics may change substantially with such funding externalities. Carry trades may become self-enforcing. A speculator is willing to enter the carry trade only if she believes that other speculators will do so: Carry trades are fuelled by the belief that other speculators engage in the carry trade. In our dynamic trading game, such strategic complementarities give rise to a unique equilibrium exchange rate process with stochastic bifurcations. Extended periods of slow appreciations of the high rate currency are stochastically punctuated by endogenous crashes. Currency traders refer to such patterns as “going up by the stairs and coming down in the elevator” (see Breedon, 2001).

While focussed on this endogenously generated speculative dynamics, our paper closely relates to three contributions. Our justification of the forward discount bias is close to the explanation based on ”positive feedback trading” developed in Froot and Thaler (1990). Our modelling of speculation as a dynamic coordination game relates to Abreu and Brunnermeier (2003), although both ”bubbles” and ”crashes” are endogenous in our framework. Finally, our refinement of multiplicity of equilibria closely follows from Frankel and Pauzner’s (2000).

1 Baseline Model

Time is continuous and is indexed by $t \in [0, +\infty)$. There are two assets. One asset is denominated in Japanese yen and serves as *numéraire*. We may think of this asset as a Japanese yen deposit. The other asset is U.S. dollar denominated, and we may construe this second asset as a U.S. dollar deposit. The price (in terms of yen assets) of the dollar-denominated asset at date t is denoted p_t. These two assets are exchanged between two types of agents, speculators and dealers. Speculators bet on the evolution of p_t and dealers supply liquidity. Each type comprises a continuum of agents with unit mass.

The speculators (also called "traders" henceforth) are risk neutral. Their date-t portfolio choice consists of holding either one dollar-denominated asset or p_t units of the yen asset. It is common knowledge among the speculators that the *fundamental value* of p_t is $v \in (0, 1)$. More precisely, they know that there is a stopping time at which the market price p_t will snap back to v for exogenous reasons, and then remain there forever. The stopping time has Poisson arrival intensity ρ. The idea here is similar to the notion of a "day of reckoning" in Duffie, Gârleanu, and Pedersen (2002) on which there is an exogenous public announcement that reveals the relative value of the future consumption generated by the dollar asset to all market participants. The assumption that the price remains at v forever once it has snapped back to v is offered as a simplification. Our focus is on how traders behave *in anticipation* of this anchor to the fundamental. In this section, for simplicity, assets generate no consumption until the day of reckoning.

The situation that we aim to capture with this stylized setup is one in which the speculators expect that the Bank of Japan and the Fed will neither modify their target rates nor intervene in the foreign exchange market until the "day of reckoning". We study whether speculation stabilizes or destabilizes the exchange rate under such circumstances.

Speculators do not consume before the day of reckoning. They aim at maximizing their expected trading profits before the day of reckoning. They face a small friction in how often they can trade. A speculator can only trade at discrete designated trading dates that are generated by a Poisson process with intensity λ. The processes are independent across traders, so that a fraction λdt of the traders gets a chance to trade between t and $t + dt$. The ratio θ defined as:

$$\theta \equiv \lambda / \rho \tag{1}$$

indicates the number of times a trader may be expected to get an opportunity to trade before p_t snaps back to fundamentals. In a very active market such as the FX market, we would expect the traders to have a free hand in trading, and so for this reason our main focus will be on the limiting case where the ratio θ is large.

This small friction may be interpreted as the time it takes to a hedge fund to structure a large deal with prime brokers, or to a proprietary trader to clear internal risk controls before a large trade. Let x_t denote the fraction of traders who are invested in dollars at date t. This fraction has the following dynamics:

$$\begin{cases} \dot{x}_t = -\lambda x_t & \text{when traders sell the dollar} \\ \dot{x}_t = \lambda \left(1 - x_t\right) & \text{when traders buy the dollar} \end{cases}$$

This departure from continuous trading strategies is the key feature of the model that warrants equilibrium uniqueness in Section 3.

When she has a chance to trade at date t, a trader meets the market-making sector. This sector is comprised of a continuum of dealers who have heterogeneous valuations of the dollar asset with c.d.f. $F(.)$ until the day of reckoning. This may stem from heterogeneity in their inventories, or from heterogeneous beliefs about the fundamentals v. Like the traders, each dealer can be long up to one dollar asset. At each trading date t, the trader submits a supply or demand schedule to the dealers, and the non-filled part is cancelled. As a result, p_t solves:

$$c_t = F(p_t) \tag{2}$$

where c_t is the quantity of yens that traders have invested in the market up to date t. Equation (2) formalizes that the date t trader buys the dollar asset from the dealer who owns it and values it the least at date t, or sells it to the dealer who does not own it and values it the most at date t. This corresponds to a trade with a dealer with a valuation of $F^{-1}(c_t)$ in both cases. For expositional simplicity only, we will assume in this paper that the valuations of the dealers are uniformly distributed over $(0, 1)$.

Since we have assumed that traders are long up to one dollar, any capital gains or losses realized by a trader between two trading dates are accumulated in yen assets: There is no compounding of gains or losses. Thus, the date t price before the day of reckoning satisfies

$$p_t = x_t \tag{3}$$

where x_t is the proportion of the traders who hold one dollar asset. Note that there is ample evidence that prices respond to flows in FX markets (see, e.g., Cao, Evans, and Lyons (2006)).

At trading date t, a trader who holds the dollar asset faces a binary decision - to keep it or to sell it for p_t yen assets. For a trader who does not already hold the dollar asset, the binary decision is either to buy it at price p_t, or to maintain her yen holdings. At the time of making a decision, the trader can condition on the realized price path as well as the calendar date t. Thus, the trading strategy of a trader is a mapping:

$$\left(t, (p_u)_{u \leq t}\right) \mapsto \{\text{dollar asset, yen asset}\} \tag{4}$$

that specifies whether a trader will hold dollars or yens for all pairs of dates and price histories.

Dominance Solvable Outcome

Our baseline model allows us to draw a very strong conclusion - starting from any price p_0, the price until the day of reckoning returns to the fundamental value v at the fastest possible rate. Any other outcome can be ruled out by the iterated deletion of strictly dominated strategies. Iterated dominance is a weaker solution concept than Nash equilibrium in the sense that Nash equilibria survive iterative elimination of dominated strategies.

Suppose that the price is p_t. The most pessimistic scenario for the holder of the dollar asset is that all future traders either switch out of it, or refrain from buying it so that the price path is declining over time. Under this most pessimistic scenario, the price path is given by $\{p_{t+u}\}_{u\geq 0}$, where

$$p_{t+u} = p_t e^{-\lambda u}. \tag{5}$$

In other words, the price converges to 0 at the rate λ, as each trader whose trading date arrives switches out of the dollar asset.

Even under this most pessimistic scenario, there is a price at which a trader is better off holding the dollar asset than the yen asset. Consider a speculator who has a chance to trade at date t. If the price path from date t onward is given by $\{p_{t+u}\}_{u\geq 0}$ then the expected excess rate of return on the dollar is:

$$\int_0^\infty \frac{\lambda p_{t+u} + \rho v}{p_t} e^{-(\lambda+\rho)u} du - 1. \tag{6}$$

Thus, if the future price path is given by $\{p_{t+u}\}_{u\geq 0}$, the trader buys the dollar asset or holds on to it whenever (6) is greater than 0.

By substituting (5) into the expression for expected return given by (6) we can obtain the price $\underline{p}^0$ at which a trader is indifferent between holding dollar and yen under this most pessimistic scenario. This threshold price $\underline{p}^0$ is given by

$$\underline{p}^0 \equiv \frac{(1+2\theta)\,v}{(1+\theta)^2} \tag{7}$$

where θ is defined as the ratio λ/ρ. If the price falls below $\underline{p}^0$, then holding yen is dominated. Note that $\underline{p}^0$ tends to 0 as $\theta \to \infty$.

But then, the most pessimistic price path given by (5) is *too pessimistic* in that it assumes that some future traders may choose dominated actions. By ruling out trading strategies that are dominated the most pessimistic price path now becomes:

$$\left\{\max\left(\underline{p}^0, p_t e^{-\lambda u}\right)\right\}_{u\geq 0} \tag{8}$$

Since (8) implies strictly higher prices than (5) beyond some date in the future, we can define a new threshold price given by $\underline{p}^1$ below which holding yen is dominated. Clearly, $\underline{p}^0 \leq \underline{p}^1$. If the price is below $\underline{p}^1$, the trader will not hold yen. Thus, any trading strategy in which a trader chooses the yen asset at a price below $\underline{p}^1$ is ruled out after *two* rounds of deletion of dominated strategies.

We can iterate this argument. After $n+1$ rounds of deletion of dominated strategies, the most pessimistic price path starting from p_t is given by:

$$\left\{\max\left(\underline{p}^n, p_t e^{-\lambda u}\right)\right\}_{u\geq 0}$$

This sets a new threshold $\underline{p}^{n+1}$ for the trading strategy, in which choosing yen for any price below $\underline{p}^{n+1}$ is ruled out by $n+2$ rounds of deletion of dominated strategies. We thus obtain the increasing sequence:

$$\underline{p}^0 \leq \underline{p}^1 \leq \underline{p}^2 \leq \cdots \leq \underline{p}^n \leq \cdots$$

Since price is bounded above, this sequence converges to some limit, denoted by $\underline{p}$. No trader will choose yen below $\underline{p}$ in any rationalizable outcome, since such an action is ruled out by iterated dominance. Thus, $\underline{p}$ constitutes a floor for the price of the dollar asset in any price path $\{p_{t+u}\}_{u\geq 0}$.

Analoguously, we can define a *decreasing* sequence of thresholds that corresponds to the most *optimistic* price paths that are consistent with n rounds of deletion of dominated strategies. If the price is sufficiently close to the upper bound 1, then yen is strictly preferred since the price will never rise sufficiently to compensate for the risk that it could possibly fall to its fundamental value v. Let $\bar{p}^0$ be the price above which selling is dominant. Thus, the price path will never rise above this level. We can then iterate the argument to derive the decreasing sequence:

$$\bar{p}^0 \geq \bar{p}^1 \geq \bar{p}^2 \geq \cdots$$

Denote by $\bar{p}$ the limit of this sequence. This limit would constitute a ceiling for any price path. Clearly,

$$\underline{p} \leq \bar{p}. \tag{9}$$

We will now show that the reverse inequality must hold, too. Consider the floor price $\underline{p}$. We must have $\underline{p} \geq v$. To see this, suppose (for the sake of argument) that $\underline{p} < v$. Since no trader sells dollars below $\underline{p}$, the future path $\{p_{t+u}\}_{u \geq 0}$ lies on or above $\underline{p}$. Thus, conditional on a price $\underline{p}$, the expected return on the dollar asset is *strictly* greater than one since all possible future values of the asset are larger than $\underline{p}$. But this contradicts the fact that $\underline{p}$ is the upper limit of the sequence of indifference thresholds. Hence, we must have

$$\underline{p} \geq v. \tag{10}$$

From an exactly analoguous argument, we conclude that $v \geq \overline{p}$. Thus, we have

$$\underline{p} \geq v \geq \overline{p} \tag{11}$$

From (11) and (9), we conclude that $\underline{p} = \overline{p} = v$. We have thus proved the following.

Proposition 1

In any subgame, the only trading strategy that survives the iterated deletion of dominated strategies is to hold the dollar asset when $p_t \leq v$ and hold the yen asset when $p_t > v$.

Corollary 2 *In the unique equilibrium price path in the subgame that starts with price p_t, the price converges to the fundamental value at the maximum speed that trading constraints allow for.*

Our baseline model shows the power of the stabilizing role of speculation, as argued by Friedman (1953). No matter how loose the anchor is to the fundamentals, the speculative behavior of traders push the price to coincide with the fundamentals. This result does not rely upon any particular equilibrium concept, but on the mere assumption that traders are rational and that this is common knowledge.

Our result can be understood as the resolution of two competing externalities generated by the predecessors of the date t trader. As the predecessors throw more "weight of money" into the dollar asset, there are two effects. First, the positive externality is that the future resale values $(p_{t+u})_{u \geq 0}$ will be high, other things being equal. But the negative externality is of course that the dollar asset is currently expensive. Because of the risk that the dollar asset reverts to its fundamental value, the negative externality ultimately wins out. Thus, a trader has no incentive to join in pushing the price away from its fundamental value. Instead, the trader will seek to trade against her predecessors to bring the price back into line with fundamentals. When θ is large, fundamental risk is small compared to the risk that other speculators create an adverse price move. In this case, the competition between positive and negative externalities is more even, in the sense that very small additional positive externalities tip the balance toward conditions that are more fertile to the emergence of destabilizing speculation, as we see now.

2 Funding Externalities

We now add to this baseline model two features that capture important practical aspects of yen carry trades.

First, we introduce a positive carry. The initial motive for funding carry trades in yen is the persistence of low Japanese official rates. Accordingly, we posit that the dollar asset pays interest at a rate equal to δ. The interest is converted into yen at the exchange rate prevailing at the time it is earned.

Second, we take into account that speculation requires capital. When she enters a carry trade at date t (i.e. borrows yen to obtain a dollar asset) an investor needs to tie up some capital. More precisely, only a fraction of the dollar asset equal to

$$1 - h(p_t)$$

where $h(p_t) \in (0, 1)$ can be financed by the sale of yen assets. The remaining fraction $h(p_t)$ has to be financed by the trader's own capital. This captures the haircut that a prime broker would require as collateral from the speculator. The trader's own capital has an opportunity cost of $\Delta > 0$ per unit of time. Haircut percentages $h(.)$ are updated for each trader at each trading date. Thus, after entering a carry trade at date t, the trader subsequently incurs an opportunity cost of $\Delta \cdot h\,(p_t) \cdot p_t$ yen per unit of time until the next trading date (or the day of reckoning if it occurs before) because she ties up an amount $h\,(p_t) \cdot p_t$ of her own funds in the trade.

The binary choice faced by the traders can also be explained in the following terms. A trader faces the choice between:

- Holding JPY deposits worth h dollars ($= h \cdot p_t$ yen)
- Or, enter (or maintain) the carry trade, given by balance sheet :

Assets	Liabilities
p_t USD	$h \cdot p_t$ Equity
	$(1-h)\,p_t$ JPY

(in JPY)

Assets	Liabilities
1 USD	h Equity
	$1-h$ JPY

(in USD)

Our key assumption is that the collateral requirements to enter a yen carry trade are lower when funding liquidity is plentiful (in the sense of Brunnermeier and Pedersen (2007)). Namely, we assume that $h(p)$ decreases with respect to p. This assumption follows Brunnermeier and Pedersen (2007), who describe a variety of practical situations in which funding liquidity increases as market liquidity increases. Another way to express Brunnermeier and Pedersen's assumption is to say that leverage is high when asset prices are buoyant, or that leverage is high when balance sheets are large. Adrian and Shin (2007) document evidence of this feature, both from aggregate data and from individual broker balance sheet data. We revisit the empirical basis for this assumption later in the paper.

A plausible explanation for this feature of the haircut $h(.)$ formalized by Brunnermeier and Pedersen (2007) is that the lenders are less informed than the speculators, and thus do not know if an increase in p_t is due to a speculative flow or reflects some fundamental news. In the latter case, the collateral value of the dollar asset is enhanced in their eyes. If more cash in the market implies a possible higher collateral value of the trade in the eyes of the financiers, then speculators' leverage should increase with respect to p_t. An explicit modelling of such funding frictions is beyond the scope of this paper. Rather, we take this feature as given and study its impact on exchange rate dynamics. We assume that the speculators are protected by limited liability, and that h and h' are defined and bounded away from 0 over $(0, 1)$.[4]

Finally, we make a technical assumption to aid our formal arguments. We will assume that the anchor to the fundamental v is weaker than in the previous section, in that the exogenous correction at the day of reckoning τ cannot move the price at an arbitrarily large rate. Formally, v is a function of the market price at date τ, $v(p_\tau)$, that satisfies:

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[4] We also ignore the cost of debt. We could add interest on debt at an instantaneous rate of $s(p_t)$ without substantially affecting our results.

Condition 3 *There is a Lipschitz function* $r\left(p_\tau\right)$ *such that:*

$$v(p_\tau) = \left(1 + r\left(p_\tau\right)\right) \cdot p_\tau$$

In other words, the impact of the price change at the day of reckoning cannot be too drastic. Condition 3 is a technical assumption that has bite only in the region where the exchange rate is close to 0. It implies that a given fundamental value $v > 0$ is out of reach at the day of reckoning if p_τ is sufficiently close to 0 at this date. The precise role played by condition 3 will be apparent in section 3, and we will return to a discussion of its exact role.

In this new environment with financing constraints, the expected profit from a one dollar carry trade is the sum of two terms, as given below in (12).

$$\overbrace{\int_0^{+\infty} \left[\begin{array}{c} \lambda \max\left(p_{t+u} - p_t, -h(p_t)\cdot p_t\right) + \\ \rho \max\left(v\left(p_{t+u}\right) - p_t, -h(p_t)\cdot p_t\right) \end{array} \right] e^{-(\lambda+\rho)u} du}^{\text{capital gain or loss due to exchange rate fluctuations}} \tag{12}$$
$$+ \underbrace{\int_0^{+\infty} (\lambda+\rho)\, e^{-(\lambda+\rho)u} \left(\int_0^u \left(\delta p_{t+s} - \Delta h\left(p_t\right)\cdot p_t\right) ds \right) du}_{\text{cumulative carry minus cost of capital}}$$

The two terms have the following interpretation. The first term in (12) is the expected capital gain or loss from movements in the exchange rate, where exchange rate movement can come either from the endogenous dynamics of the exchange rate or from the arrival of the day of reckoning. Note that the capital gain or loss from the exchange rate movement takes account of the limited liability of the trader. The second term is the expected accumulated flow payoff from the carry element minus the expected accumulated cost of tying up capital of $h\left(p_t\right)\cdot p_t$.

The fact that the haircut on a carry trade decreases with p_t implies that speculators create additional positive externalities for each other by entering carry trades. We now show that these externalities may dramatically change the underlying strategic incentives as compared to the baseline model.

Proposition 4

1. There exist two functions $\underline{Z}(.)$ *and* $\overline{Z}(.)$ *such that*

i) if a trader expects that all other traders will exit the carry trades after date t*, it is optimal to enter the carry trade at date* t *if and only if* $\delta \geq \overline{Z}(p_t)$

ii) if a trader expects that all the other traders will enter the carry trade after date t*, it is optimal to exit the carry trade at date* t *if and only if* $\delta \leq \underline{Z}(p_t)$.

2. Let C *be a compact subset of* $(\Delta \cdot h(1), +\infty)$*. If* λ *is sufficiently large and* ρ *sufficiently small, then for any* $\delta \in C$*, and starting from any price* p_t*, there are multiple possible steady states. There is both a steady state in which all traders enter the carry trade after* t*, and also a steady state in which traders exit carry trade after* t.

Proof. See the appendix.

In the statement of proposition 4, "enter the carry trade" is a shorthand for the statement that a trader enters the carry trade if she is not already engaged in the carry trade, or maintains the carry trade if she is already engaged in the carry trade. Similarly, "exit the carry trade" means that the trader exits the carry trade if she is engaged in the carry trade, and not enter if she is not already so engaged.

Proposition 4 has the following interpretation. Part 1 of the proposition establishes the existence of dominance regions for the carry element δ such that if δ lies in one of the dominance regions then it is optimal to enter or exit the carry trade irrespective of the actions of other traders. Outside these dominance regions, the optimality of entering or exiting the carry trades depends on the actions of others. Thus, it is possible to construct a path where traders enter the carry trade in expectation that others will do so in the future, and also a path where traders exit the carry trade in expectation that all others will exit.

To gain an intuition for this result, we can decompose (12) into three terms. First, there is the capital gain or loss due to the endogenous movement in the exchange rate. Second, there is the capital gain or loss due to the arrival of the day of reckoning. Finally, there is the accumulated carry minus cost of capital. When λ is large and ρ is small, traders engaging in the carry trade do not put much weight on the deviation of exchange rates from fundamentals, unless possibly when the exchange rate p_t is very high relative to its fundamental value v. However, in this case, the flow payoff from the carry element is large also, since $h\left(p_t\right)$ is decreasing in p_t. In other words, when the risk from the exchange rate snapping back to fundamental value is large, the carry payoff is large, also. Thus, it may be possible to sustain a path where entering the carry trade becomes mutually reinforcing, even though there is fundamental risk that the exchange rate reverts to its fundamental value.

Note that by taking λ sufficiently large and ρ sufficiently small, the set of parameters δ for which either steady state can be sustained can be made arbitrarily large within $(\Delta \cdot h(1), +\infty)$. Unlike in the benchmark case, Proposition 4 shows that speculation can be destabilizing in this new environment. In effect, when λ becomes large, we get closer to a single-shot game between the traders since they can trade very frequently. The two extreme steady states (all enter, all exit) resemble the Nash equilibria of a binary action game between the traders.

The fact that the two extreme steady states resemble Nash equilibria in the single-shot game suggests that trading decisions are strategic complements - that is, the more other traders enter, the greater my incentive is to enter (and conversely, the greater the other traders exit, the more I want to exit). Thus, the strategic incentives become inverted, as compared to the benchmark case. We commented after our benchmark Proposition in the previous section that the reason why speculation is stabilizing comes from the fact that the *negative* externalities created by previous buyers outweigh the *positive* externalities. In Proposition 4, the roles are reversed. If λ is sufficiently large and ρ sufficiently small, the positive externality of raising the price higher is larger than the negative externality even if the sensitivity of the haircut to the price h' is arbitrarily close to (but bounded away from) 0. This is because in this case, the probability that the asset price will snap back to v during the current trade is so small that the positive funding externalities always offset the risk of holding an overvalued asset.

When funding constraints create such strategic complementarities, the price path itself will influence expected payoffs, and we cannot come to any firm conclusions regarding predictable outcomes without additional argument. In general, we can envisage very complicated dynamic strategies that try to balance the negative and positive externalities between traders, and we cannot say much more without additional structure on the problem. Rather than going further in investigating complex dynamics, we will now go in a different direction. We will now examine what happens when the carry itself is stochastic.

3 Stochastic Fundamentals

It turns out that the multiplicity of equilibria in Proposition 3 is not robust to the addition of some variation in the carry δ. Adding (possibly arbitrarily small) shocks on δ, we obtain a unique dominance-solvable equilibrium. Such shocks may be interpreted as liquidity trading in domestic markets or noise in monetary policies. We draw on the work of Burdzy, Frankel and Pauzner (2001) and Frankel and Pauzner (2000), who showed that in binary action coordination games with strategic complementarities, the addition of small stochastic shocks to the fundamentals of the payoffs generates a unique, dominance solvable outcome. The arguments in these papers are similar to the global game arguments of Carlsson and van Damme (1993) and Morris and Shin (1998). We return to an interpretation of the results later in the paper.

Formally, we assume in this section that the carry obeys the process:

$$\delta_t = \delta + \sigma W_t,$$

where W_t is a standard Brownian motion, and $\sigma > 0$. The main result of the paper is the following:

Proposition 5

If the expected number of trades $\theta = \frac{\lambda}{\rho}$ is sufficiently larger than unity, there is a Lipschitz downward-sloping function $Z(.)$ such that in any subgame starting at date t with a carry δ_t and an exchange rate p_t, there is a unique, dominance solvable solution to the trading game. In this solution, a trader who trades at date t engages in the carry trade if and only if $\delta_t \geq Z(p_t)$.

Note that Proposition 5 does not impose any restriction on σ or $h(.)$, but only on the expected number of trades $\theta = \frac{\lambda}{\rho}$. Note that this restriction requires either that λ be sufficiently large *or* ρ sufficiently small. This is weaker than the condition in Proposition 4 (λ sufficiently large *and* ρ sufficiently small). It is actually interesting to contrast Proposition 5 with the results in Proposition 4. With a deterministic carry, Proposition 4 states that for λ arbitrarily large and ρ arbitrarily small (and thus for $\theta = \frac{\lambda}{\rho}$ arbitrarily large), there is an arbitrarily large set of values of δ for which there are multiple steady states. Adding Brownian shocks to δ, *even arbitrarily small*, dramatically changes this picture, since Proposition 5 implies in particular that for λ sufficiently large and ρ sufficiently small, the equilibrium is unique, and dominance-solvable.

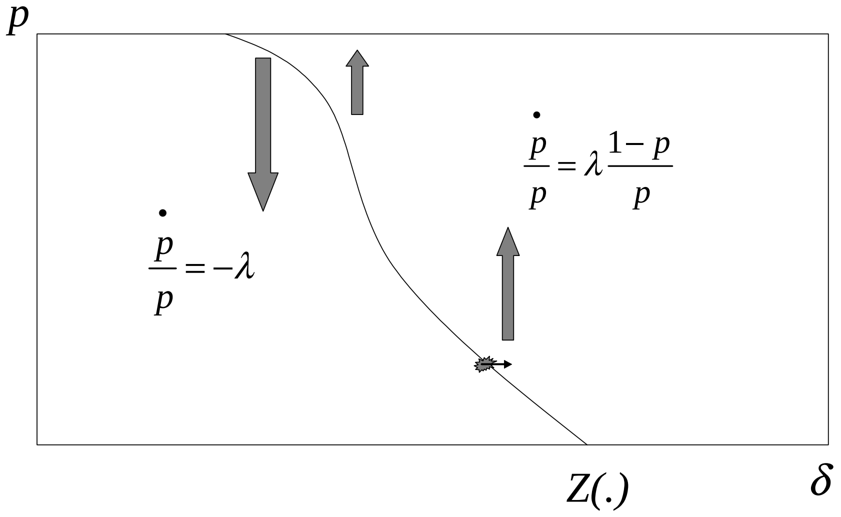

Figure 1: Unique equilibrium with stochastic δ_t

Proposition 5 can be illustrated in figure 1. The curve $Z(p_t)$ divides the (δ, p)-space into two regions. Proposition 5 states that in the unique equilibrium, any trader decides to enter the carry trade to the right of the $Z(.)$ curve, and exit the carry trade to the left of the $Z(.)$ curve. Thus, p_t will tend to rise in the right hand region, and tend to fall in the left hand region, as indicated by the arrows in figure 1.

The price dynamics implied by the unique equilibrium is given by:

$$dp_t = 1_{\{\delta_t > Z(p_t)\}} \lambda(1 - p_t)dt - 1_{\{\delta_t < Z(p_t)\}} \lambda p_t dt. \tag{13}$$

where $1_{\{.\}}$ denotes the indicator function that takes the value 1 when the condition inside the curly brackets is satisfied. These processes are known as *stochastic bifurcations,* and are studied in Bass and Burdzy (1999) and Burdzy et al. (1998). From Theorem 1 in Burdzy et al. (1998), for a given initial price p_0, and for almost every sample path of δ, there exists a unique

Lipschitz solution $(p_t)_{t\geq 0}$ to the differential equation (13) defining the price dynamics for Z Lipschitz decreasing.

Some suggestive features of the price dynamics can be seen from figure 1. When the dollar has appreciated for a while so that p_t is close to 1, the rate of return if the currency appreciates is given by

$$\frac{\dot{p}}{p} = \lambda \frac{1-p}{p} \simeq 0$$

However, if the price crosses the Z boundary, the rate of depreciation is

$$\frac{\dot{p}}{p} = -\lambda$$

In other words, when p is high and the currency crosses the Z boundary from above, there is a sharp depreciation that was preceded by a slow appreciation. Such dynamics are suggestive of the price paths of high-yielding currencies in carry trades that "go up by the stairs and come down in the elevator".

We provide a proof of Proposition 5 that follows closely the argument given by Frankel and Pauzner (2000) for their discussion of binary coordination games. The difference between our setup and the game studied in Frankel and Pauzner (2000) is that viewed from date t, the future instantaneous profits at date $t+u$ depend on p_{t+u}, but also on p_t (see expression (12)). Their proof applies identically, however, provided θ is sufficiently large. This is because for θ sufficiently large, one dominance frontier is decreasing in p_t. More precisely, let Z_0 denote the function such that if she believes that the other traders will exit the carry trade after t, a date-t trader enters the carry trade if and only if

$$\delta_t \geq Z_0 \left(p_t\right).$$

We have the following key result.

Lemma 6

$Z_0(.)$ is Lipschitz and nonincreasing for θ sufficiently large.

Proof. See the appendix.

That traders are protected by limited liability plays an important role in this result. If a trader leans against the wind and enters the carry trade, her capital losses due to other traders' run on the dollar are limited even when dollar is high because of a high leverage. Thus the prospect of a high carry overcomes the fear of downside risk when dollar is high.

Role of condition 3. Absent condition 3, $Z_0\left(.\right)$ would be Lipschitz decreasing for all values of p_t except in the vicinity of 0. It is unclear to us whether the stochastic bifurcation equation (13) would still admit uniquely defined Lipschitz paths in this case. Condition 3 essentially allows us to circumvent this open mathematical question.

Lemma 6 implies that the proof of Frankel and Pauzner (2000) applies identically to our setup. To make our paper self-contained, we now detail their argument. In what follows, we will use the shorthand of "hold dollar" to mean "enter or maintain the carry trade", and "hold yen" to mean "exit or refrain from the carry trade". Refer to figure 2. Ruling out any strategy in which the trader holds yen to the right of Z_0, we can derive a boundary Z_1 for the second-round dominance region which indicates the region where it is dominant to hold dollar in the absence of any first-round dominated trading strategies. In other words, if she knows that other traders hold dollars at least when they are on the right of Z_0 at their trading dates, a trader will be willing to hold dollar at least when she is on the right of Z_1. We skip the proof that $Z_1(.)$ is Lipschitz with at most the same constant as $Z_0(.)$ (identical to Frankel Pauzner 2000). Z_1 is decreasing. To see this, note that we know from Lemma 6 that Z_1 would be decreasing if traders were selling the dollar all the time. In the case in which the other traders use $Z_0(.)$ as a buy/sell dollar frontier, all else equal, a higher p increases the probability of future dollar buys because $Z_0(.)$ is non-increasing. In sum, if $p' \geq p$,

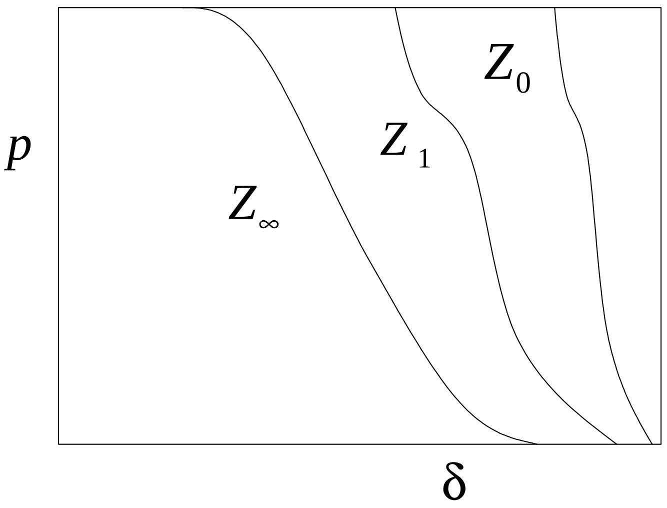

Figure 2: Iterative dominance from right. The curves Z_i are recursively defined as follows. Z_i is such that, if $\delta \geq Z_i(p)$, then a trader is willing to hold dollar if she believes that other traders hold dollar when they are on the right of Z_{i-1}.

then $Z_1(p') \leq Z_1(p)$ because i) even absent any future dollar purchases, the yen profit would be higher in p' from Lemma 6, ii) in addition there will be more future dollar purchases starting from (δ, p') than from (δ, p) since Z_0 is nonincreasing.

By iterating this process, we can obtain the boundary Z_∞ for the region where a trader holding yen can be eliminated by iterated dominance. Z_∞ is decreasing Lipschitz as a limit of decreasing Lipschitz functions with decreasing Lipschitz constants. The boundary Z_∞ defines an equilibrium strategy since, if all traders hold yen to the left and hold dollar to the right, the indifference point between dollar and yen for the trader also lies on Z_∞.

Consider now a translation to the left of Z_∞ so that the whole of the curve lies to the left of the yen-dominance region. Call this translation Z_0'. To the left of Z_0', holding yen is dominant. Then construct Z_1' as the *rightmost translation* of Z_0' such that a trader must choose yen to the left of Z_1' if she believes that other traders will play according to Z_0'. By iterating this process, we obtain a sequence of translations to the right of Z_0'. Denote by Z_∞' the limit of the sequence. Refer to figure 3. The boundary Z_∞' does not necessarily define an equilibrium strategy, since it was constructed as a translation of Z_0'. However, we know that if all others were to play according to the boundary Z_∞', then there is at least one point A on Z_∞' where the trader is indifferent between holding yen and holding dollar. If there were no such point as A, this suggests that Z_∞' is not the *rightmost* translation, as required in the definition.

We claim that Z_∞' and Z_∞ coincide exactly. The argument is by contradiction. Suppose that we have a gap between Z_∞' and Z_∞. Then, choose point B on Z_∞ such that A and B have the same height - i.e. have the same second component. But then, since the shape of the boundaries of Z_∞' and Z_∞ are identical, the stochastic bifurcation process starting from A must have the same distribution over payoffs as the process starting from B. Thus,

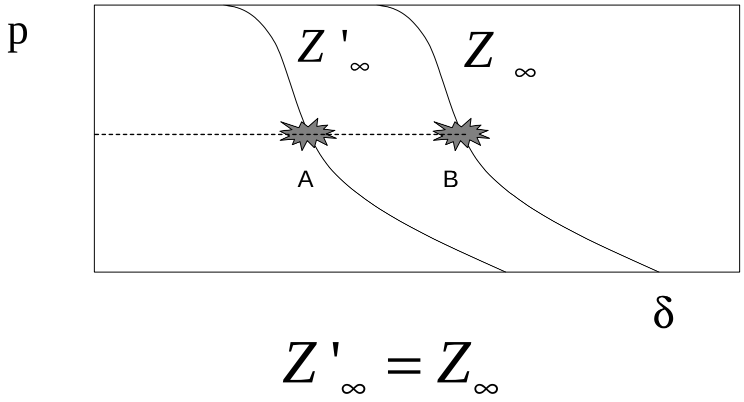

Figure 3: If a trader is in A and thinks that other traders enter the carry trade if and only if they are to the right of Z'_{∞}, then future price trajectories will just be horizontal translations of the trajectories realized when a trader is in B and thinks that other traders enter the carry trade if and only if they are to the right of Z_{∞}. Thus a trader can be indifferent between both situations only if A and B correspond to the same δ and thus $Z_{\infty} = Z'_{\infty}$.

the uncertainty governing the expected payoffs are identical at points A and B, except for the fact that B has a higher current value δ_t. This contradicts the hypothesis that a trader is indifferent between the two actions both at A and at B. If she were indifferent at A, she would strictly prefer to hold dollar at B, and if she is indifferent at B, she would strictly prefer to hold yen at A. But we constructed A and B so that traders are indifferent. Thus, there is only one way to make everything consistent, namely to conclude that $A = B$. Thus, there is no "gap", and we must have $Z'_{\infty} = Z_{\infty}$. In other words, we have the situation depicted in figure 1 as claimed.

Interpreting the Results

Proposition 5 demonstrates the impact of adding some uncertainty to the carry δ_t. The multiplicity of equilibria reported in the previous section resulted from the feature that, if the fundamentals were fixed and known, then one cannot rule out all other players trading in one direction, provided that the fundamentals were consistent with such a strategy. However, the introduction of shocks changes the picture radically. Since δ_t follows a Brownian motion, while traders must wait for their trading opportunities, the traders are far less nimble than the shifts in the fundamental value itself. Thus, choosing to enter the carry trade *versus* exiting the carry trade entails a substantial degree of commitment over time to fix one's trading strategy.

Suppose that the (δ, p) pair is close to a dominance region, but just outside it. If δ is fixed, it may be possible to construct an equilibrium for both actions, but when δ moves around stochastically, it may wander into the dominance region between now and the next opportunity that the trader gets to trade. This gives the trader some reason to hedge her bets and take one course of action for sure. But then, this shifts out the dominance region, and a new round of reasoning takes place given the new boundary, and so on. Essentially, adding Brownian shocks to the carry enables us to extend to the two dimensional space of (δ, p) pairs the dominance argument we showed in our benchmark result without funding externalities.

That $Z(.)$ is nonincreasing implies that price paths exhibit hysteresis. If the dynamic system (δ_t, p_t) is in the area where buying is dominant ($\delta_t > Z(p_t)$), then the buy pressure takes the system away from $Z(.)$, making the continuation of a bullish market even more likely, all else equal. The reader may wonder whether Brownian excursions completely swamp this effect at the proximity of $Z\left(.\right)$, so that runs never develop and the system is "trapped" in the vicinity of $Z(.)$. The next proposition shows that it is not the case provided σ is sufficiently small.

Proposition 7

Assume that the system is in the state (p_t, δ_t) *such that*

$$\delta_t = Z(p_t).$$

For any $\varepsilon > 0$*, as* $\sigma \to 0$*, the last time at which the system hits* $Z(.)$ *before* p_{t+u} *becomes larger than* $1 - \varepsilon$ *or smaller than* ε *tends to* t *in distribution. The probability that the price will go up tends to* $1 - p_t$*.*

Proof Theorem 2 in Burdzy, Frankel, and Pauzner (1998).■

The broad intuition for this result is that when σ is small, the price path around $Z(.)$ is mostly driven by changes in p_t: Liquidity flows are more important than changes in the carry δ_t. The speed at which the price goes up is $\lambda(1 - p_t)$, while it decreases with speed $-\lambda p_t$. The price path does not revert to $Z(.)$ once it has headed off towards one direction, and the ratio of the probabilities to go up or down is the ratio of the speeds at which the price goes in each direction. If the system hits $Z(.)$ when p_t is very high (low), then it is most likely to bifurcate downwards (upwards). Thus, for σ sufficiently small, the price paths will exhibit "runs", or long series of identically signed returns, with sudden and large reversals. Very small variations in traders' opinions may some times trigger very large fluctuations, depending on whether the system is close to Z or not. These price paths share features with the rational bubbles that burst stochastically in Blanchard and Watson (1982). The equilibrium is unique, however, and these endogenous "slow booms and sudden crashes" are generated by purely static externalities in an economy with finite wealth, and a finite horizon. The probabilities of reversals in our model are intrinsic, and depend on the magnitude of the deviation of the price from the "fundamental" value.

In Proposition 5, δ_t is a driftless Brownian motion. Accordingly, price paths are symmetric: long periods of yen appreciation mirror long periods of yen depreciation. It is easy to introduce asymmetry with a positive drift on δ_t. Proposition 5 still holds in this case, and dollar recovers more quickly after a reversal. A positive drift corresponds to the assumption that traders expect future relative increases in U.S. interest rate.

4 Discussion and Conclusion

Empirical predictions

The view that recent episodes of large yen appreciation are due to the disorderly unwinding of highly leveraged positions is very commonly held in the popular press, but empirical support is more scarce. An exception is the recent paper by Gagnon and Chaboud (2007), who find evidence of skewness of returns for some exchange rates. They study the relative frequencies of large upward and downward movements of three currencies relative to the dollar. They find that these frequencies are similar for the euro, which carried an interest rate close to the U.S. rate over the sample period. Conversely, large appreciations are relatively more frequent than large depreciations for the yen, while large depreciations are relatively more frequent for the Australian dollar. Interestingly, this relationship between interest rate differentials and the distribution of large exchange rate fluctuations is consistent with the stochastic bifurcations described in our setup.

Gagnon and Chaboud (2007) also find that implied volatility is negatively correlated with changes in the dollar-yen exchange rate, which is also somewhat consistent with our theory, since we expect the exchange rate to be more volatile around the bifurcation frontier. On the other hand, their study of the difference between the prices of USD/JPY calls and puts as a measure of the relative magnitudes of upside and downside risks generates more mixed findings. Our model would predict more expensive protections against yen appreciation during a long period of depreciation, which does not seem to be borne out by the data.

Carry trades and leverage

One of our key assumptions is that the haircut $h(p_t)$ is decreasing in p_t. We provide some further discussion of this assumption. Although the carry trade is often portrayed purely as a bet on the foreign exchange markets, the significance of the carry trade extends more widely. For example, a hedge fund that wishes to take on a larger position in a security obtains funding from its prime broker (a Wall Street investment bank, say) by pledging assets in a repurchase agreement. The prime broker, for its part, funds the loan to the hedge fund by borrowing from another party.

If the Wall Street bank borrows in New York, it will pay a rate closely tied to the short term US Dollar interbank rate. However, if it were to borrow in Tokyo, and in yen, it can borrow at the much lower yen overnight rate. A bank with global reach can borrow yen through its Tokyo office. The Tokyo office of the Wall Street bank then has yen liabilities to Japanese banks, but has yen assets against its New York head office. The lending by the Japan office of the Wall Street bank to its head office is captured in its "interoffice" accounts, and reported to the Bank of Japan. By monitoring the waxing and waning of the interoffice accounts of foreign banks in Tokyo, it is possible to gain a window on yen liquidity that funds general increases in balance sheets outside Japan.

For instance, until recently, foreign banks maintained a net long position in Japanese assets through its interoffice accounts. However, in the period leading up to the credit crisis of 2007, yen liabilities of foreign banks surged, leading to an unprecedented net *short* position in Japanese assets. These net short positions were sharply unwound in August 2007, coinciding with the peak of the credit crisis of 2007.

In addition, as found in Adrian and Shin (2007) for the fluctuations in US primary dealer balance sheets, Hattori and Shin find that the fluctuations in the size of the net interoffice accounts is correlated with the VIX index of implied volatility on the broader US stock market. The periods when foreign banks have large yen liabilities are also those periods with low readings of the VIX index.

Hattori and Shin also find that the difference between the yen overnight rate and a summary measure of overnight rates in developed countries mirrors closely the overall size of the net interoffice accounts. Yen liabilities are high when foreign overnight rates are high relative to overnight rates in Japan. Conversely, when foreign overnight rates are close to Japanese rates, foreign banks have low yen liabilities. During the period of exceptionally low US interest rates in 2002 to 2004, foreign banks maintained low yen liabilities, suggesting that they could satisfy their funding needs by borrowing in US dollars without tapping the yen market.

These facts seem to lend some support to the idea that the yen carry trade is associated with buoyant financial conditions when leveraged institutions lay on larger bets. Thus the fact that the haircut is falling with price is in keeping with the empirical fact that leverage is high when balance sheets are large, as shown by Adrian and Shin (2007).

Deterministic day of correction[5]

Assuming a constant ρ is meant to preserve time-homogeneity. Our results do not depend on this restriction. In fact, our results would hold even with a deterministic "day of reckoning". Since the positive funding externalities that traders create for each other are static, and not intertemporal, then for a sufficiently large λ the type of equilibrium described in Proposition 5 would still prevail at dates bounded away from the day of reckoning.

Bounds on the carry δ

That δ_t is a Wiener process implies that it takes potentially arbitrarily large values, an unpalatable property for an interest rate differential. From Theorem 2 and Corollary 1 in Burdzy et al. (1998), we can alternatively

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[5] We are indebted to our referee for this remark.

assume that δ_t obeys

$$d\delta_t = \nu.\mu\left(t, \delta_t\right) dt + \sigma dW_t,$$

where W_t is a standard Wiener process, $\mu\left(.,.\right)$ is Lipschitz in all arguments, $\mu\left(., \delta\right)$ admits an upper bound $b(\delta)$ which is a Lipschitz function of δ, and ν and σ are real numbers. Thus, δ_t can for instance be an Ornstein-Uhlenbeck process. For such δ_t, all equilibria become arbitrarily close to a unique equilibrium similar to the one described in Proposition 5 as ν and σ become arbitrarily small.

Binary portfolio choice

An important restriction in our setup is that traders choose only between being long one dollar asset or not. Our model would quickly become intractable with a larger number of options since one would have to study each pair of options and keep track of the fraction of the traders in each position. But it is worthwhile emphasizing that the absence of short sales of dollar plays no role here. We might as well have assumed that one of the options was to short dollar: only the differential return between the two options matters.

Carry trades in other markets

The main intuition that our model illustrates may be described in general terms as follows. An asset whose price is sufficiently sensitive to the flow of funds from a group of speculators would give rise to carry trades and speculative dynamics if i) short-term funds are in sufficiently elastic supply, and ii) the speculators are sufficiently leveraged that they create positive funding externalities for each other. While we view this set of assumptions as particularly plausible in the FX market, we also believe that our model can describe the destabilizing impact of carry trades in other markets such as the bond market.

We have developed a dynamic asset pricing model in which speculators face a coordination problem because of destabilizing margins. Using recent methodological advances in game theory, we obtain a unique equilibrium price that has appealing qualitative features: It implies a risk premium that is time-varying and countercyclical. The required return decreases in a highly non-linear fashion with respect to the value of the fundamentals. A natural route for future research is to improve the tractability of this baseline model in order to enrich it, and check its ability to generate quantitative features of empirical risk premia.

Appendix

Proof of Proposition 4

If a trader expects that all other traders will *exit* the carry trade, the expected profit of entering the carry trade at date t can be obtained from (12) by setting $p_{t+u} = p_t e^{-\lambda u}$.

$$\int_0^{+\infty} \left[\begin{array}{c} \lambda \max\left(p_t e^{-\lambda u} - (1-h(p_t))\, p_t, 0\right) \\ +\rho \max\left(v\left(p_t e^{-\lambda u}\right) - (1-h(p_t))\, p_t, 0\right) \end{array} \right] e^{-(\lambda+\rho)u} du - p_t \cdot h(p_t)$$
$$+ \int_0^{+\infty} (\lambda+\rho)\, e^{-(\lambda+\rho)u} \left(\int_0^u \left(\delta p_t e^{-\lambda s} - \Delta h\left(p_t\right) p_t\right) ds \right) du$$

This sum can be expressed as

$$A + B + C - p_t \cdot h(p_t)$$

where

$$\begin{aligned} A &= p_t \int_0^{-\frac{1}{\lambda}\log(1-h(p_t))} \lambda \left(e^{-(2\lambda+\rho)u} - (1-h(p_t))\, e^{-(\lambda+\rho)u}\right) du \\ &= \lambda p_t \left[\frac{-\lambda}{(2\lambda+\rho)(\lambda+\rho)} \left(1 - (1-h(p_t))^{\frac{2\lambda+\rho}{\lambda}}\right) + \frac{h(p_t)}{\lambda+\rho} \right] \\ B &= \rho \int_0^{+\infty} \max\left(v\left(p_t e^{-\lambda u}\right) - (1-h(p_t))\, p_t, 0\right) e^{-(\lambda+\rho)u} du \\ C &= \frac{\delta p_t}{2\lambda+\rho} - p_t.h(p_t) \frac{\Delta}{\lambda+\rho} \end{aligned}$$

Thus, entering the carry trade is optimal under the belief that others will exit if and only if

$$\begin{aligned}
& \lambda p_t \left[\frac{-\lambda}{(2\lambda+\rho)(\lambda+\rho)} \left(1-(1-h(p_t))^{\frac{2\lambda+\rho}{\lambda}}\right) + \frac{h(p_t)}{\lambda+\rho} \right] \\
& +\rho \int_0^{+\infty} \max\left(v\left(p_t e^{-\lambda u}\right) - (1-h(p_t))\, p_t, 0\right) e^{-(\lambda+\rho)u} du \\
& +\frac{\delta p_t}{2\lambda+\rho} - p_t.h(p_t)\frac{\Delta}{\lambda+\rho} - p_t.h(p_t) \\
\geq \; & 0
\end{aligned}$$

Simplifying the above inequality with δ on the left hand side yields:

$$\delta \geq \overline{Z}\left(p_t\right)$$

where $\overline{Z}\left(.\right)$ is defined as

$$\begin{aligned}
\overline{Z}\left(p_t\right) \equiv \; & \frac{2\lambda+\rho}{\lambda+\rho}\Delta h(p_t) + \rho\frac{2\lambda+\rho}{(\lambda+\rho)} h(p_t) \\
& + \frac{\lambda^2}{\lambda+\rho}\left(1-(1-h(p_t))^{\frac{2\lambda+\rho}{\lambda}}\right) - \rho\Phi\left(p_t\right) \qquad (14)
\end{aligned}$$

and

$$\Phi\left(p_t\right) = (2\lambda+\rho) \int_0^{+\infty} \max\left(\frac{v(p_t e^{-\lambda u}) - (1-h(p_t))\, p_t}{p_t}, 0\right) e^{-(\lambda+\rho)u} du.$$

Condition 3 implies that $\Phi\left(.\right)$ is bounded and Lipschitz.

Now, consider the opposite case. A trader who expects that all other traders will *enter* the carry trade after date t finds it optimal to *exit* the carry trade when the expected profit to the carry trade is negative under the price path:

$$p_{t+u} = 1-\left(1-p_t\right) e^{-\lambda u}$$

For this price path, the expected profit to the carry trade is given by

$$\begin{aligned}
& \int_0^{+\infty} \left[\begin{array}{c} \lambda \max\left(1-\left(1-p_t\right)e^{-\lambda u} - p_t, -p_t.h(p_t)\right) \\ +\rho \max\left(v\left(1-\left(1-p_t\right)e^{-\lambda u}\right) - p_t, -p_t.h(p_t)\right) \end{array} \right] e^{-(\lambda+\rho)u} du \\
& + \int_0^{+\infty} (\lambda+\rho)\, e^{-(\lambda+\rho)u} \left(\int_0^u \left(\delta\left(1-\left(1-p_t\right)e^{-\lambda s}\right) - \Delta h\left(p_t\right) p_t\right) ds \right) du
\end{aligned}$$

We can write this payoff as the sum:

$$D+E+\rho\int_0^{+\infty}\max\left(v\left(1-\left(1-p_t\right)e^{-\lambda u}\right)-p_t,-p_t\cdot h(p_{t)})\right)e^{-(\lambda+\rho)u}du$$

where

$$\begin{aligned}D &= \frac{\lambda^2}{\left(2\lambda+\rho\right)\left(\lambda+\rho\right)}\left(1-p_t\right)\\ E &= \delta\frac{\lambda+\left(\lambda+\rho\right)p_t}{\left(\lambda+\rho\right)\left(2\lambda+\rho\right)}-p_t\cdot h(p_t)\frac{\Delta}{\lambda+\rho},\end{aligned}$$

From the condition that the expected profit is negative, we can obtain the inequality:

$$\delta\leq\underline{Z}\left(p_t\right)$$

where

$$\underline{Z}\left(p_t\right)=\frac{2\lambda+\rho}{\lambda+\left(\lambda+\rho\right)p_t}\Delta h(p_t)-\frac{\lambda^2}{\lambda+\left(\lambda+\rho\right)p_t}\left(1-p_t\right)-\rho\Psi\left(p_t\right)\tag{15}$$

and

$$\Psi\left(p_t\right)=\frac{\left(2\lambda+\rho\right)\left(\lambda+\rho\right)}{\lambda+\left(\lambda+\rho\right)p_t}\int_0^{+\infty}\max\left(v\left(1-\left(1-p_t\right)e^{-\lambda u}\right)-p_t,-p_t.h(p_{t)})\right)e^{-(\lambda+\rho)u}du.$$

2. Let C be a compact subset of $(\Delta h(1),+\infty)$. From part 1 of Proposition 4, it suffices to establish that for λ sufficiently large and ρ sufficiently small, whenever $(\delta,p)\in C\times(0,1)$, we have

$$\underline{Z}\left(p\right)<\delta<\overline{Z}\left(p\right).\tag{16}$$

To establish (16), note first that

$$\lim_{\rho\to0}\overline{Z}\left(p\right)=2\Delta h(p)+2\lambda h(p)\left(1-\frac{h(p)}{2}\right)\tag{17}$$

and that convergence is uniform over $(0,1)$. Thus, for λ sufficiently large and ρ sufficiently small,

$$\forall\delta\in C,\delta<2\lambda h(1)\left(1-\frac{h(1)}{2}\right)<\min_{p\in(0,1)}\overline{Z}\left(p\right)$$

Similarly,

$$\lim_{\rho\to 0}\underline{Z}\left(p\right)=\frac{2}{1+p}\Delta h(p)-\lambda\frac{1-p}{1+p}$$

and convergence is uniform over $(0,1)$. Thus, for λ sufficiently large and ρ sufficiently small,

$$\forall\delta\in C,\delta>\Delta h(1)\geq\max_{p\in(0,1)}\underline{Z}\left(p\right), \tag{18}$$

So, from (17) and (18), we have

$$\max_{p\in(0,1)}\underline{Z}\left(p\right)<\delta<\min_{p\in(0,1)}\overline{Z}\left(p\right)$$

which establishes part 2 of Proposition 4.■

Proof of Lemma 6

Note that

$$\forall\left(t,u\right)\in\mathbb{R}_{+}^{2},E_{t}\left(\delta_{t+u}\right)=\delta_{t}.$$

Thus, a trader who expects at date t that all other traders will unwind their carry trades after date t finds it optimal to enter the carry trade if and only if

$$\delta_{t}\geq\overline{Z}\left(p_{t}\right),$$

where $\overline{Z}(.)$, defined in (14), is clearly Lipschitz. In other words,

$$Z_{0}=\overline{Z}.$$

We need to show that $\overline{Z}$ is strictly decreasing for θ sufficiently large. To see this, note that terms (1), (2), and (3) defined in (19) below are strictly decreasing. This is obvious for (1) and (2). For (3), condition 3 implies that $\Phi\left(.\right)$ is Lipschitz. That $\Phi\left(.\right)$ is Lipschitz implies that for θ sufficiently large, $\frac{1}{\theta}\Phi\left(p_{t}\right)$ has a derivative that is arbitrarily small, while $\frac{\theta}{\theta+1}\left(1-\left(1-h(p_{t})\right)^{\frac{2\theta+1}{\theta}}\right)$

is decreasing with a derivative bounded away from 0. As a result, $\overline{Z}\left(.\right)$ is strictly decreasing for θ sufficiently large.

$$\overline{Z}\left(p_t\right) = \underbrace{\frac{2\theta+1}{\theta+1}\Delta h(p_t)}_{(1)} + \lambda \left(\begin{array}{c} \underbrace{\frac{2\theta+1}{\theta\left(\theta+1\right)}h(p_t)}_{(2)}+ \\ \underbrace{\frac{\theta}{\theta+1}\left(1-\left(1-h(p_t)\right)^{\frac{2\theta+1}{\theta}}\right)-\frac{1}{\theta}\Phi\left(p_t\right)}_{(3)} \end{array} \right). \tag{19}$$

■

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